DRAW POKER ODDS

The Mathematics of Classical Poker

$$\Sigma \quad \Pi$$

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Introduction

Over the past few decades, gamblers have begun taking mathematics into account more seriously than ever before.

While probability theory is the only rigorous theory that can model the hazards of gaming, even in idealized conditions, numerical probabilities are viewed not only as purely theoretical information, but also as a decision-making criterion, especially in gambling.

For a math person, the games of chance appear to be pure applications of probability calculus. And this is exactly what they are: experiments generating various types of aleatory events, the probability of which can be calculated by using the properties of probability on a finite field of events.

For a gambler interested in the mathematics of games of chance, these games are more than pure probability applications.

They are a way of life, a kind of living companion a gambler interacts with in order to squeeze profit from them by using strategies that may or may not include mathematics.

Even though the randomness inherent in games of chance is would seem to ensure their fairness (at least with respect to the players around a table—shuffling a deck or spinning a wheel do not favor any player except if they are fraudulent), gamblers always search and wait for irregularities in this randomness that will allow them to win.

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Among all games of chance, poker and its variations are the most predisposed to probability-based decisions because the structure and dynamics of the game cause the odds of an expected event to change significantly from one gaming moment to another.

These probability-based decisions have a double criterion: the odds of events related to your own play and the odds of events regarding opponents' play.

For the poker player who takes probability into account, these odds are subject to weighting and comparison before gaming decisions are made.

These decisions can be calling, raising a bet, folding or discarding certain cards.

The permanent interaction with opponents, whether it is about the technical process of the game or the betting dialogue, together with the power of one single card changing hands and the predictions make poker such a spectacular game.

Classical poker (*draw poker* or *five draw poker*) is probably the oldest form of poker.

All the variations of poker played around the world are derived from this primary form of poker.

This is often the first poker variant learned by most players, and is very common in home games although it is now quite rare in casino and tournament play.

Draw poker is any poker variant in which each player is dealt a complete hand before the first betting round, and then develops the hand for later rounds by replacing cards.

In draw poker, players do not get to see the opponents' cards.

The only information directly available about their hands is how they bet and how many cards they draw.

But there is extra information any player has just in front of his or her eyes—it is about the seen cards, namely the initial hand (the five cards dealt) plus the replacements to be made.

This information can be quantified through mathematical methods and turned into a finite probability space on which basis we can make predictions about opponents' hands.

These predictions are numerical probabilities returned by compact formulas and are attached to events such as someone holding a certain card formation or someone holding a formation higher than yours at any moment in a game.

This is the main subject of this book: the probability formulas for events regarding opponents, their numerical returns and their practical usage.

These formulas were built with a large enough range of variables to cover each possible card distribution at any moment in a game.

Of course, the probabilities of the events regarding your own hand were also worked out and are presented in the first part of the book.

The whole presentation is focused on the practical aspect of the application of probability theory in draw poker, and all the sections are structured to allow direct usage of the numerical results.

This is why every section is packed with tables, some of which fill dozens of pages.

This is not a math book, even if the supporting mathematics are present throughout. Instead, it is a guidebook addressed to poker players, who can skip the math parts at any time and pick the results they need from tables.

The main goal of this guide is a learning one for the poker player who wants to use the probability information provided.

All he or she has to do is to study it at home, putting on paper as many gaming situations as possible, searching or calculating the associated odds and trying to memorize as many of them as possible.

Obviously, the process of memorizing is hard in this case. Because we deal with millions of card distributions, even if they are grouped conveniently into categories, memorization of all possible distributions is impossible.

But long-term practice may lead at least to some good results and players may be able to retain some of the most frequent situations along with the associated odds.

In addition, memorizing the numerical results is not absolutely necessary because players can bear in mind only their order and some ranges of variation.

The order of probabilities of achieving various formations, in specific gaming situations, is the fact that generates the probability-based strategy.

Of course, you cannot refer to this book while playing, not even on line, but just to do homework.

For those who want to use the results from this book while playing on line, we recommend *Draw Poker Odds Calculator*, which is a software program for simulating and analyzing hands based on the formulas from this book.

You can read more about it in the chapter titled *Using a Software Program*.

