THE MATHEMATICS OF LOTTERY

Odds, Combinations, Systems

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Introduction

Lottery is by far the oldest and the most widely known game of chance, having been practiced since antiquity. In its various forms, the lottery preserves a basic structure and technical procedure that makes it the easiest and most popular game of chance: the random draw from an urn of some objects (balls, tickets, lots, plates, slips, etc.) containing predefined symbols (numbers, images, words, etc.), followed by the distribution of prizes for players who made correct predictions regarding this draw, according to some pre-established rules. Particularizing this definition, we find forms of lottery even in the simple procedures of drawing lots or organizing tombolas.

Nowadays, the most prevalent form of lottery is that with randomly selected numbers; winning categories are based on the number of numbers correctly predicted on the playing ticket. The most popular forms of these games are the national and state lotteries.

The early history of the lottery can be traced back to the second millennium B.C. In the Chinese *The Book of Songs* there is a reference to a game of chance known as "the drawing of wood", which in context appears to describe the drawing of lots. The first signs of a lottery trace back the Han Dynasty between 205 and 187 B.C., where ancient Keno slips (a form of lottery also practiced nowadays) were discovered. The first known European lottery occurred during the Roman Empire. The earliest record of a public lottery offering tickets for sale is the lottery organized by the Roman Emperor Augustus Caesar. Proceeds went for repairs to the city of Rome, and the winners were given prizes in the form of valuable articles.

The first recorded lotteries to offer tickets for sale with prizes in the form of money were held in the Low Countries during the period 1443–1449, and their funds were used for town fortifications. In the 17th century, it was quite common in the Netherlands to organize lotteries to collect money for the poor; the prizes were paintings. The Dutch were the first to shift the lottery to solely monetary prizes and to base prizes on odds (roughly about 1 in 4 tickets winning a prize). Even the English word *lottery* stems from the Dutch word *loterij*, which is derived from the Dutch noun *lot* meaning *fate*.

The first known lottery in France was created by King Francis I in or around 1505. After that first attempt, lotteries were forbidden for two centuries. They reappeared at the end of the 17th century, as a "public lottery" for the Paris municipality. Lotteries then became one of the most important resources for religious congregations in the 18th century.

In England, the first recorded official lottery was chartered by Queen Elizabeth I and drawn in 1569. This lottery was designed to raise money for public reparations within the Kingdom. The English State Lottery ran from 1694 until 1826. Many private lotteries were held as well.

Lotteries in colonial America played a significant part in the financing of both private and public ventures. It has been recorded that more than two hundred lotteries were sanctioned between 1744 and 1776, contributing heavily toward financing roads, libraries, churches, colleges, canals, bridges, and other public and private endeavors. In the 1740s, Princeton and Columbia Universities had their beginnings financed by lotteries. Toward the end of the 19th century, a large majority of state constitutions banned lotteries; however, many have reappeared and developed since 1964.

The popularity of this game comes not only from its history, but also from other technical and psychological elements related to its rules and progress.

One important reason for the game's popularity is its transparency. All of its components are visible: the urn from which the balls are drawn, the shuffling device, the numbers on the played ticket; there are no opponents to read their intentions, no dealer managing the game, and no strategies to influence the course of the game. Participants just choose the numbers to play, buy the ticket, and wait at home for the draw (which usually is broadcast in the media), this being a commodity that isn't seen in casino games.

The multitude of variations of the game at local, national, and international levels, due to various sets of rules, allows the players to choose a lottery matrix by either objective or subjective criteria. The complete freedom of choice manifests in the options to choose, combine, and play any number of simple or compound lines.

Another feature which makes the lottery popular is its consistency. The allocation of pre-established fixed percentages from the sales to the prizes represents a guarantee and a stability element that differentiates lottery from other games of chance in which the possible winnings depend exclusively on the played (betted) stakes.

The regularity of the game (usually weekly in state and national lotteries) represents another element of stability for the lottery companies, as well for the players, who are forced to respect that interval, with the effect of reducing the risk of addiction. Studies show that this unwanted phenomenon has the lowest rate among lottery players. Of course, the rate of addictive behavior might change with the expansion of private and online lotteries, which offer additional opportunities for playing the lottery.

But the most important element contributing to the public's fascination with lottery games is the amount of the prizes, especially for the highest winning category. The possibility (physically real, mathematically too improbable) of getting "the big hit" – winning the big prize – provides a motivation with complex psychological roots that often overlooks the practical aspects, such as the investments in lottery tickets and the mathematical aspects of the game, especially the winning probabilities.

We face here a contradiction in the behavior of most lottery players. While lottery offers the highest amounts of winnings, it also offers the lowest winning probabilities. Players take into account those high amounts of winning, but not those low winning probabilities, and paradoxically, they persevere instead of quitting. And this happens with both categories of players: those who don't know the basic mathematics of lottery matrices and experience a long series of games without success, and those who do know the mathematical probabilities of the game and have the same experience of consecutive failures.

It has been proven mathematically that in ideal conditions of randomness, no long-term regular winning is possible for players of games of chance; therefore, gambling is not a good way to make a living. Most gamblers accept this premise, but still work on strategies in hopes of multiple wins over the long run.

In the case of lottery, most players are interested in the isolated winnings, because their experience and/or mathematical knowledge strengthens their intuitive belief that in the long-run, the game cannot bring them regular positive results. Rather, a single win can vanquish the previous cumulative losses, and moreover, can make them rich overnight, when, theoretically, they will stop playing. The permanent hunt for this “hit” provides the primary motivation for perseverance in the lottery game.

By its simplicity, the lottery is not predisposed to objective strategies. The only game strategy is that of choosing a lottery matrix, the financial management of the player, and choosing the numbers and the combinations of numbers to play, taking into account various parameters, among which there is also risk. Moreover, any strategy must include the personal criteria of the player (time terms, amounts at disposal, the accepted level of risk, etc.), which automatically turns it into a subjective one. Regardless of the chosen options and playing strategies, the players will be always interested in the amount of that risk, and this means *probability*.

This is not a lottery strategy book, because an optimum strategy does not exist. Only the choice of the lottery matrices and the combinations to play exist. What does exist, however, is a collection of numerical probabilities and mathematical parameters specific to each lottery matrix, and those are generated by the mathematical model of these games of chance.

The mathematical approach of this book is mainly oriented toward generalities; thus, all formulas were obtained by using a group of variables covering all possible lottery matrices.

A large number of numerical results returned by these formulas have been listed in tables and cover the most popular lottery matrices in the world. The examples refer most frequently to the 6/49 lottery, which is the most widespread matrix.

The mathematical and probability models of lottery provide information that players should know before they launch into a long-run play. This information, along with its mechanism and the possibilities it offers, is part of the entire ensemble of the lottery game. On the basis of these data, decisions can be made, decisions that might also change the customary play.

The structure and content of the major chapters follow.

The Rules of Lottery

This chapter gives readers the definition of the lottery matrices and the entire ensemble of their rules. Their variations, structure, technical progress of the game, winning categories, and prizes are presented.

Supporting Mathematics

Here, the processes of lottery drawings and choosing the combinations to play are converted into probability experiments that generate aleatory events. We will see the same space, the field of events, and the probability space in which the numerical probabilities of lottery are worked out. We also present as theoretical support the probability properties and formulas used; these appear as statements without mathematical proofs.

We will confront the whole mathematical model of the lottery matrices and see what the parameters are of such constructs. Within this model, we pay special attention to the combinatorics applied to these various matrices.

Anyone with a minimal mathematical background can follow this chapter because it requires only basic arithmetic, algebraic and combinatorial skills, and the basics of set theory and probability theory. Alternatively , readers who are interested only in direct results can skip this chapter and go to the tables of results which follow.

Number Combinations

In this chapter, readers can become familiar with the entire combinatorics applied in lottery. It is a chapter with applications regarding counting the combinations specific to a draw and to the act of choosing the lines to play, as well as unfolding the combinations from a given set of numbers.

Probabilities of Winning with Simple Lines

Here lottery players will face the real mathematical odds involved in playing simple lines. We present not only the probabilities specific to each winning category, but also the cumulated probabilities for several winning categories, as well as intermediary probabilities corresponding to the various moments in the draw.

We have deduced and presented the general formulas of these probabilities, applicable to any lottery matrix. We have given to the variables of these formulas values from a large enough range to generate tables of values of the probabilities specific to the known lottery matrices.

Enhancing the Winning Probability

This chapter explains the phenomenon of increasing the mathematical odds of winning by increasing the number of played combinations and the use of playing systems. Since the number of the played lines is limited not only by the investment available, but also by the profitability of the success, we presented numerical results that are obtained through the correlation of several parameters.

Compound Lines

Here we define and explain the compound lines and the probabilities involved in a play with such lines. The numbers of possible multiple winnings are also presented.

Bridgehead Systems

Here are presented in general terms the bridgehead systems, along with all their mathematical parameters.

Reduced Systems

Here we define and explain the reduced systems, their structure, and the way of obtaining them, as well as their mathematical parameters.

Probabilities of Repeated Events

In this chapter, we discuss and calculate probabilities of some events containing several drawings belonging to different games.

The Strategy of Choosing

This chapter discusses the options of choosing numbers, lines, systems, and lottery matrices from a strategic point of view.

The Rules of Lottery

Generally, a lotto game consists of the random draw from an urn of objects holding predefined symbols, followed by establishing the prizes for the players who made correct predictions regarding this draw, according to some predefined rules. The objects to be extracted from the urn can be balls, tickets, lots, plates, etc., and the symbols inscribed on these objects can be numbers, icons, images, letters, words, etc. The technical progress of the game assumes the existence of a shuffling and drawing procedure for the objects in the urn (can be manual or performed by a mechanical or electronic device), the display of the sequence of the drawn objects (called *the draw* from now on), the registration of the predictions before the drawing, and their comparison with the outcomes.

This general definition places the lottery game in the category of games of chance, because the prediction of the player with respect to the draw is, in practicality, equivalent to placing a bet whose stake is the price paid for participation in the game (the player's cost for the lottery tickets). Particularizing this definition, we find primary forms of lottery such as the drawing of a certain marked object from a bag (the equivalent of drawing lots) or organizing a tombola with objects as prizes. In these particular cases, the players are those who perform the draw, and their bet is on drawing tickets holding the various winning objects with their own hands. (In fact, the prediction is no longer made before the drawing, but simultaneously with it).

The most popular form of lottery – the one we deal with in this book – is that which uses numbers as the symbols inscribed on the drawn objects (henceforth called *number lottery*) and in which the rules for giving the prizes are established based upon the quantity of correct numbers predicted by the player that occur in the random draw (the winning numbers on the playing ticket).

The chronological process of the number lottery game is as follows:

1. Players buy tickets before the draw, on which they note their predicted numbers.

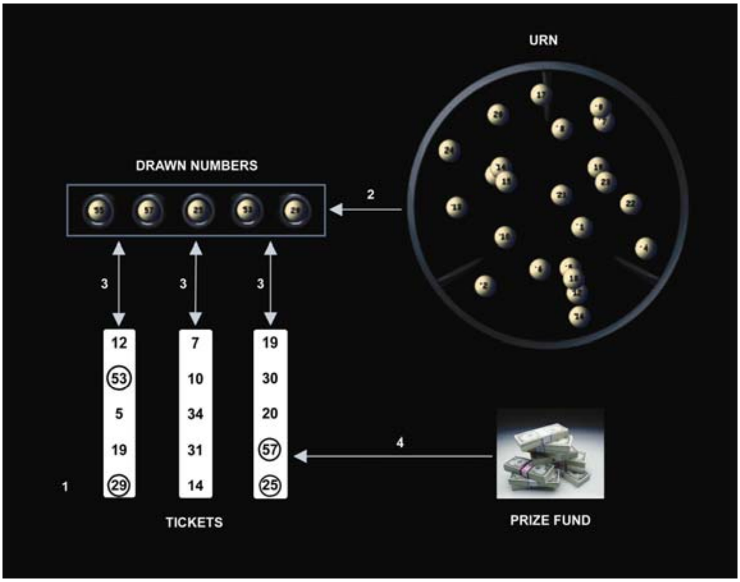
2. At the established time, the drawing of the winning numbers is performed.

3. Both players and the lottery company compare the drawn numbers with the numbers on the playing tickets.

4. If a ticket holds winning numbers according to that lottery's rules, it is declared a winner, correlated with a certain winning category, and awarded a prize according to that specific awarding system .

There are also variations of lottery in which players buy pre-filled tickets, whose numbers are not visible but must be uncovered or scratched off, and the draw is fixed from the beginning, its winning numbers being displayed. This type of lottery (running mostly in the private sector) assumes the distribution of the winning tickets in a certain pre-established proportion, which is usually made public.

