ROULETTE ODDS AND PROFITS

The Mathematics of Complex Bets

$$\Sigma$$

$$\Pi$$

Cătălin Bărboianu

Cătălin Bărboianu
PO Box 1 – 230
Craiova, Romania
http://probability.infarom.ro

INFAROM Publishing
Pure and applied mathematics
office@infarom.com
http://books.infarom.ro
http://www.infarom.com
http://www.infarom.ro

ISBN 973-87520-6-x

Publisher: **INFAROM**
Author and translator: **Cătălin Bărboianu**
Correction Editor: **Sara M. Stohl**

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Introduction

Roulette has long been the most popular game of chance. Its popularity comes not only from its history and rules, but is also related to its mathematics.

The first form of roulette was devised in 18th century France.

The game has been played in its current form since as early as 1796 in Paris. The earliest description of the roulette game in its current form is found in a French novel “La Roulette, ou le Jour” by Jaques Lablee, which describes a roulette wheel in the Palais Royal in Paris in 1796. The book was published in 1801. An even earlier reference to a game of this name was published in regulations for French Quebec in 1758.

In 1842, Frenchmen François and Louis Blanc added the “0” to the roulette wheel in order to achieve a house advantage. In the early 1800s, roulette was brought into the U.S. where, to further increase house odds, a second zero, “00”, was introduced. In some forms of early American roulette wheels, there were numbers 1 through 28, plus a single zero, a double zero, and an American Eagle. The payout on any of the numbers including the zeros and the eagle was 27 to 1. In the 1800s, roulette spread all over both Europe and the U.S., becoming one of the most famous and most popular casino games.

A legend tells François Blanc supposedly bargained with the devil to obtain the secrets of roulette. The legend is based on the fact that the sum of all the numbers on the roulette wheel (from 1 to 36) is 666, which is the “Number of the Beast”.

Of course, this history has nothing to do with the mathematics of roulette, although roulette offers the most relevant experiments in demonstrating the basic applications of probability theory. Added to which, the interpretations of probability regarding predictions have always carried a sense of the mystical.

One thing that makes roulette so popular with gamblers is the game’s transparency. All its parts are in view: the numbers on the table, on the wheel, the ball spinning and landing; no hidden cards to guess, no opponents to read their intentions, no strategy to influence the course of each game. We just place our bet and wait for the ball to land.

This transparency also allows for easy calculation of the odds involved, which also contributes to the game's popularity.

Unlike poker, in which only mathematicians can calculate the odds of some categories of events, in roulette any player can quickly calculate and memorize the probabilities of winning and losing any simple bet, and even some complex bets.

Still, for a proper betting system (one that is non-contradictory and profitable), not only the odds must be known, but also the right structure of the complex bets and the stake management.

And that is what this book deals with.

Any roulette player who plays regularly knows that each payout is approximately inversely proportional to the probability of winning that bet, regardless of the house edge. This makes roulette a fair enough game.

Another important attraction is the unrestricted possibility of combinations for the bets placed on the table.

A player may place bets wherever he or she wants, according to his or her own betting systems and criteria.

Some of the improved complex bets can increase the winning probability to over 90%, as you will see in this book. This is a winning probability you won't see in other game of chance.

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If this does not happen, a player may try to run complex bets by correlating their parameters (probabilities, basic stakes, profits and losses), and by factoring in the player's personal playing criteria to achieve regular profits in the short and medium terms.

We take that approach in this book: identify the complex bets that increase the overall winning probability, find the proper correlations between their parameters for the bets being non-contradictory and profitable and list all the numerical results in tables. From those tables, a player can choose the bets that fit the best his or her playing criteria.

This is not a roulette strategy book because such a strategy does not exist: only betting systems exist.

It is rather a collection of odds and figures attached to a large range of complex bets, revealed in their entire mathematical structure. This book provides just mathematical facts and not so-called winning strategies.

The structure and content of the major chapters follow.

The Rules of Roulette

This chapter gives readers the entire ensemble of rules of roulette: structure, betting, categories of bets, and payouts.

Supporting Mathematics

Here, the actions of roulette are converted into probability experiments that generate aleatory events. You will see the sample space, the field of events and the probability space in which the numerical probabilities of roulette are worked out.

We also present the probability properties and formulas used, as theoretical support.

In addition, you will see a mathematical model of a bet and the parameters a bet depends on.

We define complex bets and improved complex bets with respect to probability and to all parameters involved in a bet.

Anyone with a minimal mathematical background can follow this chapter because it requires only basic arithmetic and algebraic skills. On the other hand, readers who are only interested in direct results can skip this chapter and go to the tables of results to come.

Further, the next chapters present specific **improved complex bets**, each chapter dealing with one category of such bets.

All numerical results are presented in tables and the probability values are worked out for both American and European style roulette.

Repeated bets

The last chapter deals with repeated bets in the various categories, including the martingale, and lists the probabilities for each of the possible events involved in runs up to 100.

The Rules of Roulette

Roulette is a simple, easy to learn game. It offers a wide variety of bets and combinations of bets.

The Roulette wheel has 36 numbers from 1 to 36, a “0”, and usually a “00”. Most U.S. casinos have a “00” as well as the “0”, so they have 38 numbers. This is called American roulette.

Most European casinos have only “0”, without “00”, so they have 37 numbers. This is called European roulette.

The players place bets on numbers or groups of numbers; a player’s goal is to predict the winning number or other of properties of that number (colour, evenness, size, or place on the roulette table).

Each game consists of placing bets and waiting for a number, which is randomly generated by a spinning ball coming to rest inside a disk on which numbers are inscribed (the roulette wheel) that is also spinning, but in the opposite direction.

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Depending on how the chips should be placed on roulette table, these bets are of two categories: inside and outside bets.

The next two tables note all possible bets, along with their brief description and corresponding payouts.

Inside bet	Description	Payout
Straight Up	A bet directly on any single number	35 to 1
Split Bet	A bet split between any two numbers	17 to 1
Street Bet	A bet on a row of 3 numbers	11 to 1
Corner Bet	A bet on 4 numbers	8 to 1
Line Bet	A bet on 6 numbers over 2 rows	5 to 1

Outside bet	Description	Payout
Column Bet	A bet covering 12 numbers from a column	2 to 1
Dozen Bet	A bet covering a set of 12 numbers low (1-12) mid (13-24) and high (25-36)	2 to 1
Colour Bet	A bet on either red or black	1 to 1
Even/Odd Bet	A bet on either even or odd	1 to 1
Low/High Bet	A bet on either 1-18 or 19-36	1 to 1

The payout is written in odds format and represents the coefficient to multiply the stake of a won bet.

For example, if you bet $3 on a column (payout 2 to 1) and a number from that column wins, you will receive $3 x 2 = $6, together with your initial $3 stake.

If you place a split bet (payout 17 to 1) of $2 and one of those two numbers wins, you will receive $2 x 17 = $34, together with your initial $2 stake.

If you place a colour bet (payout 1 to 1) and your colour wins, you will get your stake plus an amount equal to that stake.

The next figure gives examples of how the various bets are placed on a roulette table.

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The Supporting Mathematics

The application of probability theory in gambling is a simple process because a finite sample space can be attached to any game of chance. In some games, probability calculations for some events can become harder because of their structure, but applying the theory is very natural and simple everywhere in this field.

The finite sample space and the randomness of the events (whether it is about rolling dices, drawing cards or spinning a wheel) allow us to build a simple probability model to work within to find the numerical probabilities of the events involved in that game.

This model assumes a finite probability field in which the field of events is the set of parts of the sample space (and, implicitly, is finite) and the probability-function is given by the classical definition of probability.

In this probability field, any event, no matter how complex, can be decomposed into elementary events.

Therefore, finding the probability of a compound event means applying some properties of probability and doing some algebraic calculations.

Among all games of chance, roulette holds the distinction of being the easiest game with respect to probability calculations. This is because the elementary events are one-dimensional elements, the numbers on the roulette wheel. Even dice do not allow easier calculations because games that require the use of dice deal with combinations of numbers.

At the opposite pole, card games (draw poker, for example) are known for their hardest-to-calculate probabilities.

In roulette, anyone with a minimal mathematical background can perform its probability applications and calculations. All the basic calculations involve only arithmetic and basic algebraic operations, but at some point some problems become a matter of math skill, especially those involving repeated events.

For those interested in improving their probability calculus skills and figuring out correct probability results for any game of chance, we recommend the beginner's guide, *Understanding and Calculating the Odds*, which is full of gambling applications.

The math chapter is devoted principally to the mathematical model of a roulette bet: the definition of complex bets, the profit function and its properties, and the equivalence relation between bets and its properties.

Let us see now how probability theory can be applied in Roulette and how the numerical probability results from this book were obtained.

The probability space

As in every game of chance, we are interested in making predictions for the events regarding the outcomes of roulette.

In roulette, there are no opponents or a dealer in the game, so the only events to deal with are the outcomes of the machine—the roulette wheel.

These events can be described as the occurrences of certain numbers or groups or numbers having a specific property (colour, evenness, size, or place on the roulette table).

Every spin of the wheel is an experiment generating an outcome: a number from 1 to 36 plus 0 (in European roulette), or plus 0 and 00 (in American roulette).

The set of these outcomes is the *sample space* attached to this experiment.

The sample space is the set of all elementary events (i.e., events than cannot be decomposed as an union of other non-empty events).

It is natural to take as the elementary events any number that could occur as the result of a spin.

This choice is convenient because it allows us make the following idealization: *the occurrences of the elementary events are equally possible*.

In our case, the occurrence of any number is possible in the same measure (if we assume a random spin and nonfraudulent conditions).

Without this *equally possible* idealization, the construction of a probability model to work within is not possible.

We have established the elementary events and the sample space attached to a spin as being the set of all possible elementary events.

Thus, the sample space in European roulette is the set:
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36}
and the sample space in American roulette is the set:
{00, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36},
which are finite sets.

The field of events is then the set of parts of the sample space and is implicitly finite.

As a set of parts of a set, the field of events is a Boole algebra.

Any event belonging to the field of events, no matter how complex, can be decomposed as a union of elementary events.

Because the events are identified with sets of numbers and the axioms of a Boole algebra, the operations between events (union, intersection, complementary) revert to the operations between sets of numbers.

Therefore, any counting of elementary events (for example, the elementary events a compound event consists of) reverts to counting numbers.

As examples:

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The probability properties and formulas used

Because we deal in all applications with a finite probability space with equally possible elementary events, the probability calculus uses a few basic properties of probability, starting with the classical definition:

(F1) $P = m/n$ (the probability of an event is the ratio between the number of cases favorable for that event to occur and the number of equally possible cases) (the classical definition of probability)

This formula is used on a large scale throughout the book, especially to calculate probabilities of events involving simple bets. It is applied by dividing the number of favorable numbers for a respective event to occur by the number of all possible numbers (37 or 38).

(F2) $P(A_1 \cup A_2) = P(A_1) + P(A_2)$, for any $A_1, A_2 \in \Sigma$ with $A_1 \cap A_2 = \phi$.

and its generalization:

(F3) $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$, for any finite family of mutually exclusive events $(A_i)_{i=1}^{n}$ (finite additivity in condition of incompatibility)

These properties of finite additivity were used to calculate the overall winning or losing probabilities for complex bets.

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(F8) Bernoulli scheme

Consider that n independent experiments are performed. In each experiment, event A may occur with probability p and does not occur with probability $q = 1 - p$.

We express the probability for event A to occur exactly m times in the n experiments.

Let B_m be the event *A occurs exactly* m *times in the* n *experiments.* We denote by $P_{m,n}$ the probability $P(B_m)$.