The presentation of odds is structured as follows:

- *Initial Probabilities of the First Card Distribution for Your Own Hand* – this chapter lists the odds of receiving various valuable formations or parts of them from the first five cards dealt. Probability calculus and counts of combinations are given in detail.
- *Prediction Probabilities After the First Card Distribution and Before the Second for Your Own Hand* – this chapter presents the odds of receiving various card combinations at the second distribution, after you have been dealt the first five cards and have made your discards (or not). It is built on a list of 17 possible types of dealt combinations that require making a decision to keep or discard cards the most. All probability and combinatorial calculations are still shown in detail, and the end of each sub-section contains comparisons of the odds corresponding to the discarding options with conclusions, along with recommendations for discarding based on these odds.
- *Prediction Probabilities for Opponents' Hands* – this is the most complex chapter. It lists the odds for one or several of your opponents to hold various card formations at any moment of the game. Each section starts by framing the problem, establishing the variables that reflect the hypothesis and presenting the formula of the number of favorable card combinations for the predicted event and the overall probability formula. The numerical returns of the latter formula are given in tables, some of which are quite long. All the applied probability matters are shown in detail, including the final formulas. Players may skip all the math discussions and go directly to the tables to find the desired situation and the associated odds. Counting favorable combinations is not shown in detail to keep the chapter from becoming overloaded with math expressions. Readers interested in these calculations can consult the book *Understanding and Calculating the Odds*, a probability calculus guide for beginners focused on these kinds of exercises. The order of the sections does not reflect the hierarchy of the valuable formations as predicted events, but rather gives combined criteria regarding the increasing levels of difficulty of the formulas and the types of card distributions (value, symbol or exact distributions).

As we said earlier, following entire mathematics discussions is not necessary with respect to the practical goal of a player interested in the numerical odds and how to use them – players can skip all the math and go directly to the tables with values.

The players who do not want to take the time to read the math discussions or review the tables can use the software *Draw Poker Odds Calculator*, which is entirely based on the formulas in this book.

The applied mathematics supporting the calculations are presented not only for anyone interested in probability applications, but also as proof that credits the formulas and numerical results. This is what differentiates an odds software from a book: users never know what is behind the odds calculator or how the returns are figured out and given to the user.

The chapter containing the rules of Draw Poker is followed by a chapter titled *The Supporting Mathematics*, which contains all probability matters that ground this book and its applications: the probability model, the properties of probability, formulas and the way all these are applied in Draw Poker.

This chapter is addressed mainly to those interested in both poker and math. These readers may then follow the mathematics in each section and rediscover the results. They may also work out for themselves the probabilities in poker or other card games.

The last chapter, titled *Examples*, is a collection of exercises consisting of concrete gaming situations in which all kinds of odds are figured out by using the precalculated results in the tables.

About using the probabilities provided in players' strategy: we can state that odds are necessary for informational purposes, but they are optional within a subjective strategy and this is true in any game of chance.

Although it has been mathematically proven that a probability-based strategy is optimal with respect to a long-running or even an isolated goal, a lot of personal criteria could significantly reduce the role of odds within a strategy.

Moreover, in poker, unlike in other games, the interaction with other players generates additional criteria that may prevail over of the card odds. We talk here especially about the psychology of poker.

The information given by cards (the seen cards) is the only information mathematics can quantify in order to build a solid probability model on which to base predictions.

If we could factor in additional (and, in fact, also real) information such as the way opponents discard and their psychological profile, the probability field would change significantly and thus lead to different probabilities.

But rigorously quantifying all these data is impossible. Even if the number of cards discarded can be stated statistically with respect to target formations, there are psychological elements that might intervene and render the statistics irrelevant. And psychology cannot be quantified.

Therefore, all that mathematics can do is provide odds, and this is the goal of this book.

The way these odds are taken as a criterion or combined with other criteria within a player's strategy is an act of subjective choice, and only partly of skill.

This possible combination of objective and subjective criteria may often generate positive results.

But we must bear in mind that probability theory is the only tool we have for making objective predictions and mathematics cannot take any subjective hypotheses into account.

The Rules of Classical Poker

Five card draw poker is usually played with 52 cards. All five cards are dealt face down.

A player can discard some or all of these five cards and receive a corresponding number of new cards.

The player with the best high hand wins the pot.

Five card draw is played with blinds. Before the cards are dealt, the first two players to the left of the dealer post a small and a big blind, respectively, to create a starting pot.