The progress of the game corresponding to the most popular number lotteries is illustrated in the figure below:



In some variations of lottery, the main draw is combined with a second draw (generally of a single number, as in *Powerball*), and the rules of assigning the winning categories take into account winning numbers from the main draw, as well as from both draws (for specific winning categories).

Another variation of lottery is based on the winning numbers showing on the playing ticket as the result of a draw that runs until a winner comes up. *Keno* and *Bingo* are examples of this special form of lottery. In such games, the tickets or cards have the form of a matrix of chosen numbers which are checked progressively as they come up in the draw. The player whose numbers form a particular configuration on the playing ticket (row, column, square, etc.) is announced the winner. Such games are not the subject of study in this book, although some of the obtained results will apply partially in these games as well.

With respect to the management and juridical status of a lottery, there are private and public, local, national, and international, physical, and online lotteries. The most common are the state and national number lotteries, offered and managed by all states whose regulations allows this type of game of chance.

Each lottery has its own rules for establishing the prize fund, its distribution in winning categories, and establishing prizes for the winners. The prize fund is usually calculated as a fixed percentage of the ticket sales. Its distribution in winning categories, as well as establishing the amounts to be awarded are usually determined by formulas specific to each lottery. In most cases, these distributions are still made by fixed percentages, but there are also lotteries offering fixed amounts as individual prizes.

The lottery offers unlimited possibilities of choosing the played lines, in combinations of numbers or combinations of lines. Depending on each lottery, players fill in the playing tickets either by checking the chosen numbers or writing them down in the specific fields of the ticket. For most lotteries, the design of the ticket allows combined play with several simple lines, compound lines, combinations of them, or encoded playing systems, on the same ticket.

Each lottery is defined by certain basic numerical parameters that represent the set of rules specific to that lottery, with respect to the progress of the game and the awarding system as well. These parameters are as follows:

- the total number of the numbers from the urn, denoted by m;
- the number of numbers in a draw, denoted by n;
- the number of numbers on a simple playing ticket (or the number of numbers of a simple line), denoted by p;

- the number of winning categories, denoted by q;
- the minimum number of winning numbers for each winning category;
- the price of a simple ticket (simple line);
- the percentage from sales for the prize fund;
- the percentages in which the prize fund is distributed to each winning category.

We will come back to these parameters in the chapter dedicated to the applied mathematics, where we will also see the initial conditions these parameters must fit.

The possibilities of choice within these parameters are practically unlimited, this also being the explanation for the existence of so many lotteries worldwide. For most of them, m takes values in the range 20–90; n takes values in the range 3–20; p in the range 3–20; q in the range 1–5. For the rest of the parameters, the range of their values varies locally and is too large to identify the most frequent values.

Any numerical choice of the parameters enumerated above represents a *lottery matrix*. The most important parameters for a basic classification of the lotteries that exist worldwide are m, n, p, and q, so we can identify any lottery matrix with a numerical instance of the quadruple (m, n, p, q).

For example, the lottery matrix (49, 6, 6, 3) represents the 6/49 lottery (a draw has 6 numbers from a total of 49), with a simple line consisting of 6 numbers and 3 winning categories.

In common speech, n/m represents a certain lottery matrix. The lottery matrices 6/49, 6/42, 6/52, 5/40, 5/31, 4/77 and so on, are considered individual lottery matrices, although within the same matrix any variation of the rest of the parameters besides m and n in fact generate a new lottery matrix. Locally there are so many variations of the same n/m matrix that the abbreviated denotation is entirely justified.

The most common lottery matrices worldwide are the following: 6/49, 6/44, 6/48, 6/51, 6/52, 6/54, 6/40, 6/53, 5/37, 5/55, 5/32, 5/56, 5/39, 5/36, 4/77, 4/35, 7/47 (Europe, United States, Canada, Mexico); 6/36, 6/39 (Ireland); 6/90 (Italy); 6/45 (Australia, Philippines, Austria, Switzerland, Germany); 6/42 (Belgium, Bulgaria); 6/41 (Holland); and 6/47 (Hong Kong).

Supporting Mathematics

The application of mathematics, especially probability theory in gambling, is a simple process because a finite sample space can be attached to any game of chance. In some games, probability calculations for some events can become harder because of their structure, but applying the theory is quite normal and straightforward everywhere in this field.

The finite sample space and the randomness of the events (whether rolling dices, drawing cards, or spinning a wheel) allow us to build a simple probability model to work within to find the numerical probabilities of the events involved in that game.

This model assumes a finite probability space in which the field of events is the set of parts of the sample space (and, implicitly, is finite), and the probability-function is given by the classical definition of probability. In this probability space, any event, no matter how complex, can be broken down into elementary events. Therefore, finding the probability of a compound event means applying some properties of probability and doing some algebraic calculations.

In lottery, anyone with a minimal mathematical background can perform its probability applications and calculations. All the main calculations involve only arithmetic and basic algebraic operations, but at some point, some problems, especially those involving unions of events and probability distributions, require math skill. For those interested in improving their probability calculus skills and figuring out correct probability results for any game of chance, we recommend the beginner's guide, *Understanding and Calculating the Odds*, which also contains a number of gambling applications.

Let us see now how probability theory and combinatorics can be applied in lottery and how the numerical probability results from this book were obtained.

Probability space

As in every game of chance, we are interested in making predictions for the events regarding the outcomes of lottery, the draws. In lottery, there are no opponents or a dealer in the game, so the only events to deal with are the outcomes of the machine that performs the drawing. These events can be described as the occurrences of certain numbers or groups of numbers (combinations) having a specific property (for example, those containing certain given numbers or numbers with a specific property).

Every drawing is an experiment generating an outcome: a combination of *n* different numbers from the *m* numbers in play (see the denotations of the parameters in the previous chapter).

The set of these possible combinations is the *sample space* attached to this experiment.

The sample space is the set of all elementary events (i.e., events than cannot be broken down into units of other non-empty events). Normally, an elementary event would be any number combination that could occur as the result of drawing. Thus, an elementary event is any number combination $(x_1, x_2, \ldots, x_n)$ that is possible to be drawn.

This choice is convenient because it allows us to make the following idealization: *the occurrences of all elementary events are equally possible*. In our case, the occurrence of any number combination is equally possible (if we assume a random drawing and nonfraudulent conditions). Without this *equally possible* idealization, the construction of a probability model within which to work is not possible.

We have established the elementary events and the sample space attached to a drawing as being the set of all possible elementary events (let us denote it by Ω).

This set has C_m^n elements (all combinations of *m* numbers taken *n* at a time) and these numbers are generally very large, so the elements of the set cannot realistically be enumerated. Moreover, this enumeration would not have any practical purpose from mathematical point of view.

As an example, for a 6/49 lottery, Ω has 13,983,816 elements.

For 6/42 lottery, Ω has 5,245,786 elements.

For 5/40 lottery, Ω has 658,008 elements.

For 3/90 lottery, Ω has 117,480 elements.

We consider the field of events as being the set of parts of the sample space, so this set is also finite. As a set of parts of a set, the field of events is a Boole algebra. Any event belonging to the field of events, no matter how complex, can be broken down into units of elementary events, by using the axioms of Boole algebra. Because the events are identified with sets of numbers and the axioms of a Boole algebra, the operations between events (union, intersection, complementary) revert to the operations between sets.

Therefore, any counting of elementary events (for example, the elementary events comprising a compound event) reverts to counting number combinations. Thus, the combinatorial calculus becomes an essential tool for the probability calculus applied in lottery.

Examples of events:

In the 6/49 lottery, the event *occurrence of a combination containing numbers 3, 5, 7, 11, 15* is the set of elementary events $A = \{(3, 5, 7, 11, 15, 1), (3, 5, 7, 11, 15, 2), (3, 5, 7, 11, 15, 4), \ldots\}$, or $A = \{(3, 5, 7, 11, 15, x), x \in \{1, 2, \ldots, 49\} - \{3, 5, 7, 11, 15\}\}$.
This set has 49 – 5 = 44 elements (6-number combinations).

The event *occurrence of a combination containing only numbers between 21 and 29* is the set of elementary events $B = \{(21, 22, 23, 24, 25, 26), (21, 22, 23, 24, 25, 27), \ldots\}$ or $B = \{(x, y, z, t, u, v), x, y, z, t, u, v \in \{21, 22, \ldots, 29\}\}$ (with x, y, z, t, u, v mutually distinct). This set has $C_9^6 = 84$ elements.

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Parameters of the lottery matrices

To each lottery we can attach a set of basic numerical parameters through which that lottery is uniquely identified. These parameters are as follows:

- the total number of the numbers from the urn, further denoted by m;
- the number of numbers of a draw, further denoted by n;
- the number of numbers on a simple playing ticket (or the number of numbers of a simple line), further denoted by p;
- the number of winning categories, further denoted by q;
- the minimum number of winning numbers for each winning category, further denoted by $n_1, n_2, \ldots, n_q$ (in decreasing order of the amount of the prize);
- the price of a simple ticket (simple line), further denoted by c;
- the percentage from sales for the prize fund, further denoted by f;
- the percentages in which the prize fund is distributed to each winning category, further denoted by $f_1, f_2, \ldots, f_q$.

Any numerical instance of the vector $(m, n, p, q, n_1, n_2, \ldots, n_q, c, f, f_1, f_2, \ldots, f_q)$ will be called a *lottery matrix*.

The parameters that determine the technical progress of the game in a certain lottery matrix and which are taken into account in the construction of the general probability formulas are $(m, n, p, q, n_1, n_2, \ldots, n_q)$.

The initial conditions these parameters must fit for the represented lottery matrix to be mathematically valid are:

$m, n, p, q, n_1, n_2, \ldots, n_q \in \mathbb{N}$

$1 \leq p \leq n < m$

$q \geq 1$; $n_1 > n_2 > \cdots > n_q$

(The last inequality holds for the lottery matrices in which the winning criteria are established for one single draw, non-combined with another draw of one or more additional numbers, as in Powerball.)

The other parameters are financial and must fit the following conditions:

$$c, f, \ f_1, f_2, \ldots, f_q \in \mathbb{R}_+$$

$$0 < f, f_1, f_2, \ldots, f_q \leq 1$$

$$f_1 + f_2 + \cdots + f_q = 1$$

The mathematical model of the number lottery is the entire ensemble of the above parameters that define a lottery matrix, along with the probability spaces attached to the experiments of the lottery drawing (described in a previous section), in which the structural units are the number combinations.

The goal of this chapter was to present the theoretical results and formulas used in our applications. Their detailed application is presented in the sections in which they occur. For readers who want to delve deeper into probability theory and its applications, we recommend other papers on this subject, starting with introductory and popularization books.

Number Combinations

The number combinations are the structural units of the lotto game, and their unfolding and generation represent the "engine" of every number lottery, for both the technical process of the game and placing the bets (creating the playing tickets) by the players.

The number combinations show up within the draws, being randomly generated by the shuffling and drawing device, as well as in lines played on the playing tickets, these latter being chosen by players from all possible allowed combinations.

The sizes of the number combinations involved in the lotto game, in intermediary moments of its process or as the final result of the drawing, vary from 1 up to the total number of numbers of a draw (previously denoted by *n*).

A drawing is complete when all *n* numbers of a draw have been extracted from all *m* that the urn contains. This draw represents an *n*-size combination of numbers, and the set of combinations possible to be drawn is the set of all combinations of the *m* numbers taken each *n*, which contains C_m^n elements.

By using formula (F10) we can calculate the number of all combinations possible to be drawn for any lottery matrix. In the table below are noted these numbers for a few of the most popular lottery matrices:

m* \ *n	**4**	**5**	**6**	**7**
32	35960	201376	906192	3365856
40	91390	658008	3838380	18643560
44	135751	1086008	7059052	38320568
45	148995	1221759	8145060	45379620
48	194580	1712304	12271512	73629072
49	211876	1906884	13983816	85900584
55	341055	3478761	28989675	202927725
90	2555190	43949268	622614630	7471375560

Observe that the number of all possible combinations for the entire draw increases with both *m* and *n*, when the other variable is fixed. For the 6/49 lottery matrix, this number is 13,983,816; for 6/44 matrix it is 7,059,052; and for 5/55 it is 3,478,761.

Using formula (F10), we can also calculate numbers of possible combinations for intermediary moments of a drawing, for example for the first drawn numbers.

If we choose $n' < n$, the number of possible combinations for the first n' numbers of the draw is $C_m^{n'}$.

Examples:

In the 6/49 matrix, the number of possible combinations for the first 3 drawn numbers is $C_{49}^3 = \dfrac{47 \cdot 48 \cdot 49}{2 \cdot 3} = 18424$.

In the 5/55 matrix, the number of possible combinations for the first 4 drawn numbers is $C_{55}^4 = \dfrac{52 \cdot 53 \cdot 54 \cdot 55}{2 \cdot 3 \cdot 4} = 341055$.