We then have:

$$P(B_m) = P_{m,n} = \underbrace{p^m q^{n-m} + \ldots + p^m q^{n-m}}_{C_n^m \ times} = C_n^m p^m q^{n-m} .$$

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Simple bets

When betting in roulette, we are interested in the winning or losing probability, and in the amount of profit we can gain or lose. These depend, of course, on the stake we put in. In fact, they depend on the basic stake we put on each placement.

The winning probability, losing probability, and possible profit and loss are objective criteria for a player when deciding the type of bet to make at a certain moment or what betting system to run.

Beside these objective criteria, there are also subjective criteria related to a player's personal gambling behavior. However, in this book we deal only with objective criteria, which are strictly related to mathematics.

We call a *simple bet* a bet that is made through a unique placement of chips on the roulette table.

So, if we place chips in any number or amount in a single place on the table, we have made a simple bet.

If the outcome after the spin is a number we have bet on, we win the bet and the profit on that bet, which is the stake multiplied by the payout. If the outcome is not favorable, we lose the stake. This applies to any simple bet made.

Therefore, all inside and outside bets described in the chapter titled *The Rules of Roulette* are simple bets.

The table below notes the winning probabilities for each category of simple bet, for both European and American roulette:

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The probability of winning a simple bet becomes $P(A) = \frac{|A|}{|R|}$,
where $|A|$ means the cardinality of the set A. Of course, $|R|$ could be 38 or 37, depending on the roulette type (American or European, respectively).

If a player places a simple bet $B = (A, p_A, S)$, there are two possibilities after the spin:

1) A number from A wins and the player makes a positive profit (gain) in the amount of $p_A S$ or

2) A number from $R - A$ wins and the player makes a negative profit, losing the stake S.

For a given simple bet B, we can define the following function:

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Complex bets

Roulette allows players to spread their chips anywhere on the table by following the rules of placement. In other words, a player can make simultaneously multiple placements of various stakes.

We can call *complex bets* these simultaneous placements. A complex bet consists of several placements of various stakes, so a complex bet is a family of simple bets.

The number of possible multiple placements is huge. If we identify a placement with the set of numbers it covers, this number is in fact the number of all subsets of the set $\mathcal{A}$.

While $\mathcal{A}$ has 154 elements, the number of its subsets is 2^{154}, which is a 47-digit number.

Mathematically, a complex bet can be described by the following:

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Observation 1:

If $B = (A_i, s_i)_{i \in I}$ is disjointed, then W_B is constant on each A_i.

The proof is obvious: Because sets A_i are mutually exclusive, an outcome $e \in A_i$ belongs only to A_i and does not belong to other sets A_j ($j \neq i$). Then, $W_B(e) = s_i p_i - \sum_{j \in I - \{i\}} s_j$ for every $e \in A_i$.

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Now let's consider the probabilities involved in a complex bet. First, what is the probability of winning at least one simple bet?

For a complex bet $B = (A_i, s_i)_{i \in I}$, the probability of winning at least one simple bet is $P\left(\bigcup_{i \in I} A_i\right)$.

Generally, the sets A_i are not mutually exclusive, so if we want to concretely calculate the probability of their union we must apply the inclusion-exclusion principle by considering all the intersections between them.

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Theorem A

If $\mathcal{S}$ is an algebra of sets and $(A_i)_{i \in I}$ is a finite family of sets from $\mathcal{S}$, then there exists a finite family $(E_j)_{j \in J}$ of mutually exclusive sets from $\mathcal{S}$ such that $\bigcup_{j \in J} E_j = \bigcup_{i \in I} A_i$ and, for every $j \in J$, we have:

1) E_j is included in one or more sets A_i;
2) If $E_j \not\subset A_i$, then $E_j \cap A_i = \phi$.

In other words, for any finite family of sets there exists a partition of their union such that each set of that partition is included in at least one set A_i and has no elements in common with any other set A_i that does not include it.

(A partition of a set is a family of mutually exclusive sets that exhausts the initial set.)

<u>Consequence of Theorem A</u>
Each set A_i is partitioned by a finite number of sets E_j.

We do not present the entire proof of this theorem here, but we detail the construction of a partition $\mathcal{P}$ that obeys the conditions of Theorem A. This construction is helpful in our applications in roulette:

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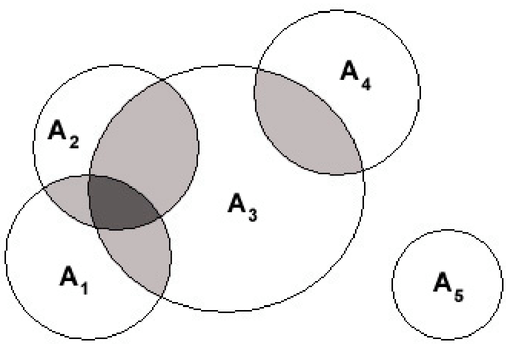
An example illustrating how the above procedure works in practice follows:



We have five sets A_i, which are not mutually exclusive, as seen in the figure. Among them, A_5 is of type 1 and the others are of type 2. So, we put A_5 in $\mathcal{P}$.

$A_1-(A_2\cup A_3\cup A_4\cup A_5)=A_1-(A_2\cup A_3)$, which is another set to put in $\mathcal{P}$.

$A_2-(A_1\cup A_3\cup A_4\cup A_5)=A_2-(A_1\cup A_3)$; put it in $\mathcal{P}$.

$A_3-(A_1\cup A_2\cup A_4\cup A_5)=A_3-(A_1\cup A_2\cup A_4)$; put it in $\mathcal{P}$.

$A_4 - (A_1 \cup A_2 \cup A_3 \cup A_5) = A_4 - A_3$; put it in $\mathcal{P}$.

Now move to the intersections between sets A_i:

$\Psi_2 = \{(A_1 \cap A_2), (A_2 \cap A_3), (A_1 \cap A_3), (A_3 \cap A_4)\}$

$\Psi_3 = \{(A_1 \cap A_2 \cap A_3)\}$

The maximum size of a non-empty intersection is 3.

Put $A_1 \cap A_2 \cap A_3$ in $\mathcal{P}$.

Now move to Ψ_2. The only 2-size intersection not containing the 3-size intersection is $A_3 \cap A_4$. Put it in $\mathcal{P}$.

For the remaining sets from Ψ_2, we have:

$(A_1 \cap A_2) \supset (A_1 \cap A_2 \cap A_3)$; put $(A_1 \cap A_2) - (A_1 \cap A_2 \cap A_3)$ in $\mathcal{P}$.

$(A_2 \cap A_3) \supset (A_1 \cap A_2 \cap A_3)$; put $(A_2 \cap A_3) - (A_1 \cap A_2 \cap A_3)$ in $\mathcal{P}$.

$(A_1 \cap A_3) \supset (A_1 \cap A_2 \cap A_3)$; put $(A_1 \cap A_3) - (A_1 \cap A_2 \cap A_3)$ in $\mathcal{P}$.

Cumulating all selected sets, we find the partition is:

$\mathcal{P} = \{A_5, A_1 - (A_2 \cup A_3), A_2 - (A_1 \cup A_3), A_4 - A_3, A_1 \cap A_2 \cap A_3,$
$(A_1 \cap A_2) - (A_1 \cap A_2 \cap A_3), (A_2 \cap A_3) - (A_1 \cap A_2 \cap A_3),$
$(A_1 \cap A_3) - (A_1 \cap A_2 \cap A_3)\}$

If sets A_i are finite, the procedure above does not need to be reproduced as theoretically described because the sets forming the partition can be easily visualized through Venn diagrams. Here are two examples:

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In this form, we can observe that W_B is well determined and is constant on E_j for every $j \in J$.

W_B is also constant on $R - \bigcup_{j \in J} E_j$, which is $R - \bigcup_{i \in I} A_i$ (and we observe that $W_B(e) = -\sum_{i \in I} s_i$ for $e \in R - \bigcup_{i \in I} A_i$).

Therefore, W_B is constant on each set of the partition consisting of $(E_j)_{j \in J}$ and $R - \bigcup_{i \in I} A_i$. That means it is a stair function:

$$W_B(e) = \sum_{t=1}^{k(j)} s_{j_t} p_{j_t} - \sum_{i \in I - \{j_1, ..., j_{k(j)}\}} s_i \text{, if } e \in E_j \text{ and}$$

$$W_B(e) = -\sum_{i \in I} s_i \text{, if } e \in R - \bigcup_{i \in I} A_i \text{ .}$$

Observation 3

If $(E_j)_{j \in J}$ is a partition of R and $W_B(e) = w_j$ for every $e \in E_j$, then the mathematical expectation of the profit of bet B is:

$$M(B) = \sum_{j \in J} \frac{|E_j|}{|R|} \cdot w_j \text{ .}$$

W_B being a stair function, its stair form allows us to calculate directly the probability of this function to take non-negative values:

On each E_j, W_B takes the same value (that is well determined), which could be negative or non-negative.

On $R - \bigcup_{i \in I} A_i$, W_B is negative.

Thus, to evaluate the probability of making a non-negative profit for a given complex bet B, we must identify those sets E_j on which W_B takes non-negative values. Let those be $E_1, ..., E_p$.

The probability to be evaluated is then

$$P\left(\bigcup_{j=1}^{p} E_j\right) = \sum_{j=1}^{p} P(E_j) = \frac{\sum_{j=1}^{p} |E_j|}{|R|}$$

Of course, we know that sets E_j are mutually exclusive, which allows us to sum their probabilities.

The algorithm is then: take the sets E_j on which the profit function takes non-negative values, add their cardinalities and divide the result to 37 or 38 (for either European or American roulette).

Examples:

1) $B = \{(1stD, 5), (High, 7), (sp\{18, 21\}, 2)\}$ in American roulette.

The complex bet B consists of three placements:
first dozen with stake 5
high numbers with stake 7
split on 18 and 21 with stake 2

(These stakes are noted without any unit of measure; players can use any currency or number of chips.)

Let us calculate the probability of making a non-negative profit.

We have three A_i sets, with corresponding payouts and stakes:

$A_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, $p_1 = 2$, $s_1 = 5$

$A_2 = \{19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36\}$

$p_2 = 1$, $s_2 = 7$

$A_3 = \{18, 21\}$, $p_3 = 17$, $s_3 = 2$

Observe that the sets A_i are not mutually exclusive because $A_2 \cap A_3 = \{21\}$.

Now we choose the partition of the coverage as follows:

$E_1 = A_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$E_2 = A_2 - (A_2 \cap A_3) =$

$= \{19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36\}$

$E_3 = A_2 \cap A_3 = \{21\}$

$E_4 = A_3 - (A_2 \cap A_3) = \{18\}$.

The sets E_j, $j = 1, 2, 3, 4$ are mutually exclusive and exhaust the coverage of B.

The numbers from E_1 belong to A_1.

The numbers from E_2 belong to A_2.

The numbers from E_3 belong to A_2 and A_3.

The numbers from E_4 belong to A_3.

On E_1, $W_B(e) = p_1 s_1 - s_2 - s_3 = 10 - 7 - 2 = 1$ (player wins 1).

On E_2, $W_B(e) = p_2 s_2 - s_1 - s_3 = 7 - 5 - 2 = 0$ (player wins and loses nothing).

On E_3, $W_B(e) = p_2 s_2 + p_3 s_3 - s_1 = 7 + 34 - 5 = 36$ (player wins 36).

On E_4, $W_B(e) = p_3 s_3 - s_1 - s_2 = 34 - 5 - 7 = 22$ (player wins 22).

On $R - (E_1 \cup E_2 \cup E_3 \cup E_4)$, $W_B(e) = -s_1 - s_2 - s_3 = -14$ (player loses 14).