When the blinds are posted, each player is dealt five cards.

When the first betting round is finished, each player remaining in the hand has the right to draw one to five new cards, or to draw no cards (stand pat).

The draw starts with the player to the dealer's left.

If more than one player remains in the hand after the second betting round, there is a showdown.

The best high hand wins the pot.

Hands are ranked as follows:

Five of a Kind

A five of a kind (which is only possible when using wild cards) is the highest possible hand.

If more than one hand has five of a kind, the higher card wins (five aces beats five kings, which beat five queens, and so on).

Straight Flush

A straight flush is the best natural hand. A straight flush is a straight (five cards in order, such as (45678)) that are all of the same suit (they have the same symbol). As in a regular straight, a player can have an ace either high (AKQJ 10) or low (5432A).

However, a straight may not wrap around (for example, (QKA23) is not a straight).

An ace-high straight flush is called a Royal Flush and is the highest natural hand.

Four of a Kind

Four of a kind is simply four cards of the same rank. If there are two or more hands that qualify, the hand with the higher ranked four of a kind wins.

If, in some bizarre game with many wild cards, there are two four of a kind hands having the same rank, then the one with the high card outside the four of the kind wins.

General Rule: When hands tie on the rank of a pair, three of a kind, and so on, the cards outside break ties following the high card rules.

Full House

A full house is a three of a kind plus a pair, such as (KKK33).

Ties are broken first by the three of a kind, then the pair.

So (KKK44) beats (QQQAA), which beats (QQQJJ). (Obviously, the three of a kind can only be similar if wild cards are used.)

Flush

A flush is a hand in which all the cards are of the same suit, such as (Q♠ 7♠ 5♠ 3♠ 2♠), all spades. When flushes ties, follow the rules for high card.

Straight

A straight is five cards in order, such as (23456). An ace may be either high (AKQJ 10) or low (5432A). However, a straight may not wrap around (for example, (QKA23) is not a straight).

When straights tie, the highest straight wins ((KQJ 10 9) beats (QJ 10 98) down to (5432A)). If two straights have the same value ((KQJ 10 9) versus (KQJ 10 9)), they split the pot.

Three of a Kind

Three cards of any rank, matched with two cards that are not a pair (otherwise, the hand would be a full house).

Again, the highest three of a kind wins. If two hands of three of a kind are the same rank, then the compare high cards.

Two Pair

This is two distinct pairs of card plus a fifth card. The highest pair wins ties. If both tie hands have the same high pair, the second pair wins. If both hands have the same pairs, the high card wins.

Pair

One pair plus three distinct cards. The high card breaks ties.

High Card

This is any hand that does not qualify as one of the hands described previously. If no player has a pair or better, then the highest card wins. If multiple players tie for the highest card, they look at the second highest, then the third highest, and so on. The high card is also used to break ties when both high hands have the same type of hand (pair, flush, straight, and the like).

Betting

In essence, when betting comes around to you (betting is typically done in a clockwise order), you have one of three choices:

Call

When you call, you bet enough to match what has been bet since the last time you bet (for instance, if you bet a dime last time, and someone else bet a quarter, you would owe 15 cents).

Raise

When you raise, you first bet enough to match what has been bet since the last time you bet (as in calling), then you raise the bet another amount (the amount is up to you, but there is typically a limit). Continuing this example, if you had bet a dime and the other person raised you fifteen cents (up to a quarter), you might raise another quarter (up to fifty cents). Because you owed the pot 15 cents for calling and 25 cents for your raise, you would put 40 cents into the pot.

Fold

When you fold, you drop out of the current hand (a give up any possibility of winning the pot), but you do not have to put any money into the pot.

Betting continues until every player calls or folds after a raise or initial bet.

There are variations of the above rules.

– Poker can be played by using 24, 28, 32, …, 52-card decks or even two decks of 52. However, many players can participate in a poker game, provided the number of cards in the deck are sufficient to cover a maximum double dealing (initially five cards for each player and then, after discarding, another five cards each).

For example, three players require a minimum of 3 x 5 + 3 x 5 = 30 cards; that means a deck with cards from 7 upward to ace.

– The game can allow wild cards (jokers) or not.

– The maximum number of cards that can be discarded can be 3, 4 or 5.

– The hierarchy of card formations can be different from one community to another, depending on pre-established game rules or the number of cards in the deck.