We can also calculate numbers of possible combinations for the last numbers of the draw, at specific moments of the drawing. In this case, we shall take into account the number of the numbers already drawn.

Still staying with n' ($n' < n$) denoting the number of the numbers drawn at a certain moment, the number of possible combinations for the rest of the numbers following to be drawn is

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The Probabilities of Winning with Simple Lines

General formula of the winning probability

A playing ticket contains one or more simple lines, grouped in compound lines or particular systems. The ticket is declared a winner if at least one of its simple lines is winning, that is, among its p numbers there exist a certain number of winning numbers ($n_1, n_2, \ldots, n_q$), according to the rules of the winning categories of that lottery matrix. Thus, given a simple line, we are interested first in what the probability of winning is with this line in one of the winning categories of that matrix. Further, we deduce a more general formula, namely of the probability that among the n drawn numbers, there is a certain number of numbers of the played line.

As readers will notice, we preserved all the denotations for the parameters of a lottery matrix mentioned in the section titled *Parameters of the lottery matrices*.

Consider a played simple line. Let $w \leq p$ and let A_w be the event "exactly w numbers from the p given numbers are drawn." We deduce now the general formula of the probability $P(A_w)$.

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$n = 3$

w / *m*	0	1	2	3
21	0.613534	0.345113	0.040602	0.000752
22	0.629221	0.333117	0.037013	0.000649
23	0.643704	0.321852	0.033879	0.000565
24	0.657115	0.311265	0.031126	0.000494
25	0.669565	0.301304	0.028696	0.000435
26	0.681154	0.291923	0.026538	0.000385
27	0.691966	0.283077	0.024615	0.000342
28	0.702076	0.274725	0.022894	0.000305
29	0.711549	0.266831	0.021346	0.000274
30	0.720443	0.25936	0.019951	0.000246
31	0.72881	0.25228	0.018687	0.000222
32	0.736694	0.245565	0.01754	0.000202
33	0.744135	0.239186	0.016496	0.000183
34	0.75117	0.233122	0.015541	0.000167
35	0.75783	0.227349	0.014668	0.000153
36	0.764146	0.221849	0.013866	0.00014
37	0.770142	0.216602	0.013127	0.000129
38	0.775842	0.211593	0.012447	0.000119
39	0.781267	0.206806	0.011817	0.000109
40	0.786437	0.202227	0.011235	0.000101
41	0.79137	0.197842	0.010694	9.38E-05
42	0.79608	0.193641	0.010192	8.71E-05
43	0.800583	0.189612	0.009724	8.1E-05
44	0.804893	0.185744	0.009287	7.55E-05
45	0.80902	0.18203	0.008879	7.05E-05
46	0.812978	0.178458	0.008498	6.59E-05
47	0.816775	0.175023	0.008141	6.17E-05
48	0.820421	0.171716	0.007805	5.78E-05
49	0.823925	0.16853	0.00749	5.43E-05
50	0.827296	0.165459	0.007194	5.1E-05
51	0.83054	0.162497	0.006915	4.8E-05
52	0.833665	0.159638	0.006652	4.52E-05
53	0.836677	0.156877	0.006403	4.27E-05
54	0.839582	0.154209	0.006168	4.03E-05
55	0.842386	0.15163	0.005946	3.81E-05

w / m	0	1	2	3
56	0.845094	0.149134	0.005736	3.61E-05
57	0.84771	0.146719	0.005537	3.42E-05
58	0.85024	0.14438	0.005347	3.24E-05
59	0.852687	0.142114	0.005168	3.08E-05
60	0.855056	0.139918	0.004997	2.92E-05
61	0.857349	0.137788	0.004835	2.78E-05
62	0.859572	0.135722	0.00468	2.64E-05
63	0.861726	0.133716	0.004533	2.52E-05
64	0.863815	0.131768	0.004392	2.4E-05
65	0.865842	0.129876	0.004258	2.29E-05
66	0.86781	0.128038	0.00413	2.19E-05
67	0.869721	0.12625	0.004008	2.09E-05
68	0.871578	0.124511	0.003891	2E-05
69	0.873382	0.122819	0.003779	1.91E-05
70	0.875137	0.121173	0.003672	1.83E-05
71	0.876844	0.11957	0.003569	1.75E-05
72	0.878504	0.118008	0.003471	1.68E-05
73	0.880121	0.116487	0.003376	1.61E-05
74	0.881695	0.115004	0.003286	1.54E-05
75	0.883228	0.113558	0.003199	1.48E-05
76	0.884723	0.112148	0.003115	1.42E-05
77	0.886179	0.110772	0.003035	1.37E-05
78	0.887599	0.10943	0.002958	1.31E-05
79	0.888984	0.10812	0.002883	1.26E-05
80	0.890336	0.10684	0.002812	1.22E-05
81	0.891655	0.105591	0.002743	1.17E-05
82	0.892943	0.10437	0.002676	1.13E-05
83	0.8942	0.103177	0.002612	1.09E-05
84	0.895428	0.102011	0.00255	1.05E-05
85	0.896629	0.100871	0.002491	1.01E-05
86	0.897801	0.099756	0.002433	9.77E-06
87	0.898948	0.098665	0.002377	9.43E-06
88	0.900069	0.097598	0.002324	9.11E-06
89	0.901166	0.096553	0.002272	8.81E-06
90	0.902239	0.095531	0.002222	8.51E-06

$n = 4$

w / m	0	1	2	3	4
21	0.397661	0.45447	0.136341	0.011362	0.000167
22	0.418319	0.446206	0.125496	0.009843	0.000137
23	0.437719	0.437719	0.115867	0.008583	0.000113
24	0.455957	0.429136	0.107284	0.007529	9.41E-05
25	0.473123	0.420553	0.099605	0.00664	7.91E-05
26	0.489298	0.41204	0.092709	0.005886	6.69E-05
27	0.504558	0.403647	0.086496	0.005242	5.7E-05
28	0.518974	0.395409	0.080879	0.004689	4.88E-05
29	0.532609	0.387352	0.075786	0.00421	4.21E-05
30	0.545521	0.379493	0.071155	0.003795	3.65E-05
31	0.557763	0.371842	0.066932	0.003432	3.18E-05
32	0.569383	0.364405	0.06307	0.003115	2.78E-05
33	0.580425	0.357185	0.059531	0.002835	2.44E-05
34	0.590931	0.350181	0.056279	0.002588	2.16E-05
35	0.600936	0.343392	0.053285	0.002368	1.91E-05
36	0.610474	0.336814	0.050522	0.002173	1.7E-05
37	0.619578	0.330441	0.047967	0.001999	1.51E-05
38	0.628273	0.32427	0.0456	0.001842	1.35E-05
39	0.636588	0.318294	0.043404	0.001702	1.22E-05
40	0.644545	0.312507	0.041361	0.001576	1.09E-05
41	0.652167	0.306902	0.039459	0.001461	9.87E-06
42	0.659475	0.301474	0.037684	0.001358	8.93E-06
43	0.666486	0.296216	0.036026	0.001264	8.1E-06
44	0.673218	0.291121	0.034475	0.001179	7.37E-06
45	0.679687	0.286184	0.033021	0.001101	6.71E-06
46	0.685909	0.281398	0.031657	0.00103	6.13E-06
47	0.691896	0.276758	0.030376	0.000964	5.61E-06
48	0.697662	0.272258	0.029171	0.000905	5.14E-06
49	0.703218	0.267893	0.028035	0.00085	4.72E-06
50	0.708576	0.263656	0.026965	0.000799	4.34E-06
51	0.713745	0.259544	0.025954	0.000752	4E-06
52	0.718737	0.255551	0.025	0.000709	3.69E-06
53	0.723558	0.251673	0.024096	0.000669	3.42E-06
54	0.728219	0.247904	0.023241	0.000632	3.16E-06
55	0.732726	0.244242	0.02243	0.000598	2.93E-06

.... missing part ....

Below we present a separate table for the Keno game, in which 20 numbers are drawn from 80, and the playing lines can contain between 4 and 20 numbers ($m = 80$, $n = 20$, $p \in \{4, 5, \ldots, 20\}$).

The formula of the winning probability becomes $P(A_w) = \dfrac{C_p^w \cdot C_{80-p}^{20-w}}{C_{80}^{20}}$, which returns the following numerical values.

In the table, in the first column are noted the values for w, and in the first row, the values for p. The existence of a dash indicates that such a case is impossible.

.... missing part ....

Examples of how to use the tables:

In the 6/49 matrix, what is the probability of occurrence of 2 specific numbers from the played 6 in a draw?

We search in the table corresponding to $n = 6$, at the intersection of row $m = 49$ with column $w = 2$ and we find the value 0.132378, that is 13.23%.

In a 3/90 matrix, what is the probability of occurrence of 3 specific played numbers in a draw?

We search in the table corresponding to $n = 3$, at the intersection of row $m = 90$ with column $w = 3$ and we find the value 8.51E-06, that is 0.00000851, and as percentage 0.000851%.

In a Keno game, what is the probability of occurrence of 7 specific numbers from 17 played numbers in a draw?

We search in the table corresponding to Keno, at the intersection of row $w = 7$ with column $p = 17$ and we find the value 0.057588, that is 5.75%.

A quick perusal of the numerical results in the tables in this section indicates that most of the probabilities are very low, some of them very close to zero, including those specific to the winning categories.

For example, in the 5/55 matrix, the occurrence of 3 numbers has the probability 0.35%, the occurrence of 4 numbers has 0.0719%, and of 5 numbers has 0.00287%. By contrast, the occurrence of 2 numbers has the probability 5.63%, the occurrence of one single number has 33.10%, while the occurrence of not having any winning number increases to over 60%!

This phenomenon can be observed in every lotto matrix. Following any row of any table, we see the probabilities showing a sudden decrease in the last columns. In the 6/49 matrix, this jump occurs from $w = 3$ up, the probability decreasing about 18 times from $w = 3$ to $w = 4$, 53 times from $w = 4$ to $w = 5$ and 257 times from $w = 5$ to $w = 6$. For all matrices, the zone of the winning categories (with respect to w) correlates closely with the zone of these probability jumps.

Let us also observe that probability does not decrease with w (by fixing the other parameters), so the maximal probabilities are not always obtained from the minimal values of w.

For example, in a Keno game with $p = 19$, the maximal probability is obtained when $w = 5$. In a 7/47 game, the maximal probability is obtained when $w = 1$, while in the 6/49, maximal probability is obtained when $w = 0$.

Cumulated winning probabilities

In the previous section, we presented the probabilities of occurrence of a fixed number of numbers from the played line, in the draw (events denoted by A_w). We saw that these probabilities are very low for superior values of w.

Within the same lottery matrix, a player will be also interested in the cumulated winning probabilities corresponding to the occurrence of a variable number of played numbers in the draw. This relates to the events of type "at least w numbers."

Although the mathematical information offered by the probabilities of these compound events is included in the results within the previous section, the new cumulated results provide us with an evaluation and decision criterion that is more practical.

Staying with the same denotations for the parameters, we want to evaluate the probability of occurrence of at least w numbers from the p of the played line, in the draw: $1, 2, \ldots, p - 1$ (the case "at least

$w = 0$" has the probability 1, and the case "at least p" is equivalent to "exactly p," which was covered in the previous section).

.... missing part ....

Enhancing the Winning Probability

In the previous chapter we evaluated the general winning probabilities corresponding to one single simple line played in any lottery matrix; now we deal with the probabilities involved in the play with several simple lines. We are especially interested in the probabilities specific to the winning categories, and we will study the increase of these probabilities with the number of the played lines.

For our purposes here, we will use the term *system* or *playing system* to indicate an independent group of simple lines played on one or more tickets, by same player. The systems can be of several types, depending on the method of choosing the simple lines and their structure:

- *aleatory system* – in which the played lines are chosen at random and not by a predefined set of rules;
- *compound line* – in which the constituent simple lines form an unfolded combination of several given numbers;
- *system of compound (complex) lines* – a group of several compound lines; the simple lines represent the result of unfolding all the constituent combinations;
- *multicriterial system* – a group of simple lines chosen by a predefined set of personal criteria;
- *reduced system* – a system obtained from unfolding a compound line, from which some combinations were removed on the basis of some predefined criteria.

We shall talk in detail about these types of systems in the sections devoted to each type. The play with any system has as its goal increasing the winning probability through the increase of the coverage of the combinations possible to be drawn, as well as the achievement of multiple prizes (in several winning categories simultaneously, in certain circumstances.

Because the lotto game assumes only two gaming events that can influence the final result – namely choosing the played lines or system, and the drawing of the winning numbers – the only way a player can increase his or her winning odds is by increasing the number of played simple lines.