The probability of making a non-negative profit is:

$$P(e \in R, W_B(e) \geq 0) = \frac{|E_1| + |E_2| + |E_3| + |E_4|}{|R|} =$$

$$= \frac{12 + 17 + 1 + 1}{38} = 81.57\%$$

The probability of losing is:

$$P(e \in R, W_B(e) < 0) = \frac{|R - (E_1 \cup E_2 \cup E_3 \cup E_4)|}{|R|} = \frac{7}{38} = 18.42\%$$

…………………… missing part ……………………

Mathematically speaking, if we have two partitions $(E_i)_{i \in I}$ and $(E_i')_{i \in I}$ of R, $E_i, E_i' \in \mathcal{A}$ and $W_B = w_i$ on E_i, $W_{B'} = w_i'$ on E_i' for every $i \in I$, and if $w_i = w_i'$ for every $i \in I$, then we say the bets B and B' are equivalent.

It is easy to verify that relation "~" is an equivalence relation on the sets of all possible bets $\mathcal{B}$, which means it is reflexive ($B \sim B$), symmetric (if $B \sim C$ then $C \sim B$) and transitive (if $A \sim B$ and $B \sim C$, then $A \sim C$).

Examples:

1) Bets $B = (sp\{16, 17\}, S)$ and $B' = (sp\{25, 28\}, S)$ are equivalent.

B and B' are simple bets with similar placements and identical stakes.

On $\{16, 17\}$, $W_B = 17S$

On $R - \{16, 17\}$, $W_B = -S$

On $\{25, 28\}$, $W_B' = 17S$

On $R - \{25, 28\}$, $W_{B'} = -S$

The conditions from the definition of equivalent bets are satisfied (the profit functions take the same values, respectively, on sets of the same length).

Generally, two simple bets having placements of the same length and the same stake are equivalent.

2) Bets $B = (sp\{19, 22\}, 2S)$ and $B' = (sp\{36\}, S)$ are not equivalent.

On $\{19, 22\}$, $W_B = 34S$

On $R - \{19, 22\}$, $W_B = -2S$

On $\{36\}$, $W_{B'} = 34S$

On $R - \{36\}$, $W_{B'} = -S$

The two partitions do not have sets of the same length. Even if we wanted to break down the partitions to obtain the same number of sets for each, we would always have different values for the profit functions. Therefore, bets B and B' are not equivalent.

The following is easily provable:

…………………. missing part ………………….

<u>Statement 4</u>

Two disjointed complex bets $B = (A_i, s_i)_{i \in I}$ and $B' = (A_i', s_i)_{i \in I}$ for which $|A_i| = |A_i'|$ for every $i \in I$ are equivalent.

In other words, if two disjointed bets have placements of equal length and the same basic stakes, respectively, then they are equivalent.

The proof is immediate:

The two families of placements form two partitions of the coverages of bets B and B' (because they are mutually exclusive).

We also have that $p_{A_i} = p_{A_i'}$ for any i (because the two placements have the same length). Then:

On A_i, $W_B = p_{A_i} s_i - \sum_{j \in I - \{i\}} s_j = p_{A_i'} s_i - \sum_{j \in I - \{i\}} s_j$.

The right-hand member is exactly the value of $W_{B'}$ on A_i' (which has the same length as A_i).

On $R - \bigcup_{i \in I} A_i$, $W_B = -\sum_{i \in I} s_i$. Function $W_{B'}$ takes the same value on $R - \bigcup_{i \in I} A_i'$ (which has the same length as $R - \bigcup_{i \in I} A_i$).

Resulting in $B \sim B'$.

If we waive the condition of the two bets being disjointed, the above statement is no longer true. Here is a counterexample:

5) Bets $B = \{(\{12\}, S), (sp\{12, 15\}, 2S)\}$ and $B' = \{(\{17\}, S), (sp\{29, 30\}, 2S)\}$ are not equivalent.

Their placements have the same length and same stakes, respectively, B' is disjointed, while B is not.

For bet B:

On $\{12\}$, $W_B = 35S + 34S = 69S$

On $\{15\}$, $W_B = 34S - S = 33S$

On $R - \{12, 15\}$, $W_B = -S - 2S = -3S$

For bet B':

On $\{17\}$, $W_{B'} = 35S - 2S = 33S$

On $\{29, 30\}$, $W_{B'} = 34S - S = 33S$

On $R - \{17, 29, 30\}$, $W_{B'} = -S - 2S = -3S$

Observe that the value $33S$ taken by W_B cannot be taken by $W_{B'}$ (except if $S = 0$, which is impossible). This results in B and B' not being equivalent.

…………………… missing part ……………………

Generally, we have the following:

Statement 5

Let $B=(A_1, S)$ be a simple bet and let $A_2, A_3 \in \mathcal{A}$ such that they form a partition of A_1 ($A_1 = A_2 \cup A_3$ and $A_2 \cap A_3 = \phi$). Then:

$$(A_1, S) \sim \{(A_2, T), (A_3, R)\} \text{ if and only if}$$

$S = T + R$ and $\dfrac{T}{R} = \dfrac{p_3+1}{p_2+1}$ (p_i is the payout of A_i).

Proof:

If we denote by B' the bet from the right-hand member of the equivalence relation, we have:

For bet B:

On A_2, $W_B = p_1 S$

On A_3, $W_B = p_1 S$

On $R-(A_2 \cup A_3)$, $W_B = -S$

…………………. missing part ………………….

Statement 6

Let $B=(A_i, s_i)_{1\le i\le n}$ be a complex bet. If $A_k = A_{k_1} \cup A_{k_2}$ is a partition of A_k with $A_{k_1}, A_{k_2} \in \mathcal{A}$ and if $(A_k, s_k) \sim \{(A_{k_1}, t_1), (A_{k_2}, t_2)\}$, then:

$$B \sim \{(A_1, s_1), \ldots, (A_{k-1}, s_{k-1}), (A_{k_1}, t_1), (A_{k_2}, t_2), (A_{k+1}, s_{k+1}), \ldots, (A_n, s_n)\}$$

Proof:

According to Statement 5, the equivalence from the hypothesis implies: $s_k = t_1 + t_2$ and $p_{k_1} t_1 - t_2 = p_{k_2} t_2 - t_1 = p_k s_k$.

Denoting by B' the bet from the right-hand member of the equivalence at the conclusion of the statement, we consider a partition $(E_j)_{j=1}^m$ of the coverage of B', according to the construction proving theorem A.

Let E_j be an arbitrary set from this partition.

We have three cases: $E_j \subset A_{k_1}$, $E_j \subset A_{k_2}$ or $E_j \subset R - \left(A_{k_1} \cup A_{k_2}\right)$.

If $E_j \subset A_{k_1}$, then $E_j \subset A_k$.

For any $e \in E_j$, we have that $e \notin A_{k_2}$.

…………………. missing part ………………….

The direct consequences of Statement 5 and Statement 6 are the following equivalences between simple bets:

$(spl\{a, b\}, 2S) \sim \{(\{a\}, S), (\{b\}, S)\}$
$(str\{a, b, c\}, 3S) \sim \{(spl\{a, b\}, 2S), (\{c\}, S)\} \sim \{(\{a\}, S), (\{b\}, S), (\{c\}, S)\}$
$(cor\{a, b, c, d\}, 4S) \sim \{(spl\{a, b\}, 2S), (spl\{c, d\}, 2S)\} \sim \{(\{a\}, S), (\{b\}, S), (spl\{c, d\}, 2S)\} \sim \{(\{a\}, S), (\{b\}, S), (\{c\}, S), (\{d\}, S)\}\}$
$(lin\{a, b, c, d, e, f\}, 6S) \sim \{(str\{a, b, c\}, 3S), (str\{d, e, f\}, 3S)\} \sim \{(spl\{a, b\}, 2S), (\{c\}, S), (str\{d, e, f\}, 3S)\}$ and so on.

In above relations, the small caps letters represent numbers on roulette table, such positioned for the above bets to make sense.

The following sentence is not true:

"If in a complex bet we replace a simple bet with another simple bet that is equivalent to the one replaced, the newly obtained complex bet is equivalent to the initial complex bet".

Example (Exercise):

Check that $(\{1, 2\}, S) \sim (\{2, 3\}, S)$, but $\{(\{1, 2\}, S), (\{3\}, T)\}$ is not equivalent to $\{(\{2, 3\}, S), (\{3\}, T)\}$.

…………………. missing part ………………….

Statement 9

For any complex bet B, there exists a disjointed bet B' such that $B \sim B'$.

Proof:

This statement is a direct consequence of Statement 5 and Statement 6.

Consider a partition $(E_j)_{j \in J}$ of the coverage of B obeying the conditions from Theorem A.

Each placement A_i of B is a union of a finite number of sets E_j, which form a partition of A_i.

Extend this partition as follows: Leave E_j as it is if $E_j \in \mathcal{A}$ and break E_j into mutually exclusive sets from $\mathcal{A}$ if $E_j \notin \mathcal{A}$ (this is possible because we can go down to sets consisting of one element that belong to $\mathcal{A}$). Let these be $E_{j_1}, ..., E_{j_{k(j)}}$.

…………………… missing part ……………………

This ends the mathematical presentation of complex bets and their properties. We have covered the construction of a rigorous mathematical model for roulette bets and provided the theoretical results necessary to identify several categories of bets and the relations between them with respect to profits and probabilities.

The whole theory goes farther with other structural properties of equivalent bets and even composition laws on the set $\mathcal{B}$ of the bets, and would fill an additional few dozens of pages.

The practical aspect of the application of the theory presented in this chapter is related to player's options in choosing the bet to run.

Improved bets

In the definition of the equivalence between bets we used the profit function, whose values are in fact possible wins and losses.

The length of a set on which the two profit functions take the same values reverts to probability. Thus, the definition catches every practical aspect of a real equivalence between bets: an equal amount to win, an equal amount to lose and equal probabilities of these happening.

These are the main objective criteria a player considers when putting an equal sign between two bets.

The equivalence relation between bets creates classes of equivalence on the set of all possible bets and generates a partition of this set (the equivalence class of a bet $B \in \mathcal{B}$ is the set of all bets B' that are equivalent to B and is denoted by $\hat{B}$; any element of $\hat{B}$ is called a *class representative* of $\hat{B}$).

Thus, set $\mathcal{B}$ is the union of all equivalence classes, and the area of choice for a bet from it is reduced to a choice between class representatives.

We also saw that each equivalence class holds a disjointed bet, so within a class we can choose a disjointed bet as the class representative. This choice is explained further.

We estimated 154 possible placements as elements of $\mathcal{A}$ (the set of allowed unique placements). The number of all possible placements for a complex bet is the number of all subsets of $\mathcal{A}$, which is 2^{154}, a 47-digit number.

So, ignoring the stakes, a player has 2^{154} possible bets (placements) to choose from for each spin.

This number of choices can be reduced significantly if we count only the mutually exclusive placements. This reduction is natural because of Statement 9, which says that any bet is equivalent to a disjointed bet.

The number of all mutually exclusive placements is in fact the number of partitions of each subset of R, all added.

This is the sum of all Bell numbers from $B(1)$ to $B(38)$ for American roulette (the Bell number $B(n)$ is the number of partitions of a set with n elements) and from $B(1)$ to $B(37)$ for European roulette.

Of course, this new number is still huge, but it has only about 36 digits instead of 47, which was the digit length of the first area of choice.

The number of choices is much fewer if we consider the equivalence between placements given by their length (two unique placements are equivalent if they have the same length and two multiple mutually exclusive placements are equivalent if they consist of unique placements of equal length, respectively). This equivalence will reduce the number of digits somewhat, but the number of choices still remains huge.