The draw poker most often practiced is one with 52 cards, without wild cards, and with the ability to discard up to five cards.

We will calculate the probabilities involved in this type of game.

We will not insist on the rules of dialogue, because we are interested only in the card distributions that generate the events.

For these distributions, the following types of probabilities are calculated:

– Initial Probabilities of the First Card Distribution for your own hand;

– Prediction probabilities after first card distribution and before the second for your own hand;

– Prediction probabilities for opponents' hands.

The events whose probability is to be measured are the occurrence of the various combinations of cards (of size 1, 2, 3, 4 or 5) in your own hand or your opponents' hands.

The game with 52 cards (from 2 upward) with up to five cards to discard allows a maximum number of five players (or six players if a maximum of three-card replacements is allowed).

The Supporting Mathematics

The application of probability theory in gambling is a simple process because a finite sample space can be attached to any game of chance.

The finite sample space and the randomness of the drawings (no matter the numbers or cards) allow us to build a simple probability model to work within to find the numerical probabilities of the events involved in that game.

This model assumes a finite probability field in which the field of events is the set of parts of the sample space (and, implicitly, is finite) and the probability-function is given by the classical definition of probability.

In this probability field, any event, no matter how complex, can be decomposed into elementary events.

Therefore, finding the probability of a compound event means applying some properties of probability and doing some algebraic calculations.

It is a step-by-step method through which we progressively write the probability of a compound event as an expression holding the probabilities of the elementary events that make up the compound event.

Of course, there are events whose literal definition makes them much more complex (for example, events involving several players and events of the "at least one..." type). These types of events are usually put in question in card games.

To find the probability of such events, the step-by-step method may lead to complex algebraic expressions, which are hardly calculable.

But such problems can be easily solved by using other probability properties or classical probability repartitions.

In fact, any problem of applied probability in gambling, no matter how difficult, can be solved by using basic theoretical results.

Anyone with a minimal mathematical background can perform such applications and calculations, but at some point, these problems become a matter of math skill, especially in terms of correctly framing the problem.

For those interested in improving their probability calculus skills and figuring out correct probability results for any game of chance, we recommend the beginner's guide, *Understanding and Calculating the Odds*, which is full of gambling applications.

Let us see now how probability theory can be applied in Draw Poker and how the numerical probability results from this book were obtained.

The probability problem

As in every card game, we are interested in making predictions for the events regarding your own hand and your opponents' hands by using the maximum of information available.

These events can be described as the occurrences of certain cards and card combinations in your hand or in your opponents' hands at various stages of the game.

Each event can be associated with a set of card combinations and decomposed into elementary events.

The only additional information available, which a player has and a neutral observer does not, is the cards that are dealt to him or her.

We then must quantify this information and establish a probability field based on it, in which to work our applications.

If we could calculate the probability of an event from a neutral observer's perspective (assuming he or she cannot see any of the cards in play), the result would be the same, no matter the person.

But if we take into account the cards that are out of play (which only a player can see), the probability of that event is changed because the additional information (the seen cards not in play) changes the probability field and implicitly the probability function and its numerical values.

Between the two probabilities, the one that takes into account the seen cards is more valuable with respect to the accuracy of the prediction because it is based on more information.

For example, if we calculate the probability of one specific player holding a *QQQ* three of a kind formation in a moment of a game from a neutral observer's perspective, we find

$C_4^3(C_{48}^2 - 12C_4^2) = 4224$ possible card combinations holding exactly three queens from $C_{52}^5 = 2598960$ possible, so the probability is P = 4224/2598960 = 0.0016252 = 0.16252%.

Now let us assume that you are in this game as a player and you hold one *Q* and you receive another *Q* after discarding.

If you have the same problem of one opponent holding a *QQQ* three of a kind formation, you know that this event is impossible because you already hold two queens out of the four possible, so only two are still in play. Therefore, the probability of the predicted event is zero (P = 0).

Although we find two different probability values for the same literally defined event, this is not a contradiction because in each case we figure the probability in a different probability field.

Of course, you will take this latter probability (zero) as the “real” one and not the neutral observer’s position.

The more cards that are seen, the more accurate the prediction.

Therefore, to obtain the most accurate results, we must change the probability field for each application in part, according to the seen cards, in order to use the most information.