Probabilities of winning with systems

Let us consider the example of the 6/49 matrix, in which a simple line has a 1/13983816 probability of winning in the first category (6 winning numbers). If we play two simple lines instead of one, thus creating a playing system, the winning probability in the first category will be doubled, becoming 2/13983816. If we play *s* simple lines ($s > 1$), that probability will increase *s* times. This happens because the events representing the occurrence in the draw of the number combinations corresponding to the played simple lines are elementary events, so they are mutually exclusive. Thus, the probability of the union of these events is the sum of the individual probabilities, which are equal. (See formula (F3).)

In this case, the probability of winning with that system grows proportionately with the number of played lines *s*. This condition holds true only for the first winning category and is true for any lottery matrix in which $n_1 = n$ (the number of winning numbers specific to the first winning category equals to the number of numbers of the draw).

But what happens with the winning probability of an lower category? Does this probability still grow proportionately with the number of played simple lines? The answers is: generally no, but in a large range of cases, yes. The same answer stands for the same question, in the case of a cumulated probability (at least a certain winning category).

Let us denote by $V_1, V_2, \dots, V_s$ the simple lines of a given system ($s > 1$). V_i are all *p*-size combinations.

For $i = 1, \dots, s$ and $1 \le k \le n$, denote by V_i^k the event "line V_i contains a minimum of *k* winning numbers."

In the previous chapter, we evaluated the cumulated winning probabilities for the simple lines, so we know the probabilities $P(V_i^k)$ for any *i* and *k*, whose values can be found in the tables of the respective section.

.... missing part ....

From these three examples, the last gives the biggest difference of probability with the case of exclusive events, which is still very low. As in the first example, the numerical results of the last two remain the same whatever other numbers we choose, through a cyclic permutation of the 49 possible numbers (though not necessarily consecutive) of that matrix.

We presented the complete solution of the three examples as both an exercise in probability calculus, and for seeing in a concrete case the evolution of the winning probability of a system when its constituent lines contain common numbers.

The number of common numbers held by the simple lines taken two at a time can provide us with a sufficient condition for the events V_i^k to be mutually exclusive, and implicitly, for the winning probability of the system to increase proportionately with the number of played lines.

One can see in the previous examples that even with only two lines, a direct calculation becomes difficult at a certain point for a person having no basic mathematical background. The direct application of formula (*) for aleatory systems with several lines is practically impossible and is not justified with respect to making decisions on choosing the simple lines, especially for the cases in which there exists the alternative of an approximation. Only a software program can perform such calculations in a short time.

.... missing part ....

This is the required sufficient condition, namely a maximal number c_{ij}: any two lines of the system will not contain more than $2k - n - 1$ common numbers. If this condition is fulfilled, then events V_i^k are mutually exclusive, and implicitly, all the intersections from the probability formula (*) are empty, so the probability of winning with minimum *k* numbers from at least one played line grows linearly with the number of played lines.

From now on, we call *the exclusiveness condition* of the lines of the system the condition that events V_i^k are mutually exclusive. Hence, we proved that a sufficient condition for the exclusiveness condition (for given k) is that the number of common numbers c_{ij} of any two lines does not exceed $2k - n - 1$.

Next we present a table with the values of the maximal number c_{ij} from the found sufficient condition, for the most frequently met values of n (the size of the draw) and k (the thresholds $n_1, n_2, ..., n$ of the winning categories).

k \ n	3	4	5	6	7
3	2	1	0	*E*	*E*
4	-	3	2	1	0
5	-	-	4	3	2
6	-	-	-	5	4
7	-	-	-	-	6

In this table, at the intersection of a column corresponding to a value of n with the row corresponding to a value of k, we find the maximal number of common numbers (c_{ij}) for any two lines of the played system. For any c_{ij} less than or equal to this maximum, the cumulated winning probability for minimum k winning numbers from the n drawn increases proportionately with the number of played lines, according to the found sufficient condition.

The existence of a line indicates that the respective case is impossible, and letter *E* indicates the impossibility of the condition of exclusiveness (the cases in which $k \leq \frac{n}{2}$).

Examples of how to use the table:

We want to build a playing system for the 6/49 lottery which will ensure the linearity of the winning probability for at least the third winning category (minimum 4 winning numbers). What is the maximal number of common numbers any two lines of the system may contain?

We follow the intersection of column $n = 6$ with row $k = 4$ and find the maximal value of c_{ij} to be 1. This means we will choose the lines of the system such that any two of them will not contain more than 1 common number.

Here is a simple example of a 11-line system in which this condition is fulfilled:

V_1:	1	2	3	4	5	6
V_2:	1	7	8	9	10	11
V_3:	12	7	13	14	15	16
V_4:	17	18	13	19	20	21
V_5:	22	23	24	19	25	26
V_6:	27	28	29	30	25	31
V_7:	32	33	34	35	36	1
V_8:	37	38	39	40	41	1
V_9:	37	42	43	44	45	2
V_{10}:	46	47	48	49	45	3
V_{11}:	12	18	24	30	36	4

Any two lines of the previous system contain no more than 1 common number; thus the exclusiveness condition is satisfied, and thus the system's winning probability, at least in the third category, grows linearly with the number of lines:

$$P\left(\bigcup_{i=1}^{11} V_i^4\right) = 11P(V_1^4) = 11 \cdot 0.000987 = 1.0857\%$$

We want to build a playing system for the 5/55 lottery which will ensure the linearity of the winning probability for at least the second winning category (minimum 4 winning numbers). What is the maximal number of common numbers any two lines of the system may contain? How about for the third winning category (minimum 3 winning numbers)?

We follow the intersection of column $n = 5$ with row $k = 4$ and find the maximal value of c_{ij} being 2. This means we will choose the lines of the system such that any two of them will not contain more than 2 common numbers.

Here is a simple example of a 10-line system in which this condition is fulfilled:

V_1:	1	3	5	7	9
V_2:	10	12	5	16	18
V_3:	19	21	23	25	26
V_4:	27	29	31	33	35
V_5:	36	38	40	42	44
V_6:	45	47	49	51	53
V_7:	54	51	50	49	48
V_8:	46	44	42	39	37
V_9:	36	35	34	33	32
V_{10}:	31	30	29	28	27

Any two lines of the previous system contain no more than 2 common numbers; thus the exclusiveness condition is satisfied, and thus the system's winning probability, at least at the third category, grows linearly with the number of lines:

$$P\left(\bigcup_{i=1}^{10} V_i^4\right) = 10P(V_1^4) = 10 \cdot 0.0000722 = 0.0722\%$$

For the very next lower winning category (minimum 3 winning numbers), we follow the intersection of column $n = 5$ with row $k = 3$ and find the maximal value of c_{ij} being 0. This means that any two variants should contain no common numbers, and the system must be built accordingly.

(In the calculations of both previous examples, we used numerical probability results from the section titled *Cumulated winning probabilities*.)

The exclusiveness condition ensures not only the linearity of the winning probability of the played system, but also a maximal probability for the same number of played lines. Therefore, whatever other lines we may choose in the same number as in the system created under the exclusiveness condition, the winning probability of the new system will be less than or equal to the probability of the initial system (for any winning category or cumulatively).

.... missing part ....

Thus, at first glance, obtaining a maximal winning probability through choosing a system of a given size, under the condition of exclusiveness, seems to prevail as a strategic criterion. Still, a system with the same number of lines, but containing more common numbers, has in addition to the disadvantage of a lower winning probability, also a theoretical advantage: in case of winning, the prizes can be multiple (several winning lines, instead of one single line in the case of the system under the exclusiveness condition).

In example 6), if the given system is winning as result of the occurrence in the draw of 2 numbers from the 4 fixed, then we will have 990 prizes in the third category, each simple line being a winning line. On the other hand, for a 990-line system under the condition of exclusiveness, only one single line can win.
We shall talk in detail about the strategic aspect of choosing the playing system by these criteria in the chapter devoted to the strategy of the lotto game.

Probability thresholds

For players who use exclusively the criterion of obtaining a maximal probability of winning for a given investment amount, the exclusiveness condition imposed on the played systems ensures attaining this maximum and allows immediate calculation of the number of simple lines required for completing a system that offers a certain fixed probability of winning. These calculations are possible due to the linearity of the winning probability.

Theoretically, this number of simple lines may increase to reach the winning probability 1, but at a practical level, it is limited by the following financial parameters:

- the investment amount at player's disposal (a);
- the price of a simple line (c);
- the amount of the prize allocated to the winning category the calculation refers to (b);

The limitation intervenes due to the given investment amount and to keep the game profitable as well – the prize must exceed the amount of the investment.

If s is the number of lines of the system, the two previous conditions are expressed through the relations: $sc \leq a$ and $sc < b$, which are equivalent to $s \leq a / c$ and $s < b / c$.

Number s is limited not only by financial considerations, but also mathematical, namely to preserve the exclusiveness condition. We saw that this condition reverts to the structure of the simple lines of the system as regards the number of common numbers. As s increases over a certain value, the lines of the system accumulate more and more common numbers (because the number of possible simple lines is finite, namely C_m^n), eventually losing the exclusiveness condition.

This constraint on the structure of the lines also answers the following paradox that might be stated by a person without a solid mathematical background: if the probability is linear with the number of played lines under conditions of exclusiveness, we can obtain a winning probability higher than 1 by adding to the system a large enough number of simple lines. As an example, given that the probability of winning at the third category in the 6/49 matrix is about 1/1032, if we choose a system with over 1033 lines, the overall winning probability would become higher than 1. The error stands in the fact that the system with over 1033 lines will no longer obey the exclusiveness condition, through the increase of their common numbers, and the winning probability is given by formula (*).

Depending on parameters *a* and *c*, players may establish their own probability thresholds for the played systems. The previous parameters, along with the chosen probability threshold, will provide through a simple calculation the minimal number of simple lines required for the system to offer the minimal probability chosen as threshold. But this number must be also limited by ensuring the profitability of the game (the eventual prize to be higher than the investment in the playing tickets).

Denoting by p_b the winning probability of a simple line in the category to which the prize of amount *b* is allocated, and by p^* the chosen winning probability threshold of the system at the respective category, the above condition writes: $\frac{p^*}{p_b} \cdot c < b$ (**)

For a given lottery matrix and location, in relation (**) parameters *b*, p_b and *c* can be considered constant, with an approximation for *b* as an average. (The prize fund and implicitly the amount of the prize allocated to a winning category changes from one game to another, depending on the ticket sales; the value of *b* can be calculated as the arithmetic mean of the amounts of the prizes in the respective category, as result of consulting the statistical history of the games on a determined previous period).

The ratio $\frac{p^*}{p_b}$ represents in fact the growing coefficient of the initial winning probability p_b (for one line), and its integer, approximated by addition, represents the minimum number of lines the system must contain for ultimately obtaining the winning probability p^*.

All the conditions mathematically expressed previously also stand for the events of cumulated winning in several categories (events of type "minimum *k* winning numbers"). In this case, p_b will represent the cumulated winning probability for all considered categories.

.... missing part ....

6 /49

k / p^*	4	5	6	min.4	min.5
1/100	11	544	139861	11	541
1/50	21	1087	279721	21	1082
1/30	35	1812	466201	34	1802
1/20	52	2718	699301	51	2703
1/10	104	5435	1398602	102	5406
1/8	129	6794	1748252	127	6757
1/7	148	7764	1998002	145	7723
1/6	172	9058	2331003	169	9010
1/5	207	10870	2797203	203	10811
1/4	258	13587	3496504	254	13514
1/3	344	18116	4662005	338	18019

6 /36

k / p^*	4	5	6	min.4	min.5
1/100	3	109	19494	3	108
1/50	6	217	38987	6	216
1/30	10	361	64978	10	359
1/20	15	542	97466	15	539
1/10	30	1083	194932	30	1077
1/8	38	1353	243665	37	1346
1/7	43	1547	278474	42	1538
1/6	50	1804	324887	49	1795
1/5	60	2165	389864	59	2153
1/4	75	2706	487330	73	2692
1/3	100	3608	649773	97	3589

.... missing part ....

Examples of how to use the tables:

In a 5/55 matrix, what is the minimal number of lines of a system under the exclusiveness condition, for the threshold of the probability of winning with exactly 4 numbers to be 1/5?

We search in the table 5/55, at the intersection of column $k = 4$ with row $p^* = 1/5$ and find the minimal value $s = 2782$.

In a 6/47 matrix, what is the minimal number of lines of a system under the exclusiveness condition, for the threshold of the probability of winning with a minimum of 5 numbers to be 1/8?

We search in the table 6/47, at the intersection of column $k = \text{min.}5$ with row $p^* = 1/8$ and find the minimal value $s = 5435$.

In a 6/90 matrix, what is the minimal number of lines of a system under the exclusiveness condition, for the threshold of the probability of winning with a minimum of 4 numbers to be 1/100?

We search in the table 6/90, at the intersection of column $k = \text{min.}4$ with row $p^* = 1/100$ and find the minimal value $s = 118$.