…………………… missing part …………………

Each criterion is listed along with the possible transformations upon the structure of the initial bet (placements and/or stakes, denoted by *P* or, respectively, *S*), the type of choice involved (over the classes, denoted by *C*, or over the representatives within a class, denoted by *R*) and its type (objective, subjective or circumstantial, denoted by *Obj*, *Sub*, or *Cir*).

Criterion	Change	Choice	Type
probability of a positive profit	P, S	C	Obj
probability of losing	P, S	C	Obj
the coverage	P	C	Obj
amount to win	P, S	C	Obj
amount to lose	P, S	C	Obj
covering “lucky” numbers	P	C, R	Sub
long-run expectation	P, S	C	Obj
amount at disposal	S	C	Obj
time to play for an expected profit	P, S	C	Obj
divisions of chips	P, S	C, R	Cir
avoiding certain numbers on the basis of statistical frequency	P	C, R	Sub
covering certain numbers on the basis of statistical frequency	P	C, R	Sub
maximum amount allowed for a bet	S	C	Cir
money management on long, medium or short term	S	C	Sub

…………………. missing part ………………….

Everyone accepts that we have no rule over the hazard. Everyone also accepts that in long run, we will never take the house edge. This is true for any game of chance.

For roulette in particular, we can only govern our own choices in placing bets. Mathematics can help players organize their choices.

The application of theory of complex bets in roulette is justified because of a technical element of the game: the numbers on the roulette wheel do not have the same status as on the betting table.

While the numbers on the wheel are arranged in a circle and the possibility of the ball landing on one number is the same for any number, on the table we cannot cover any group of numbers with unique placements of same type. For example, while 7 and 8 can be covered with a split bet, 7 and 12 cannot. They can be covered with a line bet, a First Dozen or a Red bet, which have different payouts from a split bet. The numbers 1 and 35 can be covered only with an Odd bet, while 1 and 2 can be covered by many other inside bets. The numbers 1 and 26 cannot be covered with one unique placement.

Because of table configuration, not all numbers have the same status with respect to possible placements. The mathematics of complex bets takes into account this particular configuration.

We may say that this non-equivalence between roulette numbers is the only fact a player could speculate on in developing a strategy, but only in sense of using math to organize and improve a personal betting system to fit the player's subjective and objective criteria more accurately.

As mentioned earlier, the number of choices is huge. Still, the number of complex bets players typically run is limited. Improving these bets can lead to relations between the parameters of the bets, which mathematics can easily solve.

As an example, frequently practiced complex bets are those corresponding to partially complementary events (red + few black numbers, odd + few even numbers, and so on).

If we consider the outside bet the initial bet, adding those straight-up bets can increase the odds of winning, but it can also diminish the eventual amounts won at the same time.

Example:

A player bets amount $2S$ on the colour red and amount S on number 6 (black), at American roulette.

A. In the case of a black number different from 6 winning, the player loses $3S$.

B. In the case of number 6 winning, the player wins $35S - 2S = 33S$.

C. In the case of a red number winning, the player wins $2S - S = S$.

D. In the case of 0 or 00 winning, the player loses $3S$.

The probability of winning is
$P(B \cup C) = 1 - P(A \cup D) = 1 - 19/38 = 50\%$

If, along with the bet on red, several bets are placed on black numbers, the probability of winning will increase, but the eventual winning amount will decrease.

The optimum bet combinations are those having an acceptable ratio (arithmetically, but also according to a player's personal criteria) between the winning probabilities and the eventual winning and risked amounts.

If we improve the bet in this example to cover n black numbers, each with the same basic stake T, then, to choose a bet from this category, we must impose conditions on parameters S, T and n.

The first condition is that the bet be non-contradictory.

Then, other conditions will reflect player's personal criteria (related to probability, amounts, and so forth).

All these conditions will revert to relations between the parameters described earlier, which mean equations or inequations to solve.

The values found for S, T and n will determine the improved bet the player is looking for.

The following sections present some categories of improved bets, along with the probabilities of the events involved and the afferent winnings and losses.

Each section describes a type of improved bet with respect to the objective criteria of probability of winning and of losing.

The improvement aimed at enlarging coverage enough to raise the winning probability and sufficient for the bet to avoid becoming contradictory.

Most of the sections describe disjointed bets. The act of choosing a disjointed bet within an equivalence class is based on the following strategic reasoning: while two equivalent bets in which one is disjointed have the same coverage, the non-disjointed bet involves numbers that are covered multiple times (by two or more unique placements). By choosing the disjointed bet, we can save a portion of the total stake for another bet placed at the same moment, or for a future spin. This improvement is based on criteria such as money management and enlarging the coverage.

The conditions imposed on the parameters leave a large enough range of values to allow players to choose from several other sub-categories of bets, all of which are listed in tables, along with probabilities of all the events involved and amounts to win or to lose.

The final choice within these sub-categories of bets is a matter of the player's personal strategy.

The limitations are the result of using the same basic stake for multiple similar simple bets within a complex bet, and a small increment for the ratios between the stakes.

…………………. missing part ………………….

Betting on a colour and on numbers of the opposite colour

This complex bet consists of a colour bet (payout 1 to 1) and several straight-up bets (payout 35 to 1) on numbers of the opposite colour.

To generalize the example from the previous section, let us denote by S the amount bet on each number, by cS the amount bet on the colour and by n the number of bets placed on single numbers (the number of straight-up bets).

S is a positive real number (measurable in any currency), the coefficient c is also a positive real number and n is a non-negative natural number (between 1 and 18 because there are 18 numbers of one colour).

The possible events after the spin are: A – winning the bet on colour, B – winning a bet on a number and C – not winning any bet.

These events are mutually exclusive and exhaustive, so:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 1$$

Now let us find the probability of each event and the profit or loss in each case:

A. The probability of a number of a certain colour winning is $P(A) = 18/38 = 9/19 = 47.368\%$.

In the case of winning the colour bet, the player wins $cS - nS = (c - n)S$, using the convention that if this amount is negative, that will be called a loss.

B. The probability of one of n specific numbers winning is $P(B) = n/38$.

In the case of winning a straight-up bet, the player wins $35S - (n - 1)S - cS = (36 - n - c)S$, using the same convention from event A.

…………………… missing part ……………………

It is natural to put the condition of a positive profit in both cases A and B, which results in: $n < c < 36 - n$.

This condition gives a relation between parameters n and c and restrains the number of subcases to be studied.

These formulas return the next tables of values, in which n increases from 1 to 17 and c increases by increments of 0.5.

S is left as a variable for players to replace with any basic stake according to their own betting behaviors and strategies.

Observation:

The same formulas and tables also hold true for the following complex bets: Even/Odd bet and straight-up bets on odd/even numbers; High/Low bet and straight-up bets on low/high numbers.

This happens because the bets are equivalent, respectively, if they have the same stakes.

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	c	Odds	Profit	Odds	Profit	Odds	Loss
1	1.5	47.36%	0.5 S	2.63%	33.5 S	50%	2.5 S
1	2	47.36%	S	2.63%	33 S	50%	3 S
1	2.5	47.36%	1.5 S	2.63%	32.5 S	50%	3.5 S
1	3	47.36%	2 S	2.63%	32 S	50%	4 S
1	3.5	47.36%	2.5 S	2.63%	31.5 S	50%	4.5 S
1	4	47.36%	3 S	2.63%	31 S	50%	5 S
1	4.5	47.36%	3.5 S	2.63%	30.5 S	50%	5.5 S
1	5	47.36%	4 S	2.63%	30 S	50%	6 S
1	5.5	47.36%	4.5 S	2.63%	29.5 S	50%	6.5 S
1	6	47.36%	5 S	2.63%	29 S	50%	7 S
1	6.5	47.36%	5.5 S	2.63%	28.5 S	50%	7.5 S
1	7	47.36%	6 S	2.63%	28 S	50%	8 S
1	7.5	47.36%	6.5 S	2.63%	27.5 S	50%	8.5 S
1	8	47.36%	7 S	2.63%	27 S	50%	9 S
1	8.5	47.36%	7.5 S	2.63%	26.5 S	50%	9.5 S
1	9	47.36%	8 S	2.63%	26 S	50%	10 S
1	9.5	47.36%	8.5 S	2.63%	25.5 S	50%	10.5 S
1	10	47.36%	9 S	2.63%	25 S	50%	11 S
1	10.5	47.36%	9.5 S	2.63%	24.5 S	50%	11.5 S
1	11	47.36%	10 S	2.63%	24 S	50%	12 S
1	11.5	47.36%	10.5 S	2.63%	23.5 S	50%	12.5 S
1	12	47.36%	11 S	2.63%	23 S	50%	13 S
1	12.5	47.36%	11.5 S	2.63%	22.5 S	50%	13.5 S
1	13	47.36%	12 S	2.63%	22 S	50%	14 S
1	13.5	47.36%	12.5 S	2.63%	21.5 S	50%	14.5 S
1	14	47.36%	13 S	2.63%	21 S	50%	15 S
1	14.5	47.36%	13.5 S	2.63%	20.5 S	50%	15.5 S
1	15	47.36%	14 S	2.63%	20 S	50%	16 S
1	15.5	47.36%	14.5 S	2.63%	19.5 S	50%	16.5 S
1	16	47.36%	15 S	2.63%	19 S	50%	17 S
1	16.5	47.36%	15.5 S	2.63%	18.5 S	50%	17.5 S
1	17	47.36%	16 S	2.63%	18 S	50%	18 S
1	17.5	47.36%	16.5 S	2.63%	17.5 S	50%	18.5 S
1	18	47.36%	17 S	2.63%	17 S	50%	19 S
1	18.5	47.36%	17.5 S	2.63%	16.5 S	50%	19.5 S
1	19	47.36%	18 S	2.63%	16 S	50%	20 S
1	19.5	47.36%	18.5 S	2.63%	15.5 S	50%	20.5 S
1	20	47.36%	19 S	2.63%	15 S	50%	21 S
1	20.5	47.36%	19.5 S	2.63%	14.5 S	50%	21.5 S
1	21	47.36%	20 S	2.63%	14 S	50%	22 S