For each application, we must:

- establish the probability field and its elementary events;
- quantify the information given by the seen cards in a convenient way;
- identify the events to measure as sets of card combinations;
- count the combinations associated with each event;
- decompose the compound events;
- apply the definition of probability, properties of probability and classical probability repartitions; and
- work out the numerical calculations.

The events whose probability is important from a player’s perspective and will be calculated are:

- events regarding the first distribution – these are the occurrences of certain five-card combinations in your hand and in your opponents’ hands. No additional information is available.
- events regarding the second distribution – these are the occurrences of one-, two-, three-, four- or five-card combinations in your own hand after discarding. The additional information to use is the first five cards dealt to you.

- events regarding opponents' hands (1) – these are the occurrences of five-card combinations in one or more opponents' hands after the first distribution and before the second. The additional information to use is the first five cards dealt to you.
- events regarding opponents' hands (2) – these are the occurrences of five-card combinations in one or more opponents' hands after the second distribution. The additional information to use is the first five cards dealt to you plus any replacements.

Among these events, the ones of the type "at least one opponent holds a higher formation than yours..." have been granted larger importance because their probability represents a major criterion in a probability-based strategy.

The probability space

The only way we can use the information given by the seen cards is to subtract them from any combination that could be dealt to or held by a player.

This subtraction will modify the sample space and reduce the number of its elements for each application in part.

If we call an experiment any distribution of cards to one or more players, then we must attach a sample space to each experiment.

The sample space is the set of all elementary events (i.e., events than cannot be decomposed as a union of non-elementary events).

It is natural to take as the elementary events any card combination that could occur as the result of a specific experiment.

We define a *card combination* as a combination of specific cards (as value and as symbol).

This choice is convenient because it allows us make the following idealization: *the occurrences of the elementary events are equally possible*.

In our case, the occurrences of various card combinations in a hand are possible in the same measure (if we assume a random shuffling and nonfraudulent conditions).

For example, no matter what cards you hold after the first distribution, the occurrence of the five-card combinations

(2♠ 3♦ 5♣ 7♥ J♣), (3♦ 5♣ 8♦ 8♥ 9♦) or (5♥ 7♥ 8♥ J♥ K♥) in one opponent's hand are possible in the same measure (the same probability applies to all players).

Without this *equally possible* idealization, the construction of a probability model to work within is not possible.

Thus far, we have established the elementary events – the occurrences of card combinations in player's hands. These could be one-, two-, three-, four- or five-card combinations, depending on the specific application we deal with.

For example, if we are in the moment after discarding and before the second distribution, you have discarded three cards and want to make predictions about the replacements to come, the elementary events of this experiment are the occurrences of all possible three-card combinations.

But if you want to make predictions about the hand of one opponent (what he or she holds or will hold), the elementary events are the occurrences of all possible five-card combinations.

Once we have established the elementary events, we can establish the sample space attached to each experiment as being the set of all possible elementary events. In our case, this is the set of all occurrences of possible card combinations.

The number of elements of this set depends directly on the seen cards because these cards are out of play with respect to future occurrences of the various card combinations.

For example, you have been dealt (3♦ 5♥ 8♠ Q♣ Q♦), we are in the moment before the second distribution and you want to make predictions about the hand dealt to one of your opponents. This experiment has the following sample space: the occurrences of all five-card combinations from all 52 cards, less the seen cards.

In the calculations for this experiment, we are not interested in enumerating all the elements of this set, but just in their number; and this number is $C_{52-5}^{5} = C_{47}^{5} = 1533939$.

If the seen cards are not taken into account (the problem as seen by a neutral observer), the sample space would have $C_{52}^{5} = 2598960$ elements.

Because we can assign one card combination to each elementary event, as well as the reverse, we can identify any elementary event by its attached card combination.

We have established thus far the sample space for each experiment involved in this card game and see that it is a finite set of card combinations.

The field of events is then the set of parts of the sample space and is implicitly finite.

As a set of parts of a set, the field of events is a Boole algebra.

Any event belonging to the field of events, no matter how complex, can be decomposed as a union of elementary events.

For example, if you were initially dealt (7♦ 7♣ 7♥ 9♦ K♣) and you discard (9♦ K♣) the event *you will receive the fourth 7 with your replacements* is the union of the elementary events (7♠, x), where *x* can be any card from all 52, less 7♦, 7♣, 7♥, 9♦, K♣ and 7♠.