Compound Lines

A compound (combined) line is a playing system consisting of all possible simple lines that can be formed with a given number of playing numbers. Using our general denotations, a compound line is in fact a combination of r numbers, where $p < r < m$.

Even if a simple line contains only p numbers, every lottery matrix allows the play with $r > p$ numbers forming a compound line, which is in fact a system containing several p-size simple lines. On the playing ticket, such a system is marked by checking (or writing) all the chosen r numbers, meaning that it allows all p-size combinations of numbers that can be formed with the r chosen numbers. Thus, there is no need for an explicit enumeration of all constituent simple lines of the system. Obviously, the number of the simple lines a compound line with r numbers consists of is C_r^p.

Therefore, a compound line is a combination $(a_1, a_2, \ldots, a_r)$, where a_i are arbitrary playing numbers from the m allowed by the respective lottery matrix and $p < r < m$. The combination $(a_1, a_2, \ldots, a_r)$ is in fact the condensed denotation of the playing system that contains all the simple lines that can be created with numbers a_i, $i = 1, \ldots, r$.

Number r (the number of chosen playing numbers) is called the size of the compound line.

The playing system corresponding to the compound line $(a_1, a_2, \ldots, a_r)$ will cost the equivalent of all constituent simple lines together, namely $c \cdot C_r^p$ (c being the price of a simple line).

We shall see further that the systems equivalent to the compound lines have specific properties, as concerns the structure of their constituent simple lines and the number of possible prizes. These properties differentiate them from the systems that are built at random.

.... missing part ....

Winning probabilities

For evaluating the probability of occurrence in the draw of a certain number of winning numbers in the case of the play with one compound line, let us observe that the event "we will have exactly *w* winning numbers in at least one simple line of the system" is identical with "we will have exactly *w* winning numbers among the *r* of the initial combination," where $0 \le w \le n$. This identity of events holds true because the simple lines of the system represent all possible *p*-size combinations of numbers from the played *r*.

Denoting this event by $A_{w,r}$, the probability $P(A_{w,r})$ is given by the same general formula of the winning probability deduced in the chapter titled *Probabilities of Winning with Simple Lines*, in which parameter *r* will replace parameter *p*:

$$P(A_{w,r}) = \frac{C_r^w C_{m-r}^{n-w}}{C_m^n}$$

.... missing part ....

Adding together the three partial results, we obtain a winning probability at no lower than the third category of about 6.24%.

Still related to the compound lines, we must make the following important observation: the unfolded system corresponding to a compound line does not satisfy the exclusiveness condition. This happens because, since it contains all possible *p*-size combinations of the *r* chosen numbers, in the system there will exist simple lines differing from one another by one single number, so they will have $p-1$ common numbers. For this reason, the probability of a compound line winning in a specific category (eventually cumulated) will be lower than that of a system with an identical number of simple lines which adheres to the exclusiveness condition.

There is also an advantage to the play with compound lines – namely, the number of possible prizes. We shall see in the next section that eventually winning in a certain category also implies winning in lower categories, and we will estimate rigorously the number of these prizes.

Next, we present tables containing the winning probabilities (including cumulated) for the play with one compound line, for the most popular lotteries.

In the tables, in the first row are noted the values of the number of winning numbers (w) the probability refers to (including cumulated), and in the first column are noted the values of the size of the compound line (r).

At the intersection of the column corresponding to a value of w with the row corresponding to a value of r is the probability of occurrence in the draw of w numbers from the played r.

The second column holds the number of simple lines of the unfolded compound line, for every value of r.

6/49

w / r	C_r^6	4	5	6	min.4	min.5
7	7	0.002155	6.31E-05	5.01E-07	0.002219	6.36E-05
8	28	0.004105	0.000164	2E-06	0.004271	0.000166
9	84	0.007028	0.00036	6.01E-06	0.007394	0.000366
10	210	0.011128	0.000703	1.5E-05	0.011846	0.000718
11	462	0.01659	0.001255	3.3E-05	0.017878	0.001288
12	924	0.023575	0.002096	6.61E-05	0.025737	0.002162
13	1716	0.032212	0.003313	0.000123	0.035648	0.003436
14	3003	0.042592	0.005011	0.000215	0.047818	0.005226
15	5005	0.054761	0.007301	0.000358	0.06242	0.007659
16	8008	0.068719	0.010308	0.000573	0.0796	0.010881
17	12376	0.084418	0.01416	0.000885	0.099463	0.015045
18	18564	0.101753	0.018994	0.001328	0.122074	0.020321
19	27132	0.120572	0.024946	0.00194	0.147458	0.026886
20	38760	0.140668	0.032153	0.002772	0.175592	0.034924
21	54264	0.161782	0.040745	0.00388	0.206408	0.044626

.... missing part ....

w / r	C_r^7	4	5	6	7	min.4	min.5	min.6
8	8	0.010172	0.00066	1.74E-05	1.27E-07	0.010849	0.000677	1.75E-05
9	36	0.016901	0.001408	5.08E-05	5.72E-07	0.018361	0.00146	5.13E-05
10	120	0.025945	0.002669	0.000124	1.91E-06	0.028739	0.002794	0.000125
11	330	0.037465	0.004628	0.000264	5.25E-06	0.042363	0.004898	0.00027
12	792	0.051514	0.007493	0.000514	1.26E-05	0.059534	0.00802	0.000527
13	1716	0.068031	0.01148	0.000928	2.73E-05	0.080466	0.012435	0.000955
14	3432	0.086839	0.016808	0.001576	5.46E-05	0.105277	0.018438	0.00163
15	6435	0.107652	0.023683	0.002547	0.000102	0.133984	0.026332	0.002649
16	11440	0.13008	0.032296	0.003947	0.000182	0.166505	0.036425	0.004129
17	19448	0.153642	0.0428	0.005904	0.000309	0.202655	0.049013	0.006213
18	31824	0.177786	0.055311	0.00856	0.000506	0.242163	0.064377	0.009066
19	50388	0.2019	0.069888	0.012079	0.000801	0.284669	0.082769	0.012881
20	77520	0.225335	0.086528	0.01664	0.001233	0.329736	0.104401	0.017873
21	116280	0.247426	0.105156	0.022433	0.001849	0.376864	0.129438	0.024282
22	170544	0.267516	0.125616	0.029659	0.002712	0.425503	0.157987	0.032371

In the previous tables, the upper value of parameter r has not been chosen through a specific rule; still, we took into account the total number of playing numbers, as well as the fact that, for the listed higher values, the effective play has no practical motivation, since the large numbers of played lines requires a substantial initial investment. Still, for values of r that are not listed in tables, one can calculate any winning probability directly through the general formula presented at the beginning of this section.

Examples of how to use the tables:

In a 5/55 matrix, let us find the probability of winning with a minimum of 4 numbers of a 12-number compound line. How much does such system cost, if a simple line costs $0.50?

In the table corresponding to matrix 5/55, we follow the intersection of row $r = 12$ with column $w =$ min.4 and we find the probability 0.006346, that is 0.6346%.

At the intersection with the first column, we find the number of simple lines of the system, which is 792. The total cost of the system is then $792c = 792 \times \$0.50 = \$396$.

In a 6/49 matrix, let us find the probability of winning with a minimum of 5 numbers of a 19-number compound line. How much does such a system cost, if a simple line costs $1?

In the table corresponding to matrix 6/49, we follow the intersection of row $r = 19$ with column $w = \text{min.}5$ and we find the probability 0.026886, that is 2.6886%.

At the intersection with the first column we find the number of simple lines of the system, which is 27,132. The total cost of the system would be $27132c = 27132 \times \$1 = \$27{,}132$.

In a 7/47 matrix, let us find the probability of having exactly 4 winning numbers in a 11-number compound line. How much does such system cost, if a simple line costs $0.80?

In the table corresponding to matrix 7/47, we follow the intersection of row $r = 11$ with column $w = 4$ and we find the probability 0.037465, that is 3.7465%.

At the intersection with the first column we find the number of simple lines of the system, which is 330. The total cost of the system is $330c = 330 \times \$0.80 = \$264$.

From the tables as well as the examples in this section, we clearly saw that the number of simple lines in a compound line increases very quickly with its size r, while the winning probabilities remain very low. To achieve relatively reasonable winning probabilities requires generating thousands of simple lines, and consequently, requires a huge initial investment.

Still, the play with high-size compound lines becomes reasonably possible in some lotteries that offer a fraction game. This means the playing systems can be paid in percentages of 25%, 50% or 100%, and the winning amount is divided accordingly, by applying the same percentages.

The number of prizes

The fact that the simple lines corresponding to a compound line are generated by combining a limited number of given numbers implies an accumulation of common numbers in these lines, which we generally do not see in random systems, and even less frequently in systems having the exclusiveness property. Due to this combinatorial accumulation, a compound line may ensure – in case of winning in an arbitrary category – the simultaneous existence of several winnings in lower categories.

Next, we deduce a general formula that returns the number of possible winnings of a compound line, for every lottery matrix, as a function of the size of the compound line and its number of winning numbers.

.... missing part ....

Example of calculation:

In a 6/49 matrix, from a 12-number compound line, 5 numbers are winning. How many simple lines containing exactly 5 and exactly 4 winning numbers are in that system?

We have $p = 6$, $r = 12$, $k = 5$. We apply the above formula for $i = 5$ and $i = 4$. We obtain $N_{6,12}^{5}(5) = C_5^5 C_{12-5}^{6-5+5} = C_7^6 = 7$ lines containing the 5 winning numbers and $N_{6,12}^{5}(4) = C_5^4 C_{12-5}^{6-5+4} = C_7^5 = 21$ lines containing exactly 4 from the 5 winning numbers. This means that we will have simultaneously 7 prizes in the second category and 21 prizes in the third category.

Next we present tables of values of number $N_{p,r}^{k}(i)$, returned by the general formula, for the most popular lottery matrices (each matrix being represented through a value of p) and for a large enough range of values for the size of the compound line.

In these tables, in the first row are noted the values of the number of winning numbers (k), in the second row are values of the number of winning numbers that are less than or equal to k (namely $k - i$), and in the first column are the values of the size of the compound line (r).

At the intersection of the row corresponding to a value of r with the column corresponding to a pair $(k, k - i)$ we find the number of simple lines containing exactly $k - i$ numbers from the winning k.

$p = 3$

k	2		3		
r ***k-i***	2	1	3	2	1
4	2	2	1	3	0
5	3	6	1	6	3
6	4	12	1	9	9
7	5	20	1	12	18
8	6	30	1	15	30
9	7	42	1	18	45
10	8	56	1	21	63
11	9	72	1	24	84
12	10	90	1	27	108
13	11	110	1	30	135
14	12	132	1	33	165
15	13	156	1	36	198
16	14	182	1	39	234
17	15	210	1	42	273
18	16	240	1	45	315
19	17	272	1	48	360
20	18	306	1	51	408
21	19	342	1	54	459
22	20	380	1	57	513
23	21	420	1	60	570

.... missing part ....

Example of how to use the table:

In a 7/47 matrix, we played a 15-number compound line. What is the number of simple lines containing exactly 5 winning numbers, if exactly 6 winning numbers from the played 15 occurred in the draw?

We follow the intersection of row $r = 15$ with the column corresponding to ($k = 6$; $k - i = 5$) and find 216 simple lines with exactly 5 winning numbers.

If in addition we want to find the probability of having 6 winning numbers among the played 15, we consult the table 7/47 in the previous section devoted to the probabilities of winning with compound lines (for $r = 15$ and $w = 5$) and find 0.023683, that is 2.3683%.

The general formula and the tables of this section cover all possible values of the number of winning numbers (k) from a compound line. In particular, among these values of k there are also the thresholds of the winning categories $n_1, n_2, \ldots, n_q$, for any lottery matrix. For most lotteries, we have $n_{i-1} = n_i + 1$, for every $i = 2, \ldots, q$ (passing to the next higher threshold of the winning category is done by increasing the current one with 1).

.... missing part ....

Systems of compound lines

As on the playing tickets we can inscribe several simple lines forming a playing system, so we can also inscribe several compound lines. The resulting system is equivalent to the simple-line system obtained through the unfolding of all compound lines. Therefore, a system consisting only of compound lines is still a simple-line system, the only difference between them being the way they are noted (encoded) on the playing ticket.

In mathematical denotations, a compound-line system is a group of combinations of various sizes, made of playing numbers:

$$\begin{array}{cccc}
(\ a_{11} & a_{12} & \cdots & a_{1r_1}\ ) \\
(\ a_{21} & a_{22} & \cdots & a_{2r_2}\ ) \\
\cdots & \cdots & \cdots & \cdots \\
(\ a_{s1} & a_{s2} & \cdots & a_{sr_s}\ )
\end{array}$$

$s > 1$ is the number of compound lines the system consists of, $r_1, r_2, \ldots, r_s$ represents the size of each compound line ($r_i > p,\ i = 1, \ldots, s$), and a_{ij} $(i = 1, \ldots, s;\ j = 1, \ldots, r_i)$ are numbers of the respective lottery matrix.