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	**c**	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
1	21.5	47.36%	20.5 S	2.63%	13.5 S	50%	22.5 S
1	22	47.36%	21 S	2.63%	13 S	50%	23 S
1	22.5	47.36%	21.5 S	2.63%	12.5 S	50%	23.5 S
1	23	47.36%	22 S	2.63%	12 S	50%	24 S
1	23.5	47.36%	22.5 S	2.63%	11.5 S	50%	24.5 S
1	24	47.36%	23 S	2.63%	11 S	50%	25 S
1	24.5	47.36%	23.5 S	2.63%	10.5 S	50%	25.5 S
1	25	47.36%	24 S	2.63%	10 S	50%	26 S
1	25.5	47.36%	24.5 S	2.63%	9.5 S	50%	26.5 S
1	26	47.36%	25 S	2.63%	9 S	50%	27 S
1	26.5	47.36%	25.5 S	2.63%	8.5 S	50%	27.5 S
1	27	47.36%	26 S	2.63%	8 S	50%	28 S
1	27.5	47.36%	26.5 S	2.63%	7.5 S	50%	28.5 S
1	28	47.36%	27 S	2.63%	7 S	50%	29 S
1	28.5	47.36%	27.5 S	2.63%	6.5 S	50%	29.5 S
1	29	47.36%	28 S	2.63%	6 S	50%	30 S
1	29.5	47.36%	28.5 S	2.63%	5.5 S	50%	30.5 S
1	30	47.36%	29 S	2.63%	5 S	50%	31 S
1	30.5	47.36%	29.5 S	2.63%	4.5 S	50%	31.5 S
1	31	47.36%	30 S	2.63%	4 S	50%	32 S
1	31.5	47.36%	30.5 S	2.63%	3.5 S	50%	32.5 S
1	32	47.36%	31 S	2.63%	3 S	50%	33 S
1	32.5	47.36%	31.5 S	2.63%	2.5 S	50%	33.5 S
1	33	47.36%	32 S	2.63%	2 S	50%	34 S
1	33.5	47.36%	32.5 S	2.63%	1.5 S	50%	34.5 S
1	34	47.36%	33 S	2.63%	1 S	50%	35 S
1	34.5	47.36%	33.5 S	2.63%	0.5 S	50%	35.5 S
2	2.5	47.36%	0.5 S	5.26%	31.5 S	47.36%	4.5 S
2	3	47.36%	1 S	5.26%	31 S	47.36%	5 S
2	3.5	47.36%	1.5 S	5.26%	30.5 S	47.36%	5.5 S
2	4	47.36%	2 S	5.26%	30 S	47.36%	6 S
2	4.5	47.36%	2.5 S	5.26%	29.5 S	47.36%	6.5 S
2	5	47.36%	3 S	5.26%	29 S	47.36%	7 S
2	5.5	47.36%	3.5 S	5.26%	28.5 S	47.36%	7.5 S
2	6	47.36%	4 S	5.26%	28 S	47.36%	8 S
2	6.5	47.36%	4.5 S	5.26%	27.5 S	47.36%	8.5 S
2	7	47.36%	5 S	5.26%	27 S	47.36%	9 S
2	7.5	47.36%	5.5 S	5.26%	26.5 S	47.36%	9.5 S
2	8	47.36%	6 S	5.26%	26 S	47.36%	10 S
2	8.5	47.36%	6.5 S	5.26%	25.5 S	47.36%	10.5 S
2	9	47.36%	7 S	5.26%	25 S	47.36%	11 S

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	**c**	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
2	9.5	47.36%	7.5 S	5.26%	24.5 S	47.36%	11.5 S
2	10	47.36%	8 S	5.26%	24 S	47.36%	12 S
2	10.5	47.36%	8.5 S	5.26%	23.5 S	47.36%	12.5 S
2	11	47.36%	9 S	5.26%	23 S	47.36%	13 S
2	11.5	47.36%	9.5 S	5.26%	22.5 S	47.36%	13.5 S
2	12	47.36%	10 S	5.26%	22 S	47.36%	14 S
2	12.5	47.36%	10.5 S	5.26%	21.5 S	47.36%	14.5 S
2	13	47.36%	11 S	5.26%	21 S	47.36%	15 S
2	13.5	47.36%	11.5 S	5.26%	20.5 S	47.36%	15.5 S
2	14	47.36%	12 S	5.26%	20 S	47.36%	16 S
2	14.5	47.36%	12.5 S	5.26%	19.5 S	47.36%	16.5 S
2	15	47.36%	13 S	5.26%	19 S	47.36%	17 S
2	15.5	47.36%	13.5 S	5.26%	18.5 S	47.36%	17.5 S
2	16	47.36%	14 S	5.26%	18 S	47.36%	18 S
2	16.5	47.36%	14.5 S	5.26%	17.5 S	47.36%	18.5 S
2	17	47.36%	15 S	5.26%	17 S	47.36%	19 S
2	17.5	47.36%	15.5 S	5.26%	16.5 S	47.36%	19.5 S
2	18	47.36%	16 S	5.26%	16 S	47.36%	20 S
2	18.5	47.36%	16.5 S	5.26%	15.5 S	47.36%	20.5 S
2	19	47.36%	17 S	5.26%	15 S	47.36%	21 S
2	19.5	47.36%	17.5 S	5.26%	14.5 S	47.36%	21.5 S
2	20	47.36%	18 S	5.26%	14 S	47.36%	22 S
2	20.5	47.36%	18.5 S	5.26%	13.5 S	47.36%	22.5 S
2	21	47.36%	19 S	5.26%	13 S	47.36%	23 S
2	21.5	47.36%	19.5 S	5.26%	12.5 S	47.36%	23.5 S
2	22	47.36%	20 S	5.26%	12 S	47.36%	24 S
2	22.5	47.36%	20.5 S	5.26%	11.5 S	47.36%	24.5 S
2	23	47.36%	21 S	5.26%	11 S	47.36%	25 S
2	23.5	47.36%	21.5 S	5.26%	10.5 S	47.36%	25.5 S
2	24	47.36%	22 S	5.26%	10 S	47.36%	26 S
2	24.5	47.36%	22.5 S	5.26%	9.5 S	47.36%	26.5 S
2	25	47.36%	23 S	5.26%	9 S	47.36%	27 S
2	25.5	47.36%	23.5 S	5.26%	8.5 S	47.36%	27.5 S
2	26	47.36%	24 S	5.26%	8 S	47.36%	28 S
2	26.5	47.36%	24.5 S	5.26%	7.5 S	47.36%	28.5 S
2	27	47.36%	25 S	5.26%	7 S	47.36%	29 S
2	27.5	47.36%	25.5 S	5.26%	6.5 S	47.36%	29.5 S
2	28	47.36%	26 S	5.26%	6 S	47.36%	30 S
2	28.5	47.36%	26.5 S	5.26%	5.5 S	47.36%	30.5 S
2	29	47.36%	27 S	5.26%	5 S	47.36%	31 S

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	**c**	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
2	29.5	47.36%	27.5 S	5.26%	4.5 S	47.36%	31.5 S
2	30	47.36%	28 S	5.26%	4 S	47.36%	32 S
2	30.5	47.36%	28.5 S	5.26%	3.5 S	47.36%	32.5 S
2	31	47.36%	29 S	5.26%	3 S	47.36%	33 S
2	31.5	47.36%	29.5 S	5.26%	2.5 S	47.36%	33.5 S
2	32	47.36%	30 S	5.26%	2 S	47.36%	34 S
2	32.5	47.36%	30.5 S	5.26%	1.5 S	47.36%	34.5 S
2	33	47.36%	31 S	5.26%	1 S	47.36%	35 S
2	33.5	47.36%	31.5 S	5.26%	0.5 S	47.36%	35.5 S
3	3.5	47.36%	0.5 S	7.89%	29.5 S	44.73%	6.5 S
3	4	47.36%	1 S	7.89%	29 S	44.73%	7 S
3	4.5	47.36%	1.5 S	7.89%	28.5 S	44.73%	7.5 S
3	5	47.36%	2 S	7.89%	28 S	44.73%	8 S
3	5.5	47.36%	2.5 S	7.89%	27.5 S	44.73%	8.5 S
3	6	47.36%	3 S	7.89%	27 S	44.73%	9 S
3	6.5	47.36%	3.5 S	7.89%	26.5 S	44.73%	9.5 S
3	7	47.36%	4 S	7.89%	26 S	44.73%	10 S
3	7.5	47.36%	4.5 S	7.89%	25.5 S	44.73%	10.5 S
3	8	47.36%	5 S	7.89%	25 S	44.73%	11 S
3	8.5	47.36%	5.5 S	7.89%	24.5 S	44.73%	11.5 S
3	9	47.36%	6 S	7.89%	24 S	44.73%	12 S
3	9.5	47.36%	6.5 S	7.89%	23.5 S	44.73%	12.5 S
3	10	47.36%	7 S	7.89%	23 S	44.73%	13 S
3	10.5	47.36%	7.5 S	7.89%	22.5 S	44.73%	13.5 S
3	11	47.36%	8 S	7.89%	22 S	44.73%	14 S
3	11.5	47.36%	8.5 S	7.89%	21.5 S	44.73%	14.5 S
3	12	47.36%	9 S	7.89%	21 S	44.73%	15 S
3	12.5	47.36%	9.5 S	7.89%	20.5 S	44.73%	15.5 S
3	13	47.36%	10 S	7.89%	20 S	44.73%	16 S
3	13.5	47.36%	10.5 S	7.89%	19.5 S	44.73%	16.5 S
3	14	47.36%	11 S	7.89%	19 S	44.73%	17 S
3	14.5	47.36%	11.5 S	7.89%	18.5 S	44.73%	17.5 S
3	15	47.36%	12 S	7.89%	18 S	44.73%	18 S
3	15.5	47.36%	12.5 S	7.89%	17.5 S	44.73%	18.5 S
3	16	47.36%	13 S	7.89%	17 S	44.73%	19 S
3	16.5	47.36%	13.5 S	7.89%	16.5 S	44.73%	19.5 S
3	17	47.36%	14 S	7.89%	16 S	44.73%	20 S
3	17.5	47.36%	14.5 S	7.89%	15.5 S	44.73%	20.5 S
3	18	47.36%	15 S	7.89%	15 S	44.73%	21 S
3	18.5	47.36%	15.5 S	7.89%	14.5 S	44.73%	21.5 S

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	**c**	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
3	19	47.36%	16 S	7.89%	14 S	44.73%	22 S
3	19.5	47.36%	16.5 S	7.89%	13.5 S	44.73%	22.5 S
3	20	47.36%	17 S	7.89%	13 S	44.73%	23 S
3	20.5	47.36%	17.5 S	7.89%	12.5 S	44.73%	23.5 S
3	21	47.36%	18 S	7.89%	12 S	44.73%	24 S
3	21.5	47.36%	18.5 S	7.89%	11.5 S	44.73%	24.5 S
3	22	47.36%	19 S	7.89%	11 S	44.73%	25 S
…………………… missing part ……………………							
15	19	47.36%	4 S	39.47%	2 S	13.15%	34 S
15	19.5	47.36%	4.5 S	39.47%	1.5 S	13.15%	34.5 S
15	20	47.36%	5 S	39.47%	1 S	13.15%	35 S
15	20.5	47.36%	5.5 S	39.47%	0.5 S	13.15%	35.5 S
16	16.5	47.36%	0.5 S	42.10%	3.5 S	10.52%	32.5 S
16	17	47.36%	1 S	42.10%	3 S	10.52%	33 S
16	17.5	47.36%	1.5 S	42.10%	2.5 S	10.52%	33.5 S
16	18	47.36%	2 S	42.10%	2 S	10.52%	34 S
16	18.5	47.36%	2.5 S	42.10%	1.5 S	10.52%	34.5 S
16	19	47.36%	3 S	42.10%	1 S	10.52%	35 S
16	19.5	47.36%	3.5 S	42.10%	0.5 S	10.52%	35.5 S
17	17.5	47.36%	0.5 S	44.73%	1.5 S	7.89%	34.5 S
17	18	47.36%	1 S	44.73%	1 S	7.89%	35 S
17	18.5	47.36%	1.5 S	44.73%	0.5 S	7.89%	35.5 S

The tables were designed for American roulette with 38 numbers.

For European roulette with 37 numbers, only the probability columns change, but the change is very slight: the probability of winning the colour bet will be 18/37 instead of 18/38, which is 48.64% instead of 47.36%, and the probability of winning a straight-up bet from n such bets will be $n/37$ instead of $n/38$.

The probability of not winning any bet will be then $(19 - n)/37$ instead of $(20 - n)/38$.

Let us calculate these new probabilities for the marginal values of n in the tables, or 1 and 17:

$P(A) = 48.64\%$ instead of 47.36%, as we saw, so the difference is less than 1.3%.

for $n = 1$, $P(B) = 1/37 = 2.70\%$ (instead of 2.63%)

for $n = 17$, $P(B) = 17/37 = 45.94\%$ (instead of 44.73%), so the difference is less than 1.3%.

for $n = 1$, $P(C) = 18/37 = 48.64\%$ (instead of 50%)

for $n = 17$, $P(C) = 2/37 = 5.40\%$ (instead of 7.89%), so the difference is less than 2.5%.