This union counts 52 – 6 = 46 elementary events.

In the same hypothesis, the event *you will receive a pair with your replacements* is the union of the elementary events (yy), where *y* is a possible value (for example, (2♣ 2♦), (5♥ 5♣) or (9♥ 9♠)).

To count these elementary events, we must observe that the values 2, 3, 4, 5, 6, 8, 10, *J*, *Q* and *A* are not represented within the seen cards (out of play), so we have four cards each in play. The value 7 is represented by three cards, so it is only one in play (therefore (77) cannot occur); while 9 and *K* are represented by one card each, so there are three of each in play.

Thus, the calculation is $10C_4^2 + 0 + 2C_3^2 = 66$. We then have 66 elementary events that compose the event *you will receive a pair with your replacements*.

Because the events are identified with sets of card combinations and the axioms of a Boole algebra, the operations between events (union, intersection, complementary) revert to the operations between sets of card combinations, and the size of the sample space reverts to the size of the attached set of combinations.

Therefore, any counting of elementary events (for example, the elementary events a compound event consists of) reverts to counting combinations.

In detailing the calculations, we use the term *favorable combinations* for an event to occur. This means the set of card combinations (elementary events) that make up that event.

For example, the favorable combinations for the event *holding a full house of aces* are those of (AAAxx) form.

Most of the probability calculations revert to counting favorable combinations.

At this point, we have rigorously established a sample space and a field of events for each application in part:

- the elementary events are card combinations of the size given by the experiment involved;
- the sample space (Ω) is the set of possible card combinations that occur as result of the experiment involved;
- the field of events is the set of parts of the sample space ($\Sigma = \mathcal{P}(\Omega)$) and has a Boolean structure.

This field of events is suitable as a domain for a function *P* given by the classical definition of probability on a finite field of events with equally possible elementary events:

The probability *P* of event *A* is the ratio between the number of situations favorable for *A* to occur and the number of equally possible situations.

On a finite field of events, *P* is a function $P : \Sigma \to R$ that satisfies the following axioms:

(1) $P(A) \geq 0$, for any $A \in \Sigma$;

(2) $P(\Omega) = 1$;

(3) $P(A_1 \cup A_2) = P(A_1) + P(A_2)$, for any $A_1, A_2 \in \Sigma$ that $A_1 \cap A_2 = \phi$.

Therefore, *P* is a probability-function and we have built a probability space (field) (Ω, Σ, P) that ensures a rigorous basic probability model on which to work any application for the game of Draw Poker.

As we said earlier, the information about the seen cards (the initial five plus any replacements) is the one that gives the sample space and, implicitly, the probability space in which the calculations are performed.

Depending on the problem, this information must be quantified in a convenient way to provide all parameters needed in an application.

Because a card is fully determined by its value and symbol, and because poker card formations are defined through values or symbols or both, these are the parameters that describe any card distribution.

We define a *card distribution* as a complete sequence of seen cards – it is the cumulated set of cards from your own hand, those discarded and their replacements.

Of course, the possible lengths of a card distribution are 5, 6, 7, 8, 9 or 10, depending on the number of cards discarded and the maximum number of replacements allowed by the rules of the game. The number of seen cards is denoted by *c* in all sections.

Any card distribution has a *value distribution* and a *symbol distribution*.

Value distribution is the cumulated quantitative representation of all values from 2 to *A* in a given card distribution.

It is a 13-size vector whose elements may take values from 0 to 4, under the condition that their sum is always equal to *c*.

For example, the card distribution (2♣ 3♦ 3♣ 7♦ 8♠ 8♥ J♥) has the value distribution 2-2-1-1-1-0-0-0-0-0-0-0-0, meaning that two values (3 and 8) are represented by two cards each, three values (2, 7 and *J*) are represented by one card each and the rest of the values are not represented.

Symbol distribution is the cumulated quantitative representation of all symbols (hearts, diamonds, clubs and spades) in a given card distribution.

It is a 4-size vector whose elements may take values from 0 to 10, under the condition that their sum is always equal to *c* (which can be a maximum of 10).

The card distribution from the example above has the symbol distribution 2-2-2-1, meaning that three symbols (♥, ♦ and ♣) are represented by two cards each and one symbol (♠) is represented by one card.