Denoting by W_i the constituent compound lines $(a_{i1}\ a_{i2} \ldots a_{ir_i})$, for every $i = 1, \ldots, s$, it is obvious that the total number of simple lines of the unfolded system is the sum of the numbers of simple lines of each compound line W_i, that is $\sum_{i=1}^{s} C_{r_i}^p = C_{r_1}^p + C_{r_2}^p + \cdots + C_{r_s}^p$.

A compound-line system may hold as many such lines, of any size, as the player's financial limitations allow. Also, the structure of the compound line may be influenced by the strategic criteria as regards the coverage of the playing numbers.

.... missing part ....

Bridgehead Systems

A bridgehead system is a group of simple lines of the form $(\underbrace{a_1\ a_2\ \ldots a_h}_{h}\ \underbrace{x_{h+1}\ x_{h+2}\ \ldots x_p}_{p-h})$, where numbers a_i are h distinct fixed numbers (showing in each line), and the last $p-h$ numbers (x_j) are obtained through the unfolding of all $p-h$ -size combinations of the remaining playing numbers ($m-h$ numbers).

Obviously, $1 \le h \le p$. Parameter h will refer to the number of fixed numbers of the system; $p-h$ will refer to the number of variable numbers of the system (being in fact the size of the combinations of numbers x_j).

For counting the number of simple lines of such a system, we shall count the combinations of numbers x_j, which number C_{m-h}^{p-h}.

Let us make the following immediate observations:

1) Numbers a_i and x_j cover all the m playing numbers.

2) The set of numbers (a_i) and (x_j) are exclusive (any number a_i is not found among numbers x_j and conversely).

Here is an example (already presented in a previous chapter) of a bridgehead system in the 6/49 matrix:

1 2 3 4 5 6
1 2 3 4 5 7
1 2 3 4 5 8
…………………..
1 2 3 4 5 49
1 2 3 4 6 7
1 2 3 4 6 8
1 2 3 4 6 9
…………………
1 2 3 4 6 49
…………………
1 2 3 4 48 49

It is a bridgehead system with $h = 4$ fixed numbers and $p - h = 2$ variable numbers. The last 2 places of the lines were filled by unfolding all the 2-size combinations of the numbers from 5 to 49. This system contains $C_{49-4}^{6-4} = C_{45}^{2} = 990$ simple lines. Any equivalent system obtained through a cyclic permutation of the 49 numbers will have the same parameters and number of simple lines.

Let us observe that as the number of the variable numbers increases, the number of constituent simple lines increases much faster. For example, still for the 6/49 matrix, a bridgehead system with 3 variable numbers will have $C_{49-3}^{6-3} = C_{46}^{3} = 15180$ simple lines, while for 4 variable numbers, the system will have $C_{49-2}^{6-2} = C_{47}^{4} = 178365$ simple lines, which, practically speaking, makes it inapplicable.

The bridgehead systems have the advantage of quickly checking the winning numbers and establishing the winning categories after the drawing, as well as the possibility of achieving multiple winnings, as we shall see further.

Winning probabilities

In this section, we refer to the cumulated winning probabilities for the bridgehead systems.

Denoting by B_k the event "we will have minimum k winning numbers in at least one simple line of the system," we aim to evaluate $P(B_k)$. We have two cases:

.... missing part ....

Below, we present tables with numerical values of the probabilities returned by the previous general formula, for a large range of classical lottery matrices (in which $p = n$). We built a table for each value of n. For a given lottery matrix, the tables display the cumulated winning probabilities for any type of played bridgehead system, as a function of the number h of its fixed numbers. In a table, at the intersection of the row corresponding to a value of m with the column corresponding to a pair (k, h) we find the probability of winning with minimum k numbers, for a played bridgehead system with h fixed numbers, at the matrix n/m.

.... missing part ....

Examples of how to use the tables:

In a 6/90 matrix, what is the probability of winning with a minimum of 5 numbers, using a bridgehead system with 3 fixed and 3 variable numbers?

We have $n = 6$, $m = 90$, $h = 3$, $k = 5$. In table $n = 6$, we follow the intersection of row $m = 90$ with the column corresponding to the pair ($k = 5$; $h = 3$), and we find the probability 0.010895, that is 1.0895%.

In a 5/32 matrix, what is the probability of winning with a minimum of 4 numbers, using a bridgehead system with 3 fixed and 2 variable numbers?

We have $n = 5$, $m = 32$, $h = 3$, $k = 4$. In table $n = 5$, we follow the intersection of row $m = 32$ with the column corresponding to the pair ($k = 4$; $h = 3$), and we find the probability 0.056452, that is 5.6452%.

In a 6/42 matrix, what is the probability of winning with minimum 4 numbers, using a bridgehead system with 4 fixed and 2 variable numbers?

We have $n = 6$, $m = 42$, $h = 4$, $k = 4$. In table $n = 6$, we follow the intersection of row $m = 42$ with the column corresponding to the pair ($k = 4$; $h = 4$), and we find the probability 0.090994, that is 9.0994%.

We immediately observe that the tables of this section contain much higher probabilities than those listed in the previous chapters.

But is these specific cases, at a practical level the high winning probability is counterbalanced by the very large number of simple lines (hence a large initial investment) that the unfolded system consists of, as well as the risk of a cumulated winning that is lower than the amount invested in playing that system.

The number of prizes

.... missing part ....

Example:

In a 6/49 matrix with the minimal winning category having the threshold of 4 numbers, we played a bridgehead system with fixed numbers 7, 12, 21, 32 and 2 variable numbers. The draw came up 5, 12, 19, 21, 29, 32. How many prizes did we win in each category?

We have $n_q = 4$ and $h = 4$. Observe that $k' = 3$ (winning numbers 12, 21, 32). The condition $k' \ge n_q - p + h$ is equivalent to $3 \ge 4 - 6 + 4$, so it is satisfied. Hence we have prizes from the third category upward, as follows (for k from 4 to $k' + p - h = 5$):

For $k = 4$, $N_{4,3} = C_{6-3}^{4-3} C_{49-4}^{6-4-4+3} = C_3^1 C_{45}^1 = 135$.

For $k = 5$, $N_{4,3} = C_{6-3}^{5-3} C_{49-4}^{6-4-5+3} = C_3^2 C_{45}^0 = 3$.

For $k = 6$, we have $N_{6,3} = 0$.

.... missing part ....

***h* = 2**

k \ *k'*	0	1	2	3	4	5	6
1	97290	178365	0	0	0	0	0
2	16215	81075	178365	0	0	0	0
3	940	10810	64860	178365	0	0	0
4	15	470	6486	48645	178365	0	0
5	0	5	188	3243	32430	178365	0
6	0	0	1	47	1081	16215	178365

h = 5

k' / k	0	1	2	3	4	5	6
1	6	44	0	0	0	0	0
2	0	5	44	0	0	0	0
3	0	0	4	44	0	0	0
4	0	0	0	3	44	0	0
5	0	0	0	0	2	44	0
6	0	0	0	0	0	1	44

Example of how to use the tables:

In a 6/49 lottery, we played a bridgehead system with the fixed numbers 8, 11, 23, 32, and 2 variable numbers. The draw came: 11, 32, 37, 41, 42, 48. How many of the simple lines of the played system do contain exactly 2 winning numbers? How about exactly 4?

We have: $h = 4$, $k' = 2$ (numbers 11, 32). In table h = 4, at the intersection of row k = 2 with column $k' = 2$, we find 990 simple lines containing exactly 2 winning numbers; at the intersection of row k = 4 with column $k' = 2$ we find 6 simple lines containing exactly 4 winning numbers.

Reduced Systems

A reduced system is a system obtained through the unfolding of a compound line (from here on called *the base compound line*), from which some simple lines have been removed on the basis of some pre-established criteria. The goal of these removals is to reduce the number of simple lines of the playing system, having an immediate effect on the initial investment. Since generally the number of simple lines of an unfolded compound line is quite large, obtaining a reduced system is a compromise between using the same playing numbers (but with a different distribution within the new system) and keeping costs acceptable.

The criteria for removing lines can be multiple, objective, and/or subjective: properties of numbers and groups of numbers within a simple line, the frequency of certain numbers, reduction of the number of possible multiple winnings while keeping some minimal guarantees, etc. We shall come back to these criteria in the section titled *Obtaining the reduced systems*.

The reduced systems are noted on the playing tickets either encoded or manually. Encoded notation is possible on the tickets only for those reduced systems that are registered officially by the issuing lottery. It is done by checking/writing all played numbers (the base compound line) and mentioning the code of the played reduced system, according to the official encoding. Manual notation assumes writing all the simple lines of the reduced system. Manual notation is obligatory for the systems that are not officially encoded.

A reduced system is declared a winner if after the draw the existence of at least one winning constituent simple line (containing the minimal threshold of winning numbers) is ascertained.

If *k* winning numbers have been found in the base compound line, this does not mean that we automatically have *k* winning numbers (and even fewer than *k*) in the reduced system, because it is possible that no simple line containing them will exist, according to the structure of the system and the distribution of the numbers in the lines remaining after removal. For winning with the *k* numbers, they must “fit” in the played reduced system.

Example of reduced system:

In a 4/77 matrix, let us consider the base compound line (11 20 22 45 57 59 73). If we unfold this compound line (see the unfolding procedure and the similar example from the section titled *Unfolding compound lines*) and remove all simple lines containing several numbers from the same numerical range of tens (20 and 22, as well 57 and 59), we obtain a reduced system on the basis of a subjective criterion.

From the 35 simple lines of the base compound line remained 16 simple lines of the reduced system:

11 20 45 57	11 22 45 59	20 45 57 73
11 20 45 59	11 22 45 73	20 45 59 73
11 20 45 73	11 22 57 73	22 45 57 73
11 20 57 73	11 22 59 73	22 45 59 73
11 20 59 73	11 45 57 73	
11 22 45 57	11 45 59 73	

If, for example, the winning draw is (20 22 57 73), we have in the reduced system no simple line containing all these numbers, so we have no winners in the first category.

Notice that a hypothetical play with the base compound line would ensure this winning.

Still, we have 4 winnings in the second category (3 winning numbers), corresponding to simple lines (11 20 57 73), (11 22 57 73), (20 45 57 73) and (22 45 57 73), compared to 12 that we would have in the case of the hypothetic play with the base compound line.

The numerical parameters attached to a reduced system are the following:

r' – the size of the base compound line;

s – the number of simple lines of the reduced system;

g – the coefficient of reduction of the base compound line (the ratio between the number of simple lines of the reduced system and the number of simple lines of the unfolded base compound line).

These parameters fit the following equality and order relations:

$p < r' < m$, $s < C_{r'}^{p}$, $0 < g < 1$, $s = gC_{r'}^{p}$.

.... missing part ....

Winning probabilities

In the compound and bridgehead lines we obtained, for the winning probabilities, general formulas whose variables were only the parameters that define the respective systems. This is not possible for reduced systems because not only the parameters, but also the particular structure of each system influences the probability of a certain winning event.

Thus, we cannot obtain a general formula on the basis of which to calculate the winning probabilities for a given reduced system. Depending on the removal criterion, this calculation can theoretically be made with the help of a computer, on the base of some complex mathematical algorithms.

The only situation in which a direct calculation is possible is that in which the reduced system satisfies the exclusiveness condition with respect to the winning event to be measured. In this case, the winning probability is linear with the number of simple lines in the system. But this situation is very rare for the most common lottery matrices and for their winning thresholds.

Next we shall find a convenient approximation of the winning probability for any reduced system, which can replace a very laborious exact calculation.

.... missing part ....

Examples of approximations of the winning probability:

1) In the 6/49 matrix, we play a reduced system with 30 simple lines, coming from a 10-number compound line. Let us approximate the probability of winning with exactly 4 numbers.

a. $r'=10$ and $s = 30$.

b. In table 6/49 from the section devoted to the probabilities of the compound lines, for $w = 4$ and $r'=10$ we find $P(A'_4) = 0.011128$.

c. In the same table, in the second column, the number closest to 30 and less than or equal to 30 is $s'' = 28$.

d. At the intersection of the row corresponding to $s'' = 28$ with column $w = 4$ we find $P(A''_4) = 0.004105$.

e. We have then: $0.004105 < P(A_4) < 0.11128$.

The probability is somewhere between 0.4% and 1.2%.

.... missing part ....