Overall, we can state that the probabilities of events A, B and C vary between the two types of roulette within a range of 0 – 2.5%.

This range does not greatly influence a betting decision, so players of European roulette can use these tables, as well. In addition, the amount columns are identical in both cases.

How should we interpret these tables and how do we choose the right betting options?

The basic observations are: as n increases, the probability of winning increases; as c increases with the same n, the possible profit increases, but so does the eventual loss.

Each zone has its own interpretation. For example, the last row says we have a huge probability of winning (44.36% + 44.73% = 89.1%); in case of a win, the profit would be $0.5S$ or $1.5S$. It also says we have a low probability of losing (7.89%), but in case of a lose, the loss would be $35.5S$.

This means that if we choose a low amount S and run such bets with the goal of making a small profit under the safety of a high winning probability, we need considerable time to make a reasonable gain, but just one loss could ruin all previous efforts.

If we raise the stake and run this bet with a high amount S, we should expect to gain something in a reasonable time, but a loss would be more destructive than in the previous situation, even if it is under the same probability.

The mathematical expectation for a long-run bet of this type is

$$M = 47.36\% \cdot 1.5S + 44.73\% \cdot 0.5S - 7.89\% \cdot 35.5S = -1.42S$$

For a \$1 amount for S, a player could expect to lose on average \$1.42 for every \$35.5 bet.

Let's take an example from the first rows at the beginning of the table: $n = 3$, $c = 4$.

The winning probability is 47.36% + 7.89% = 55.25% and the probability of losing is 44.73%, almost 10% lower.

The most probable gain in case of a win is $1S$ and in case of a loss, the loss is $7S$.

This bet seems to be suitable for a player who wants to make a regular profit in a short- or medium-run bet: playing for that $1S$ gain with odds over 50% and waiting for the big win of $29S$ if a straight bet is won. An eventual $7S$ loss will take the player back few games but this might not be a ruin, especially if the player's previous win was a straight-up bet ($29S$).

The mathematical expectation for a long-run bet of this type is

$$M = 47.36\% \cdot S + 7.89\% \cdot 29S - 44.73\% \cdot 7S = -0.37S$$

For a \$1 amount for S, a player could expect to lose on average \$0.37 for every \$7 bet.

Betting on a column and on outside numbers

This bet consists of one column bet (payout 2 to 1) and several straight-up bets (payout 35 to 1) on numbers outside that column.

We use the same denotations from the previous section: S is the amount bet on each number, cS is the amount bet on the column and n is the number of straight-up bets.

n could be between 1 and 24 because there are 24 numbers outside a column.

The possible events after the spin are: A – winning the bet on the column, B – winning a bet on a number and C – not winning any bet.

…………………. missing part ………………….

The overall winning probability is $P(A)+P(B)=\dfrac{12+n}{38}$.

It is natural to put the condition of a positive profit in both cases A and B, which results in:

$$\begin{cases} 2c-n>0 \\ 36-c-n>0 \end{cases} \Leftrightarrow \begin{cases} c>\dfrac{n}{2} \\ c<36-n \end{cases} \Rightarrow \frac{n}{2}<c<36-n \quad (1)$$

$$\begin{cases} 2c-n>0 \\ 36-c-n>0 \end{cases} \Leftrightarrow \begin{cases} 2c-n>0 \\ 72-2c-2n>0 \end{cases} \Rightarrow 3n<72 \Leftrightarrow n<24 \quad (2)$$

The relations (*1*) and (*2*) are the conditions on the parameters n and c that restrain the number of subcases to be studied.

These formulas return the next tables of values, in which n increases from 1 to 23 and c increases by increments of 0.5.

S is left as a variable for players to replace with any basic stake according to their own betting behaviors and strategies.

As in previous section, the tables were designed for American roulette.

Observation:

The same formulas and tables also hold true for the complex bets consisting of a dozen bet and straight-up bets on numbers outside that dozen.

This happens because the bets are equivalent, respectively, if they have the same stakes.

		Winning the bet on column		Winning a bet on a number		Not winning any bet	
n	**c**	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
1	1	31.57%	1 S	2.63%	34 S	65.78%	2 S
1	1.5	31.57%	2 S	2.63%	33.5 S	65.78%	2.5 S
1	2	31.57%	3 S	2.63%	33 S	65.78%	3 S
1	2.5	31.57%	4 S	2.63%	32.5 S	65.78%	3.5 S
1	3	31.57%	5 S	2.63%	32 S	65.78%	4 S
1	3.5	31.57%	6 S	2.63%	31.5 S	65.78%	4.5 S
1	4	31.57%	7 S	2.63%	31 S	65.78%	5 S
1	4.5	31.57%	8 S	2.63%	30.5 S	65.78%	5.5 S
1	5	31.57%	9 S	2.63%	30 S	65.78%	6 S
1	5.5	31.57%	10 S	2.63%	29.5 S	65.78%	6.5 S
1	6	31.57%	11 S	2.63%	29 S	65.78%	7 S
1	6.5	31.57%	12 S	2.63%	28.5 S	65.78%	7.5 S
1	7	31.57%	13 S	2.63%	28 S	65.78%	8 S
1	7.5	31.57%	14 S	2.63%	27.5 S	65.78%	8.5 S
1	8	31.57%	15 S	2.63%	27 S	65.78%	9 S
1	8.5	31.57%	16 S	2.63%	26.5 S	65.78%	9.5 S
1	9	31.57%	17 S	2.63%	26 S	65.78%	10 S
1	9.5	31.57%	18 S	2.63%	25.5 S	65.78%	10.5 S
1	10	31.57%	19 S	2.63%	25 S	65.78%	11 S
1	10.5	31.57%	20 S	2.63%	24.5 S	65.78%	11.5 S
1	11	31.57%	21 S	2.63%	24 S	65.78%	12 S
1	11.5	31.57%	22 S	2.63%	23.5 S	65.78%	12.5 S
1	12	31.57%	23 S	2.63%	23 S	65.78%	13 S
1	12.5	31.57%	24 S	2.63%	22.5 S	65.78%	13.5 S
1	13	31.57%	25 S	2.63%	22 S	65.78%	14 S
1	13.5	31.57%	26 S	2.63%	21.5 S	65.78%	14.5 S
1	14	31.57%	27 S	2.63%	21 S	65.78%	15 S
1	14.5	31.57%	28 S	2.63%	20.5 S	65.78%	15.5 S
1	15	31.57%	29 S	2.63%	20 S	65.78%	16 S
1	15.5	31.57%	30 S	2.63%	19.5 S	65.78%	16.5 S
1	16	31.57%	31 S	2.63%	19 S	65.78%	17 S
1	16.5	31.57%	32 S	2.63%	18.5 S	65.78%	17.5 S
1	17	31.57%	33 S	2.63%	18 S	65.78%	18 S
1	17.5	31.57%	34 S	2.63%	17.5 S	65.78%	18.5 S
1	18	31.57%	35 S	2.63%	17 S	65.78%	19 S
1	18.5	31.57%	36 S	2.63%	16.5 S	65.78%	19.5 S
1	19	31.57%	37 S	2.63%	16 S	65.78%	20 S
1	19.5	31.57%	38 S	2.63%	15.5 S	65.78%	20.5 S
1	20	31.57%	39 S	2.63%	15 S	65.78%	21 S
1	20.5	31.57%	40 S	2.63%	14.5 S	65.78%	21.5 S

		Winning the bet on column		Winning a bet on a number		Not winning any bet	
n	**c**	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
1	21	31.57%	41 S	2.63%	14 S	65.78%	22 S
1	21.5	31.57%	42 S	2.63%	13.5 S	65.78%	22.5 S
1	22	31.57%	43 S	2.63%	13 S	65.78%	23 S
1	22.5	31.57%	44 S	2.63%	12.5 S	65.78%	23.5 S
1	23	31.57%	45 S	2.63%	12 S	65.78%	24 S
1	23.5	31.57%	46 S	2.63%	11.5 S	65.78%	24.5 S
1	24	31.57%	47 S	2.63%	11 S	65.78%	25 S
1	24.5	31.57%	48 S	2.63%	10.5 S	65.78%	25.5 S
1	25	31.57%	49 S	2.63%	10 S	65.78%	26 S
1	25.5	31.57%	50 S	2.63%	9.5 S	65.78%	26.5 S
1	26	31.57%	51 S	2.63%	9 S	65.78%	27 S
1	26.5	31.57%	52 S	2.63%	8.5 S	65.78%	27.5 S
1	27	31.57%	53 S	2.63%	8 S	65.78%	28 S
1	27.5	31.57%	54 S	2.63%	7.5 S	65.78%	28.5 S
1	28	31.57%	55 S	2.63%	7 S	65.78%	29 S
1	28.5	31.57%	56 S	2.63%	6.5 S	65.78%	29.5 S
1	29	31.57%	57 S	2.63%	6 S	65.78%	30 S
1	29.5	31.57%	58 S	2.63%	5.5 S	65.78%	30.5 S
1	30	31.57%	59 S	2.63%	5 S	65.78%	31 S
1	30.5	31.57%	60 S	2.63%	4.5 S	65.78%	31.5 S
1	31	31.57%	61 S	2.63%	4 S	65.78%	32 S
1	31.5	31.57%	62 S	2.63%	3.5 S	65.78%	32.5 S
1	32	31.57%	63 S	2.63%	3 S	65.78%	33 S
1	32.5	31.57%	64 S	2.63%	2.5 S	65.78%	33.5 S
1	33	31.57%	65 S	2.63%	2 S	65.78%	34 S
1	33.5	31.57%	66 S	2.63%	1.5 S	65.78%	34.5 S
1	34	31.57%	67 S	2.63%	1 S	65.78%	35 S
1	34.5	31.57%	68 S	2.63%	0.5 S	65.78%	35.5 S
2	1.5	31.57%	1 S	5.26%	32.5 S	63.15%	3.5 S
2	2	31.57%	2 S	5.26%	32 S	63.15%	4 S
2	2.5	31.57%	3 S	5.26%	31.5 S	63.15%	4.5 S
2	3	31.57%	4 S	5.26%	31 S	63.15%	5 S
2	3.5	31.57%	5 S	5.26%	30.5 S	63.15%	5.5 S
2	4	31.57%	6 S	5.26%	30 S	63.15%	6 S
2	4.5	31.57%	7 S	5.26%	29.5 S	63.15%	6.5 S
2	5	31.57%	8 S	5.26%	29 S	63.15%	7 S
2	5.5	31.57%	9 S	5.26%	28.5 S	63.15%	7.5 S
2	6	31.57%	10 S	5.26%	28 S	63.15%	8 S
2	6.5	31.57%	11 S	5.26%	27.5 S	63.15%	8.5 S
2	7	31.57%	12 S	5.26%	27 S	63.15%	9 S
2	7.5	31.57%	13 S	5.26%	26.5 S	63.15%	9.5 S

		Winning the bet on column		Winning a bet on a number		Not winning any bet	
n	**c**	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
2	8	31.57%	14 S	5.26%	26 S	63.15%	10 S
2	8.5	31.57%	15 S	5.26%	25.5 S	63.15%	10.5 S
2	9	31.57%	16 S	5.26%	25 S	63.15%	11 S
2	9.5	31.57%	17 S	5.26%	24.5 S	63.15%	11.5 S
2	10	31.57%	18 S	5.26%	24 S	63.15%	12 S
2	10.5	31.57%	19 S	5.26%	23.5 S	63.15%	12.5 S
2	11	31.57%	20 S	5.26%	23 S	63.15%	13 S
2	11.5	31.57%	21 S	5.26%	22.5 S	63.15%	13.5 S
2	12	31.57%	22 S	5.26%	22 S	63.15%	14 S
2	12.5	31.57%	23 S	5.26%	21.5 S	63.15%	14.5 S
2	13	31.57%	24 S	5.26%	21 S	63.15%	15 S
2	13.5	31.57%	25 S	5.26%	20.5 S	63.15%	15.5 S
2	14	31.57%	26 S	5.26%	20 S	63.15%	16 S
2	14.5	31.57%	27 S	5.26%	19.5 S	63.15%	16.5 S
2	15	31.57%	28 S	5.26%	19 S	63.15%	17 S
2	15.5	31.57%	29 S	5.26%	18.5 S	63.15%	17.5 S
2	16	31.57%	30 S	5.26%	18 S	63.15%	18 S
2	16.5	31.57%	31 S	5.26%	17.5 S	63.15%	18.5 S
2	17	31.57%	32 S	5.26%	17 S	63.15%	19 S
2	17.5	31.57%	33 S	5.26%	16.5 S	63.15%	19.5 S
2	18	31.57%	34 S	5.26%	16 S	63.15%	20 S
2	18.5	31.57%	35 S	5.26%	15.5 S	63.15%	20.5 S
2	19	31.57%	36 S	5.26%	15 S	63.15%	21 S
2	19.5	31.57%	37 S	5.26%	14.5 S	63.15%	21.5 S
2	20	31.57%	38 S	5.26%	14 S	63.15%	22 S
2	20.5	31.57%	39 S	5.26%	13.5 S	63.15%	22.5 S
2	21	31.57%	40 S	5.26%	13 S	63.15%	23 S
2	21.5	31.57%	41 S	5.26%	12.5 S	63.15%	23.5 S
2	22	31.57%	42 S	5.26%	12 S	63.15%	24 S
2	22.5	31.57%	43 S	5.26%	11.5 S	63.15%	24.5 S
2	23	31.57%	44 S	5.26%	11 S	63.15%	25 S
2	23.5	31.57%	45 S	5.26%	10.5 S	63.15%	25.5 S
2	24	31.57%	46 S	5.26%	10 S	63.15%	26 S
2	24.5	31.57%	47 S	5.26%	9.5 S	63.15%	26.5 S
2	25	31.57%	48 S	5.26%	9 S	63.15%	27 S
2	25.5	31.57%	49 S	5.26%	8.5 S	63.15%	27.5 S
2	26	31.57%	50 S	5.26%	8 S	63.15%	28 S
2	26.5	31.57%	51 S	5.26%	7.5 S	63.15%	28.5 S
2	27	31.57%	52 S	5.26%	7 S	63.15%	29 S
2	27.5	31.57%	53 S	5.26%	6.5 S	63.15%	29.5 S

		Winning the bet on column		Winning a bet on a number		Not winning any bet	
n	**c**	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
2	28	31.57%	54 S	5.26%	6 S	63.15%	30 S
2	28.5	31.57%	55 S	5.26%	5.5 S	63.15%	30.5 S
2	29	31.57%	56 S	5.26%	5 S	63.15%	31 S
2	29.5	31.57%	57 S	5.26%	4.5 S	63.15%	31.5 S
2	30	31.57%	58 S	5.26%	4 S	63.15%	32 S
2	30.5	31.57%	59 S	5.26%	3.5 S	63.15%	32.5 S
2	31	31.57%	60 S	5.26%	3 S	63.15%	33 S
2	31.5	31.57%	61 S	5.26%	2.5 S	63.15%	33.5 S
2	32	31.57%	62 S	5.26%	2 S	63.15%	34 S
2	32.5	31.57%	63 S	5.26%	1.5 S	63.15%	34.5 S
2	33	31.57%	64 S	5.26%	1 S	63.15%	35 S
2	33.5	31.57%	65 S	5.26%	0.5 S	63.15%	35.5 S
3	2	31.57%	1 S	7.89%	31 S	60.52%	5 S
3	2.5	31.57%	2 S	7.89%	30.5 S	60.52%	5.5 S
3	3	31.57%	3 S	7.89%	30 S	60.52%	6 S
3	3.5	31.57%	4 S	7.89%	29.5 S	60.52%	6.5 S
3	4	31.57%	5 S	7.89%	29 S	60.52%	7 S
3	4.5	31.57%	6 S	7.89%	28.5 S	60.52%	7.5 S
3	5	31.57%	7 S	7.89%	28 S	60.52%	8 S
3	5.5	31.57%	8 S	7.89%	27.5 S	60.52%	8.5 S
3	6	31.57%	9 S	7.89%	27 S	60.52%	9 S
3	6.5	31.57%	10 S	7.89%	26.5 S	60.52%	9.5 S
3	7	31.57%	11 S	7.89%	26 S	60.52%	10 S
3	7.5	31.57%	12 S	7.89%	25.5 S	60.52%	10.5 S
3	8	31.57%	13 S	7.89%	25 S	60.52%	11 S
3	8.5	31.57%	14 S	7.89%	24.5 S	60.52%	11.5 S
3	9	31.57%	15 S	7.89%	24 S	60.52%	12 S
3	9.5	31.57%	16 S	7.89%	23.5 S	60.52%	12.5 S
3	10	31.57%	17 S	7.89%	23 S	60.52%	13 S
3	10.5	31.57%	18 S	7.89%	22.5 S	60.52%	13.5 S
3	11	31.57%	19 S	7.89%	22 S	60.52%	14 S
3	11.5	31.57%	20 S	7.89%	21.5 S	60.52%	14.5
3	12	31.57%	21 S	7.89%	21 S	60.52%	15 S
3	12.5	31.57%	22 S	7.89%	20.5 S	60.52%	15.5 S
3	13	31.57%	23 S	7.89%	20 S	60.52%	16 S
3	13.5	31.57%	24 S	7.89%	19.5 S	60.52%	16.5 S
3	14	31.57%	25 S	7.89%	19 S	60.52%	17 S
3	14.5	31.57%	26 S	7.89%	18.5 S	60.52%	17.5 S
3	15	31.57%	27 S	7.89%	18 S	60.52%	18 S
3	15.5	31.57%	28 S	7.89%	17.5 S	60.52%	18.5 S

		Winning the bet on column		Winning a bet on a number		Not winning any bet	
n	c	Odds	Profit	Odds	Profit	Odds	Loss
3	16	31.57%	29 S	7.89%	17 S	60.52%	19 S
3	16.5	31.57%	30 S	7.89%	16.5 S	60.52%	19.5 S
3	17	31.57%	31 S	7.89%	16 S	60.52%	20 S
3	17.5	31.57%	32 S	7.89%	15.5 S	60.52%	20.5 S
…………………… missing part ……………………							
21	14	31.57%	7 S	55.26%	1 S	13.15%	35 S
21	14.5	31.57%	8 S	55.26%	0.5 S	13.15%	35.5 S
22	11.5	31.57%	1 S	57.89%	2.5 S	10.52%	33.5 S
22	12	31.57%	2 S	57.89%	2 S	10.52%	34 S
22	12.5	31.57%	3 S	57.89%	1.5 S	10.52%	34.5 S

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When moving to European roulette, the following changes must be made:
- the probability of winning the column bet will be 12/37 = 32.43%;
- the probability of winning a bet on a number will be $n/37$;
- the probability of not winning any bet will be $1 – 12/37 – n/37 = (25 – n)/37$.

The amount columns are left unchanged.

Let us take two examples of bets from the tables above.

1) $n = 19$, $c = 10$

The probability of winning the column bet is 31.57% and the correspondent win is S, the probability of winning a straight-up bet is 50% and the corresponding win is $7S$, while the probability of losing all is 18.42% and the corresponding loss is $29S$.

Thus, we have an overall winning probability of 81.57%, which has a very good probability of winning a straight-up bet (50%).

This type of bet can be made in both the long and short run, depending on the initial amount at the player’s disposal (the entire amount bet all at once is $29S$)

The mathematical expectation for a long-run bet of this type is
$M = 31.57\% \cdot S + 50\% \cdot 7S - 18.42\% \cdot 29S = -1.52S$

For a \$1 amount for *S*, a player could expect to lose on average \$1.52 for every \$29 bet (at American roulette, of course).

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Betting on corners and on the opposite of the predominant colour

This complex bet derives from the observation that there are some corners joining three black numbers and one red number each. These are the corners joining the following groups of numbers: (7, 8, 10, 11), (10, 11, 13, 14), (25, 26, 28, 29) and (28, 29, 31, 32).

Observe that these sets are not mutually exclusive. We can extract from them a maximum of two exclusive sets; for example (7, 8, 10, 11) and (25, 26, 28, 29).

Two of such corners contain six black and two red numbers.

By betting on these two corners (payout 8 to 1) and on the colour red (payout 1 to 1), we enlarge the coverage and implicitly increase the probability of winning.

The denotations we use here are: S is the amount bet on one corner and cS is the amount bet on the colour red. The possible events after the spin are: A – a red number from the chosen corners occurs, B – a black number from a chosen corners occurs, C – a red number from outside the chosen corners occurs and D – a black number from outside the chosen corners or 0 or 00 occurs.

These events are mutually exclusive and exhaustive, so we have:

$$P(A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D) = 1$$

Let us find the probability of each event and the profit or loss in each case:

A. The probability of a red number from the two corners occurring is $P(A) = 2/38 = 1/19$.

In the case this happens, the player wins
$8S + cS - S = (7 + c)S$.

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	Red number in one of the 2 corners		Black number in one of the 2 corners		Red number outside the 2 corners		Not winning any bet	
c	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
2	5.26%	9 S	15.78%	5 S	42.10%	0	36.84%	4 S
2.5	5.26%	9.5 S	15.78%	4.5 S	42.10%	0.5 S	36.84	4.5 S
3	5.26%	10 S	15.78%	4 S	42.10%	1 S	36.84	5 S
3.5	5.26%	10.5 S	15.78%	3.5 S	42.10%	1.5 S	36.84	5.5 S
4	5.26%	11 S	15.78%	3 S	42.10%	2 S	36.84	6 S
4.5	5.26%	11.5 S	15.78%	2.5 S	42.10%	2.5 S	36.84	6.5 S
5	5.26%	12 S	15.78%	2 S	42.10%	3 S	36.84	7 S
5.5	5.26%	12.5 S	15.78%	1.5 S	42.10%	3.5 S	36.84	7.5 S
6	5.26%	13 S	15.78%	1 S	42.10%	4 S	36.84	8 S
6.5	5.26%	13.5 S	15.78%	0.5 S	42.10%	4.5 S	36.84	8.5 S
7	5.26%	14 S	15.78%	0	42.10%	5 S	36.84	9 S

For the European roulette, the probabilities change as follows:
A: red number in one of the two corners: $P(A) = 2/37 = 5.40\%$
B: black number in one of the two corners: $P(B) = 6/37 = 16.21\%$
C: red number outside the two corners: $P(C) = 16/37 = 43.24\%$
D: not winning any bet: $P(D) = 13/37 = 35.13\%$

Because the same probabilities apply for the noted events, choosing a value for c is only a matter of money management.

If a player has a low amount at his or her disposal, the player may choose to reduce c and the eventual loss, which starts from $4S$ (in the first row of the table).

In this case, the overall winning probability is about 21% (with profits of $9S$ or $5S$), 42.10% is the probability of achieving zero profit (with no loss) and 36.84% is the probability of losing ($4S$).

Even if the probability of losing is higher than the probability of winning, each of the possible profits is higher than the possible loss, so this bet may work in a short or medium run.