Probabilities of Repeated Events

All probabilities obtained thus far refer to events corresponding to the same probabilistic experiment, namely the drawing of the winning numbers in a single lotto game. We can also calculate probabilities of some events containing several drawings belonging to different games. These drawings, from a probability point of view, represent independent experiments. These probabilities provide useful information, especially for players who study the draw statistics as part of a personal strategy of choosing which numbers to play and/or whether to play the same numbers or groups of numbers repeatedly.

Generally, moving from the probability of occurrence of an event in a singular game to the probability of its repeated occurrence in several games requires a simple calculation based on Bernoulli's scheme (F8):

Assume that t independent experiments (drawings) are performed. In each experiment, event A can occur with probability u. Then the probability for event A to occur exactly v times in the t experiments is $P_{v,t}(A) = C_t^v u^v (1-u)^{t-v}$.

Calculating the probability of the repeated event assumes knowing in advance the probability of the singular event, namely u.

Next we present the probabilities of some particular repeated events, expressed through general formulas and accompanied by numerical examples for matrix 6/49.

a) We consider the singular event A as being *the occurrence in the draw of a certain number*.

According to the calculus formula from the hypergeometric scheme (F9), we have $u = P(A) = \dfrac{C_{m-1}^{n-1}}{C_m^n}$. This is the probability that the chosen number will occur in a draw.

The next table lists the numerical results of this formula for the various lottery matrices.

***P*(*A*)**

n / *m*	3	4	5	6	7
21	0.142857	0.190476	0.238095	0.285714	0.333333
22	0.136364	0.181818	0.227273	0.272727	0.318182
23	0.130435	0.173913	0.217391	0.26087	0.304348
24	0.125	0.166667	0.208333	0.25	0.291667
25	0.12	0.16	0.2	0.24	0.28
26	0.115385	0.153846	0.192308	0.230769	0.269231
27	0.111111	0.148148	0.185185	0.222222	0.259259
28	0.107143	0.142857	0.178571	0.214286	0.25
29	0.103448	0.137931	0.172414	0.206897	0.241379
30	0.1	0.133333	0.166667	0.2	0.233333
31	0.096774	0.129032	0.16129	0.193548	0.225806
32	0.09375	0.125	0.15625	0.1875	0.21875
33	0.090909	0.121212	0.151515	0.181818	0.212121
34	0.088235	0.117647	0.147059	0.176471	0.205882
35	0.085714	0.114286	0.142857	0.171429	0.2
36	0.083333	0.111111	0.138889	0.166667	0.194444
37	0.081081	0.108108	0.135135	0.162162	0.189189
38	0.078947	0.105263	0.131579	0.157895	0.184211
39	0.076923	0.102564	0.128205	0.153846	0.179487
40	0.075	0.1	0.125	0.15	0.175
41	0.073171	0.097561	0.121951	0.146341	0.170732
42	0.071429	0.095238	0.119048	0.142857	0.166667
43	0.069767	0.093023	0.116279	0.139535	0.162791
44	0.068182	0.090909	0.113636	0.136364	0.159091
45	0.066667	0.088889	0.111111	0.133333	0.155556
46	0.065217	0.086957	0.108696	0.130435	0.152174
47	0.06383	0.085106	0.106383	0.12766	0.148936
48	0.0625	0.083333	0.104167	0.125	0.145833
49	0.061224	0.081633	0.102041	0.122449	0.142857
50	0.06	0.08	0.1	0.12	0.14
51	0.058824	0.078431	0.098039	0.117647	0.137255
52	0.057692	0.076923	0.096154	0.115385	0.134615
53	0.056604	0.075472	0.09434	0.113208	0.132075
54	0.055556	0.074074	0.092593	0.111111	0.12963
55	0.054545	0.072727	0.090909	0.109091	0.127273

56	0.053571	0.071429	0.089286	0.107143	0.125
57	0.052632	0.070175	0.087719	0.105263	0.122807
58	0.051724	0.068966	0.086207	0.103448	0.12069
59	0.050847	0.067797	0.084746	0.101695	0.118644
60	0.05	0.066667	0.083333	0.1	0.116667
61	0.04918	0.065574	0.081967	0.098361	0.114754
62	0.048387	0.064516	0.080645	0.096774	0.112903
63	0.047619	0.063492	0.079365	0.095238	0.111111
64	0.046875	0.0625	0.078125	0.09375	0.109375
65	0.046154	0.061538	0.076923	0.092308	0.107692
66	0.045455	0.060606	0.075758	0.090909	0.106061
67	0.044776	0.059701	0.074627	0.089552	0.104478
68	0.044118	0.058824	0.073529	0.088235	0.102941
69	0.043478	0.057971	0.072464	0.086957	0.101449
70	0.042857	0.057143	0.071429	0.085714	0.1
71	0.042254	0.056338	0.070423	0.084507	0.098592
72	0.041667	0.055556	0.069444	0.083333	0.097222
73	0.041096	0.054795	0.068493	0.082192	0.09589
74	0.040541	0.054054	0.067568	0.081081	0.094595
75	0.04	0.053333	0.066667	0.08	0.093333
76	0.039474	0.052632	0.065789	0.078947	0.092105
77	0.038961	0.051948	0.064935	0.077922	0.090909
78	0.038462	0.051282	0.064103	0.076923	0.089744
79	0.037975	0.050633	0.063291	0.075949	0.088608
80	0.0375	0.05	0.0625	0.075	0.0875
81	0.037037	0.049383	0.061728	0.074074	0.08642
82	0.036585	0.04878	0.060976	0.073171	0.085366
83	0.036145	0.048193	0.060241	0.072289	0.084337
84	0.035714	0.047619	0.059524	0.071429	0.083333
85	0.035294	0.047059	0.058824	0.070588	0.082353
86	0.034884	0.046512	0.05814	0.069767	0.081395
87	0.034483	0.045977	0.057471	0.068966	0.08046
88	0.034091	0.045455	0.056818	0.068182	0.079545
89	0.033708	0.044944	0.05618	0.067416	0.078652
90	0.033333	0.044444	0.055556	0.066667	0.077778

According to Bernoulli's scheme, the probability that the chosen number will occur exactly v times in t draws is

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The next table lists the numerical values of probabilities $P_{v,t}(A)$ of the repeated occurrences of a certain number, for the 6/49 matrix, for values of t up to 100, listed in increments of 10.

t v	10	20	30	40	50	60	70	80	90	100
1	0.377926	0.20472	0.083172	0.030036	0.010169	0.003305	0.001044	0.000323	9.85E-05	2.96E-05
2	0.237302	0.271373	0.168277	0.081725	0.034763	0.013604	0.005027	0.001782	0.000612	0.000205
3	0.088299	0.227196	0.219152	0.144444	0.077611	0.0367	0.015901	0.006464	0.002503	0.000933
4	0.021561	0.134733	0.206411	0.186434	0.127246	0.072974	0.037164	0.017362	0.007597	0.003158
5	0.00361	0.06016	0.149768	0.187301	0.163348	0.114043	0.06845	0.036825	0.018234	0.00846
6	0.00042	0.020986	0.087074	0.152454	0.170946	0.145869	0.103471	0.064229	0.036044	0.018691
7	3.35E-05	0.005857	0.041657	0.103325	0.149933	0.157015	0.132003	0.094743	0.060352	0.035022
8	1.75E-06	0.001328	0.016711	0.059472	0.112449	0.145148	0.14505	0.120632	0.08737	0.056809
9	5.43E-08	0.000247	0.0057	0.029505	0.073223	0.117019	0.139428	0.134659	0.111076	0.08103
10	7.58E-10	3.79E-05	0.00167	0.012763	0.04189	0.083274	0.118676	0.133407	0.125541	0.102889
11		4.81E-06	0.000424	0.004857	0.021255	0.052816	0.090324	0.118458	0.127399	0.117463
12		5.03E-07	9.36E-05	0.001638	0.009639	0.030093	0.061966	0.095042	0.11703	0.121561
13		4.32E-08	1.81E-05	0.000492	0.003931	0.015504	0.038577	0.069369	0.097978	0.11482
14		3.02E-09	3.06E-06	0.000132	0.00145	0.007263	0.021916	0.046323	0.075193	0.099561
15		1.68E-10	4.56E-07	3.2E-05	0.000486	0.003108	0.011417	0.02844	0.053159	0.079649
16		7.34E-12	5.97E-08	6.98E-06	0.000148	0.00122	0.005476	0.016122	0.03477	0.059042
17		2.41E-13	6.86E-09	1.38E-06	4.14E-05	0.00044	0.002427	0.008469	0.021119	0.040708
18		5.6E-15	6.91E-10	2.45E-07	1.06E-05	0.000147	0.000997	0.004136	0.011951	0.026192
19		8.23E-17	6.09E-11	3.96E-08	2.49E-06	4.53E-05	0.000381	0.001883	0.006319	0.015773
20		5.74E-19	4.67E-12	5.81E-09	5.38E-07	1.3E-05	0.000135	0.000801	0.00313	0.008913
21			3.11E-13	7.72E-10	1.07E-07	3.44E-06	4.5E-05	0.00032	0.001456	0.004738
22			1.77E-14	9.3E-11	1.97E-08	8.52E-07	1.4E-05	0.00012	0.000637	0.002374
23			8.6E-16	1.02E-11	3.35E-09	1.96E-07	4.07E-06	4.21E-05	0.000263	0.001123
24			3.5E-17	1E-12	5.26E-10	4.22E-08	1.11E-06	1.39E-05	0.000102	0.000503
25			1.17E-18	8.96E-14	7.63E-11	8.49E-09	2.86E-07	4.36E-06	3.77E-05	0.000213
26			3.15E-20	7.22E-15	1.02E-11	1.59E-09	6.9E-08	1.29E-06	1.32E-05	8.59E-05
27			6.5E-22	5.22E-16	1.27E-12	2.8E-10	1.57E-08	3.59E-07	4.35E-06	3.28E-05
28			9.72E-24	3.38E-17	1.46E-13	4.61E-11	3.36E-09	9.48E-08	1.37E-06	1.19E-05
29			9.36E-26	1.95E-18	1.54E-14	7.09E-12	6.8E-10	2.37E-08	4.08E-07	4.14E-06
30			4.35E-28	9.99E-20	1.5E-15	1.02E-12	1.3E-10	5.63E-09	1.16E-07	1.37E-06
31				4.5E-21	1.35E-16	1.38E-13	2.33E-11	1.27E-09	3.12E-08	4.31E-07
32				1.76E-22	1.12E-17	1.75E-14	3.97E-12	2.71E-10	8.03E-09	1.3E-07
33				5.97E-24	8.54E-19	2.07E-15	6.38E-13	5.49E-11	1.97E-09	3.73E-08
34				1.71E-25	5.96E-20	2.29E-16	9.68E-14	1.06E-11	4.61E-10	1.02E-08
35				4.1E-27	3.8E-21	2.37E-17	1.39E-14	1.94E-12	1.03E-10	2.7E-09
36				7.95E-29	2.21E-22	2.3E-18	1.89E-15	3.39E-13	2.19E-11	6.79E-10
37				1.2E-30	1.17E-23	2.08E-19	2.42E-16	5.62E-14	4.47E-12	1.64E-10
38				1.32E-32	5.57E-25	1.76E-20	2.93E-17	8.88E-15	8.69E-13	3.79E-11
39				9.45E-35	2.39E-26	1.38E-21	3.35E-18	1.33E-15	1.62E-13	8.41E-12
40				3.3E-37	9.17E-28	1.01E-22	3.63E-19	1.91E-16	2.88E-14	1.79E-12
41					3.12E-29	6.9E-24	3.7E-20	2.6E-17	4.9E-15	3.65E-13

Examples of how to use the table:

1) In the 6/49 matrix, let us find the probability of occurrence of the same number exactly 15 times in 80 draws.

In the table, we follow the intersection of row $v = 15$ with column $t = 80$ and we find the probability 0.02844 = 2.844%.

2) In the 6/49 matrix, let us find the probability of occurrence of the same number for a minimum of 8 times in 20 draws.

In the table we follow the column $t = 20$ and add together all the values in this column standing between the rows $v = 8$ and $v = 20$ inclusive. We obtain:

0.001328 + 0.000247 + 3.79E-05 + 4.81E-06 + 5.03E-07 + 4.32E-08 + 3.02E-09 + 1.68E-10 + 7.34E-12 + 2.41E-13 + 5.6E-15 + 8.23E-17 + 5.74E-19 = 0.001618 = 0.1618%

.... missing part ....

In the same way, we can obtain calculus formulas for the probabilities of repeated events of any type – for example, for the repeated occurrence of more than 2 given numbers in the draws or events related to various numerical properties (consecutiveness, parity, divisibility, etc.).

If we want to find the probability of a repeated event in several consecutive draws, we can use directly the formula from the definition of independent events (F7), applied several times successively: If the singular event A has probability u for one draw, then it occurs in each of t independent drawings (in particular, consecutive drawings) with probability u^t.

This result can be also obtained by choosing $v = t$ in Bernoulli's formula; therefore, it is a particular case of the results of this chapter.

Example:

In a 5/36 matrix, what is the probability of occurrence of the number 14 in 3 consecutive draws?

For one draw, we have $u = 0.138889$ (according to the table from case a) of the event of occurrence of one single number). For 3 consecutive draws, the probability of occurrence of that number in all 3 draws will be $u^3 = 0.002679 \simeq 0.268\%$.

In the same way, we can calculate the probability of repeated occurrences of any singular event, if we know its probability of occurrence in one draw.

The numerical probabilities obtained in this chapter can be used as a guide by players who use the draw statistics for building a strategy of choosing the numbers to play.

For example, in observing the frequency of occurrence of a given number in a certain number of previous draws, a player might notice certain differences between probability and relative frequency of that event, one being greater or lesser than the other, which in turn might influence a decision regarding playing that number in the next draw or draws. In addition, the decision of repeated play of a number or group of numbers for a pre-established interval in future games can be based on the probabilities of the repeated events presented in this chapter.

We shall talk about these choosing criteria in the next chapter.

The Strategy of Choosing

Every strategy for any game of chance, whether elaborated for a long or a short term, is a pre-established algorithmic plan based on a player's set of personal criteria, which can be objective or subjective.

The existence of subjective criteria in a personal gaming strategy automatically turns it into a subjective one, and the personal and subjective criteria cannot be avoided. Therefore, there is no absolute strategy for any game of chance, and implicitly for lottery, much less an optimal one, because both its means and its goals are related exclusively to the individual who runs it.

Even though the randomness inherent in games of chance would seem to ensure their fairness (at least by not favoring a player to other player's detriment), gamblers always search and wait for irregularities in this randomness that will allow them to win and in which to embed in a personal strategy.

It has been demonstrated mathematically that under conditions of absolute randomness, regular winning over the long term is impossible for games of chance; clearly, gambling is not a reliable source of income. Most gamblers realize this, yet they continue to try to develop strategies which will result in their eventual winning over the long run. But using a long-run strategy to achieve a cumulated positive result means ignoring the randomness and skipping the experiments which yield negative results. This strategy is possible only if a player has access to some paranormal information – someone has to have prior knowledge and be able to tell the player when to play and when not to! Until this magic help becomes possible, probability theory remains the only tool that provides some information about gaming events, even if only as an idealized relative frequency.

Gamblers, especially lotto players, are also interested in isolated winnings, achieved in a single game or a short-run play, with or without a pre-established strategy.

As we mentioned in the beginning, this is not a lotto strategy book of "how to win" because such a strategy in fact does not exist. It is rather a collection of probabilities and figures attached to the lottery matrices and playing systems, which covers all that relates to the practical applications of the mathematical model of the lottery. It provides information a player must know and to which s/he must relate when building a so-called personal strategy of playing, because mathematics provides the player with the only objective tools available to manage the risk.

The simplicity of the lottery game in fact eliminates the strategic needs of players. The progress of the game is not influenced by players' actions; there are no opponents to read or dealers to confront. The game consists only of placing the bets (buying the tickets containing the chosen lines or systems) and comparing them with the draw. Thus, the only playing plan we might term "strategic" consists of the player's choices.

In other words, the act of choice is the only strategy of the lotto game. This strategy of choosing works through choosing the lottery matrix, choosing the lottery games, choosing the numbers to play, choosing the systems to play, and even choosing not to play.

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Like the strategy based on relative frequency, the strategy of repeated play of the same number or group of numbers is also questionable for the same theoretical reasons related to the physical possibility that the series of games will not offer the expected relative frequency as an approximation of probability.

Another strategy consists of choosing or avoiding certain arithmetical correlations between the played numbers, expressed through properties of consecutiveness, divisibility, size orders, etc.

Many players avoid playing lines whose numbers verify, for example, one of the conditions:

- all numbers are consecutive;
- the numbers are all odd or all even;
- the numbers belong to the same interval of tens (1 – 10, 11 – 20, 21 – 30, etc.).
- all numbers are divisible by 10.

Those practicing this strategy of choosing erroneously consider such lines to be unusual or exceptional in some way, and implicitly assign them a different measure of the possibility of occurrence in the draw, while in fact the winning probability is the same for any of the C_m^p simple lines. For this reason, the choosing criteria exemplified above are subjective.

Still, there is an exception to that motivation, which can change the attribute of such criteria from subjective to objective. That is the situation in which the player is convinced (by some technical studies) that the technical process of drawing the numbers influences some arithmetical correlations between those numbers. Such correlations might include all physical aspects of the process of shuffling and extracting, from introduction of the balls into the urn, to the order of introduction, their initial position in the urn, the structure and shape of the urn, etc. Even if these kinds of convictions can be scientifically combated quite easily, the player who does not accept the argument can still create his or her own objective strategy of choosing based on such correlation criteria.

From a mathematical point of view, each simple line has the same winning probabilities, and on this principle is developed the entire probability calculus applied to the lotto game. A simple line represents an elementary event in the built probability space, and without the "equally possible" idealization, the probability calculus would not make sense.

At the practical level, there exists a real tendency of players to avoid certain so-called "unique" combinations of numbers, as those previously exemplified. For example, in a $6/m$ matrix, most players avoid playing the line (1, 2, 3, 4, 5, 6). Although mathematics says that this combination has absolutely the same winning probabilities as any other one, it is still avoided by players who convince themselves that "it's impossible for this line to be drawn." This statement is not far from the truth, the line being "almost impossible" to be drawn, in the sense that its probability of occurrence of 1/13983816 is almost null, but this probability stands for any other played line.

No combination has a preferential status compared to the others, and all have the same winning probabilities. If in a certain lottery, we study the statistics of the draws over its entire history and find that the line (1, 2, 3, 4, 5, 6) has never won, this does not contradict the previous statement regarding preferentiality. If we have the required technical means, we will find in the same statistics that hundreds of thousands, even millions of other “non-particular” combinations met the same fate.

Moreover, this subjective reasoning can be considered through an objective strategy: if you play this particular line and it wins, the cash amount will be larger than that for another played line, because the prize fund will be divided among fewer winners. Thus, the optimal decision would be to play this kind of particular lines. Obviously, this argument remains valid as long as the minority (those playing particular lines) does not become majority!

All the strategies of choosing the numbers to play contain more or less subjective criteria, as we saw in the previous exposition.

From a mathematical point of view, any number has the same probability of occurrence in a draw as any other, and any combination has the same probability of occurrence as any other one. No matter what the previous frequency has been, we have the same mathematical probability of occurrence in the next draw, because the probabilities are calculated under the hypothesis that the experiments are independent. Thus, from a probability point of view, it does not matter whether we play a number (or group of numbers, or a complete line) repeatedly or we change them with every game; the mathematical chances of winning with them in an isolated game or cumulatively (in several games) are the same for both situations.

In mathematics, the strategies of choosing do not exist; they could be eliminated, leaving only the random choice of the number to play.

Remaining devoted to this non-preferentiality principle, we cannot make recommendations as to any of the strategies of choosing the numbers presented in this chapter for illustrative purposes. On the other hand, neither we can declare as somehow incorrect the criteria these strategies are based on, nor recommend their elimination. Whether we choose the numbers at random or based on some personal strategies, the final result will be equally possible for both situations. In addition, choosing the numbers seems to be part of the enjoyment of the game, and this is a sufficient reason not to oppose it.

But there also exist strategies of choosing based on objective criteria, as we shall see further. These strategies relate to choosing the playing systems.

Going from the play with one simple line to the play with systems containing several lines is done mainly to enhance the winning probabilities. Thus, probability becomes an important criterion in the strategy of choosing the playing systems.

The criterion of winning probability, along with the criterion of possible multiple winnings relate to the goal of the game and are objective criteria due to their mathematical essence. In the case of systems, there are also financial criteria, related to the means of achieving the goal of the game. These criteria take into account the cost of the playing systems, depending upon the number of constituent simple lines.

All parameters taken into account within a strategy of choosing the playing system are correlated. For example, the probability of winning in a certain category cannot be raised in any level without raising the total cost of the system, which is limited by the investment fund at player's disposal).

For the sake of simplicity, we shall refer hereafter to one category of winning; the results can be extended for the case of cumulated winnings. This choice is natural, because in a personal strategy of choosing, the player follows particularly a certain category of winning, the eventual additional winnings in other lower or higher categories being optional but welcome.

The next analysis uses extensively the theoretical results and applications obtained in previous chapters.

If the player holds the winning probability (achieving maximal probability for the same cost), as the sole criterion of the strategy of choosing, he or she may choose an aleatory system created under the condition of exclusiveness, because such a system offers the maximal probabilities of winning for the same number of constituent simple lines. In addition, the parameters of such system can be managed easily in correlation with the aimed probability.

Having chosen a threshold of the winning probability and the corresponding number of lines that will verify the profitability condition (see section titled *Probability thresholds* in the chapter titled *Enhancing the Winning Probability*), the player can build the playing system by using the sufficient condition for exclusiveness that refers to the number of common numbers (see section titled *Probabilities of winning with systems* in the same chapter). The generation of the aleatory system can be done manually or with the help of a software program.

Within a strategy of choosing based on probability, it is not recommended to play an aleatory system that does not respect the exclusiveness condition, it is preferable to play a compound line or a reduced system instead. This recommendation is made because for these types of systems, we know the values of the winning probabilities (from formulas, tables or approximations) and the number of possible multiple winnings (the winning guarantees, in case of the reduced systems).

If the player has the number of possible multiple winnings (the most possible winnings for the same cost of the system) as the sole criterion of the strategy of choosing, obviously he or she should play a compound line, whose size should be chosen according to his or her own financial limitations, because the compound lines offers the largest numbers of possible multiple winnings (see section *The number of prizes* in the chapter *Compound Lines*).

Of course, depending on the chosen criteria and their method of aggregation, there is, in addition to aleatory systems and compound lines, also the option of playing reduced systems (including here the bridgehead lines as well).

In the following section, we present a comparative analysis of the three types of systems – aleatory systems under the exclusiveness condition, compound lines, and reduced systems – for two situations of aggregation of the criteria for choosing (the winning probability, the number of possible multiple winnings, and the total cost of the system):

a) *winning probability* and *number of possible multiple winnings* for the same total cost of the system (thus the same number of played simple lines);

b) *number of possible multiple winnings* and *total cost of the system* for the same winning probability of the system.

This analysis presents the order relation between the values of the parameters; the statements are direct consequences of the theoretical results obtained in previous chapters.

For the parameters we use the following denotations:

P_1 - the winning probability of the aleatory system under the exclusiveness condition;

P_2 - the winning probability of the compound line;

P_3 - the winning probability of the reduced system;

N_1 - the number of possible multiple winnings of the aleatory system under the exclusiveness condition;

N_2 - the number of possible multiple winnings of the compound line;

N_3 - the number of possible multiple winnings of the reduced system;

C_1 - the total cost of the aleatory system under the exclusiveness condition;

C_2 - the total cost of the compound line;

C_3 - he total cost of the reduced system.

(Probabilities P_i and numbers N_i, $i = 1, 2, 3$, are corresponding to the same winning category fixed through the simplification convention mentioned before.)

We have respectively the following relations between parameters P_i, N_i, C_i, $i = 1, 2, 3$:

a) for the same total cost of the systems ($C_1 = C_2 = C_3$)

$P_1 > P_3 > P_2$

$N_2 > N_3 > N_1 = 1$

The aleatory system under the exclusiveness condition offers the maximal probability of winning, while the compound line offers the minimal. However, the largest number of possible multiple winnings is offered by the compound line, and the smallest (namely 1) by the aleatory system under the exclusiveness condition. The reduced system (obtained from a compound line other than the one which we compare with) holds the intermediary values of the two parameters.

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We can observe that in both situations, the reduced system could offer a compromise between the other two types of systems, which hold the minimal and maximal values for each of the parameters that comprise the decision criteria.

All of the strategies involved in the lotto game, either subjective or objective, are built by every player according to his or her personal needs, goals, means through which these goals can be achieved, and last but not least, a player's own convictions and gaming behaviors. As we mentioned before, any strategy is personal and predominantly subjective.

This is also the reason this chapter does not make absolute strategic recommendations. The correlation of all parameters considered within the criteria of a strategy of choosing, from choosing the lottery matrix and the numbers to play, to creating a playing system, represents a action of the player with a strictly personal motivation.

None of the information in this book, though obtained through exact mathematical computations, can create an optimum playing strategy, even at the choice level, simply because such a strategy does not exist..

On the other hand, this information can motivate players to better manage their own playing actions and personal playing systems and to confront their own convictions (often based on false perceptions specific to the psychology of gambling) with the incontestable reality provided by the direct applications of the mathematical model of the lottery game.

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