The mathematical expectation for a bet of such type is:

$$M = 5.26\% \cdot 9S + 15.78\% \cdot 5S - 36.84\% \cdot 4S = -0.21S$$

For a \$1 amount for S, someone should expect to lose on average \$0.21 at every \$4 bet.

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Betting on first and third columns and on the colour black

This complex bet derives from the observation that the first and third columns contain the most red numbers (first column has six red numbers, the third column has eight, while the second column has only four). By combining the column bets with a bet on a black colour we enlarge the coverage and implicitly increase the probability of winning.

This bet consists of two bets on the first and third columns (payout 2 to 1) and a bet on the colour black (payout 1 to 1).

The denotations we use here are: S is the amount bet on each column, cS is the amount bet on the colour black.

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It is natural to put the condition of a non-negative profit in a maximum number of cases from A to G.

Observe that if $c \geq 2$ there are three cases with non-negative profits (A, C and E); if $c \leq 1$ there are four cases with non-negative profits (A, B, C and D); and if $c \in (1, 2)$ there only two cases with non-negative profits (A and C).

Therefore, we choose parameter c in the interval [0, 1].

These formulas return the next table of values, in which c increases by increments of 0.2.

S is left as a variable for players to replace with any basic stake according to their own betting behaviors and strategies.

As in previous sections, the table was designed for American roulette.

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Repeated bets

A frequent betting behavior of many players is to place the same simple bet for several spins, with a constant or increasing stake.

The motivation for such behavior can be subjective, but it can also be an act of pure gambling choice.

It is the perfect bet for a player with a *hit and run* profile, the player who doesn't want to wait to run elaborate betting schemes that could bring small regular profits.

This player puts everything into a short session, expecting to catch a lucky series that will bring enough profit to allow the player leave the table quickly.

Let us find the odds involved in such repeated bets.

Consider that n independent spins are performed.

Denote by A the event given by the expected outcome of a simple bet (for example: red colour, high number, number 15, column 1, and so on).

After each spin, the event A may occur with probability p and does not occur with probability $q = 1 - p$.

The probability for event A to occur exactly m times in n spins ($m \leq n$) is given by the Bernoulli formula:

Let B_m be the event *A occurs exactly* m *times in the* n *experiments*. Then $P(B_m) = C_n^m p^m q^{n-m}$.

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Repeated colour bet – European roulette

After each spin, the event A may occur with probability p = 18/37 and does not occur with probability $q = 1 - p$ = 19/37.

The probability for event A to occur exactly m times in n spins ($m \le n$) is given by the formula: $P(B_m) = C_n^m \left(\frac{18}{37}\right)^m \left(\frac{19}{37}\right)^{n-m}$.

The next table note the numerical returns of this formula for n increasing from 10 up to 100 spins, with an increment of 10.

n / m	10	20	30	40	50	60	70	80	90	100
1	12.079E-3	30.803E-6	58.911E-9	100.15E-12	159.62E-15	244.22E-18	363.29E-21	529.37E-24	759.33E-27	1.0757E-27
2	51.495E-3	277.22E-6	809.25E-9	1.8502E-9	3.7048E-12	6.8253E-15	11.874E-18	19.81E-21	32.012E-24	50.446E-27
3	130.09E-3	1.5758E-3	7.1555E-6	22.202E-9	56.157E-12	125.01E-15	254.97E-18	487.94E-21	889.59E-24	1.5612E-24
4	215.68E-3	6.3446E-3	45.757E-6	194.56E-9	625.12E-12	1.6877E-12	4.046E-15	8.8985E-18	18.33E-21	35.866E-24
5	245.2E-3	19.234E-3	225.42E-6	1.3271E-6	5.4484E-9	17.907E-12	50.596E-15	128.14E-18	298.69E-21	652.39E-24
6	193.58E-3	45.555E-3	889.8E-6	7.3339E-6	38.712E-9	155.51E-12	519.28E-15	1.5174E-15	4.0087E-18	9.7858E-21
7	104.79E-3	86.314E-3	2.8902E-3	33.747E-6	230.53E-9	1.1365E-9	4.4978E-12	15.197E-15	45.572E-18	124.49E-21
8	37.229E-3	132.88E-3	7.8719E-3	131.88E-6	1.1739E-6	7.133E-9	33.556E-12	131.37E-15	447.93E-18	1.3711E-18
9	7.8377E-3	167.85E-3	18.23E-3	444.23E-6	5.1898E-6	39.044E-9	219E-12	995.68E-15	3.8663E-15	13.278E-18
10	742.52E-6	174.91E-3	36.268E-3	1.3046E-3	20.158E-6	188.64E-9	1.2656E-9	6.6973E-12	29.669E-15	114.47E-18
11		150.64E-3	62.47E-3	3.3708E-3	69.444E-6	812.33E-9	6.5398E-9	40.376E-12	204.42E-15	887.26E-18
12		107.04E-3	93.706E-3	7.7174E-3	213.81E-6	3.1425E-6	30.462E-9	219.94E-12	1.2749E-12	6.2342E-15
13		62.402E-3	122.92E-3	15.747E-3	592.1E-6	10.992E-6	128.75E-9	1.0899E-9	7.2469E-12	39.979E-15
14		29.559E-3	141.4E-3	28.771E-3	1.4825E-3	34.96E-6	496.62E-9	4.9415E-9	37.76E-12	235.37E-15
15		11.201E-3	142.89E-3	47.245E-3	3.3707E-3	101.57E-6	1.7565E-6	20.598E-9	181.25E-12	1.2784E-12
16		3.3161E-3	126.91E-3	69.935E-3	6.9853E-3	270.63E-6	5.7201E-6	79.276E-9	804.89E-12	6.4341E-12
17		739.2E-6	99.012E-3	93.536E-3	13.235E-3	663.58E-6	17.213E-6	282.74E-9	3.3192E-9	30.119E-12

…………………… missing part ……………………

65							1.9736E-15	1.3797E-9	3.0786E-6	370.13E-6
66							141.64E-18	297.06E-12	1.1048E-6	185.95E-6
67							8.0113E-18	58.805E-12	374.91E-9	89.397E-6
68							334.84E-21	10.65E-12	120.13E-9	41.101E-6
69							9.1946E-21	1.7548E-12	36.288E-9	18.058E-6

70							124.44E-24	261.24E-15	10.313E-9	7.5762E-6
71								34.857E-15	2.7523E-9	3.0327E-6
72								4.1278E-15	688.06E-12	1.1572E-6
73								428.56E-18	160.73E-12	420.5E-9
74								38.406E-18	34.981E-12	145.35E-9
75								2.9107E-18	7.0699E-12	47.737E-9
76								181.42E-21	1.3219E-12	14.876E-9
77								8.9282E-21	227.7E-15	4.3928E-9
78								325.32E-24	35.953E-15	1.2271E-9
79								7.8025E-24	5.1737E-15	323.75E-12
80								92.398E-27	673.95E-18	80.51E-12
81									78.824E-18	18.833E-12
82									8.1961E-18	4.134E-12
83									748.41E-21	849.35E-15
84									59.085E-21	162.85E-15
85									3.9512E-21	29.04E-15
86									217.63E-24	4.7985E-15
87									9.4793E-24	731.53E-18
88									306.15E-27	102.38E-18
89									6.5177E-27	13.078E-18
90									68.607E-30	1.5142E-18
91										157.64E-21
92										14.61E-21
93										1.1906E-21
94										83.996E-24
95										5.0258E-24
96										247.98E-27
97										9.6879E-27
98										280.96E-30
99										5.3772E-30
100										50.942E-33

Observation:

The same tables also hold true for repeated bets on High/Low or Even/Odd because identical probabilities are involved.

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The martingale

Here are some probabilities of consecutive occurrences of the same colour (in American roulette):

2 times in a row: $P_2 = \frac{9}{19} \cdot \frac{9}{19} = 22.43\%$

3 times in a row: $P_3 = \frac{9}{19} \cdot \frac{9}{19} \cdot \frac{9}{19} = 10.62\%$

4 times in a row: $P_4 = \frac{9}{19} \cdot \frac{9}{19} \cdot \frac{9}{19} \cdot \frac{9}{19} = 5.03\%$

5 times in a row: $P_5 = \frac{9}{19} \cdot \frac{9}{19} \cdot \frac{9}{19} \cdot \frac{9}{19} \cdot \frac{9}{19} = 2.38\%$

Generally, the probability for the same colour to win n times in a row is $P_n = \left(\frac{9}{19}\right)^n$.

Here are some probabilities of consecutive occurrences in the same column:

2 times in a row: $P_2 = \frac{6}{19} \cdot \frac{6}{19} = 9.97\%$

3 times in a row: $P_3 = \frac{6}{19} \cdot \frac{6}{19} \cdot \frac{6}{19} = 3.14\%$

4 times in a row: $P_4 = \frac{6}{19} \cdot \frac{6}{19} \cdot \frac{6}{19} \cdot \frac{6}{19} = 0.99\%$

5 times in a row: $P_5 = \frac{6}{19} \cdot \frac{6}{19} \cdot \frac{6}{19} \cdot \frac{6}{19} \cdot \frac{6}{19} = 0.31\%$

Generally, the probability for the same column to win n times in a row is $P_n = \left(\frac{6}{19}\right)^n$.

More generally, if A is an event corresponding to a certain group of numbers, the probability that A will occur n times in a row is $P_n = p^n$, where p is the probability of event A.

As n increases, the probability P_n becomes lower.

Obviously, the lowest probabilities are for consecutive occurrences of the same number.

For European roulette, the figures are not much different.

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We have:

$$S + 2S + 4S + \ldots + 2^{n-1}S = S(1 + 2^2 + 2^3 + \ldots + 2^{n-1}) =$$
$$= S(2^n - 1) < 2^n S$$

The first sum is the loss from previous games and the last term is the amount received when the winning colour occurs.

The difference between the two amounts is $2^n S - S(2^n - 1) = S$.

Of course, this calculus also stand for High/Low or Even/Odd repeated bet, which have the same payout (1 to 1).

The inequality is also valid for other larger-than-2 multipliers of S (3, 4, etc.), and can also generate bigger winnings.

Based on this mathematical certainty and on the low probability of the consecutive repetition of colour, the system seems to be infallible.

However, it assumes the major risk of consuming the entire cash amount available before the winning bet, as well as the roulette ball consistently landing on one colour during a very long series of plays.

The same proof method also holds true for other repeated bets with different payouts (column, dozen, line, corner, street, split or number) because their payout is higher, which increases the last term in the above inequality. For example, for a repeated column bet, we have:

$$S + 2S + 4S + \ldots + 2^{n-1}S = S(1 + 2^2 + 2^3 + \ldots + 2^{n-1}) =$$
$$= S(2^n - 1) < 2^n S < 2^{n+1} S = 2 \cdot 2^n S$$

The last term is exactly the amount received when the chosen column wins.

In this case, the profit after n spins is

$$2^{n+1} S - S(2^n - 1) = S(2^n + 1)$$

Unlike the colour bet, in a column bet the profit depends on n and is significantly higher. But keep in mind that the probability for a column to win is lower than that for a colour.

Of course, the highest profit will correspond to a straight-up bet.

In this case, the profit after n spins is

$35 \cdot 2^n S - S(2^n - 1) = S(34 \cdot 2^n + 1)$

Knowing the probability for that number to win being 1/38, we should expect this to happen on average once in every 38 spins.

Assuming just 20 consecutive failures, the cumulated loss would be $S(2^{20} - 1) = 1048575S$. For just a \$1 for a basic stake S, a player would need more than \$1 million to sustain that loss.

If you had it, would you use it to run such a bet?

If your answer to that question is yes, would the house let you place a bet of over \$1 million on one number? Probably not.

These are some of the non-mathematical reasons why the house always wins.

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