You can read more about these distributions and how their parameters are inputted in the formulas when they first appear in our calculations.

We quantified this information because the predicted events are identified with the various types of valuable card formations, and the number of favorable combinations for a certain type of formation to occur depends directly on the parameters of the value or symbol distribution, as follows:

- *one pair* is a formation made of values (symbols do not count) and the number of favorable combinations for one pair to occur in one player's hand is a function of the value distribution of the seen cards;

- the same is true for *two pair*, *three of a kind*, *straight*, *full house* and *four of a kind*;

- *flush* (regular flush, not straight flush) is a formation made of symbols (values do not count), and the number of favorable combinations for a flush to occur in one player's hand is a function of the symbol distribution of the seen cards;

- *straight flush* is a formation made of values and symbols, and the number of favorable combinations for a flush to occur in one player's hand is a function of the exact card distribution of the seen cards (each card as value and as symbol).

The probability properties and formulas used

Because we deal in all applications with a finite probability space with equally possible elementary events, the probability calculus uses a few basic properties of probability, starting with the classical definition:

(F1) P = m/n (the probability of an event is the ratio between the number of cases favorable for that event to occur and the number of equally possible cases) (the classical definition of probability)

This formula is used on a large scale throughout the book, especially to calculate probabilities of events involving one player. It is applied by dividing the number of favorable combinations for a respective event to occur by the number of all possible combinations.

For one player (whether that player is you or an opponent), the probability of that player receiving or holding a certain sequence of cards or type of sequence of cards is calculated by the formula $P = \frac{F}{C_{52-c}^{s}}$, where *F* is the number of all favorable combinations for that (type of) sequence to occur, *c* is the number of seen cards and *s* is the length of that sequence (*s* could be 1, 2, 3, 4 or 5).

(F2) $P(A \cup B) = P(A) + P(B)$, for any $A, B \in \Sigma$ with $A \cap B = \phi$ and its generalization:

(F3) $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$, for any finite family of mutually exclusive events $(A_i)_{i=1}^{n}$ (finite additivity in condition of incompatibility)

These properties of finite additivity were used when calculating the overall probabilities of occurrences of several card formations in one player's hand.

The formulas allow us to sum the probabilities of one player holding several types of formations because these types cannot occur simultaneously (i.e., the events are mutually exclusive).

For example, if we want to find the probability of a player holding *three of a kind or a full house*, we are allowed to add the partial probabilities:

P(*three of a kind or full house*) = P(*three of a kind*) + P(*full house*).

If we want to include more players, this formula no longer applies because the events are no longer mutually exclusive.

As a direct application, we use the generalized formula (F3) to compute the probability of one player holding a higher formation than a given one (in particular, higher than the one you hold).

(F4) $P(A^C) = 1 - P(A)$ (probability of a contrary event)

This primary property is used everywhere we know the probability of the event that is contrary to the event to measure.

(F5) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (general formula of probability of union of two events) and its generalization:

(F6) $P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{j<i} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) + \ldots + (-1)^{n-1} P(A_1 \cap A_2 \cap \ldots \cap A_n)$

(inclusion-exclusion principle or the general formula of probability of finite union of events)

Formula (F6) is applied as follows:

We have *n* events and want to calculate the probability of their union.

We consider in succession all 1-size combinations, 2-size combinations and so on until n-size combinations of the *n* events (we have a single 1-size combination and a single n-size combination).

We add the probabilities of unions of each group of same-size combinations. For the groups of combinations having an even number as a dimension, the total result is add with minus (subtracted). For those with an uneven number as a dimension, the total result is add with plus (addition).

We used the inclusion-exclusion principle to work out the probabilities of events of the type *at least one opponent is holding a formation higher than yours.*

In this application, this formula saves us from very complex calculations:

If we tried to use the partial results for *one opponent* to obtain the probabilities for *at least one opponent*, we would have to compute *F* as the sum of all numbers of favorable combinations one opponent could hold as a higher formation than yours, then input the result into a classical probability scheme (see the formula derived from F(8)).

This procedure would lead to so many hard expressions as to abort it.

Instead of computing a very complex formula of *F* and composing it with a complex expression from (F8), we can use the inclusion-exclusion principle to write the probability of at least one opponent holding a higher formation than yours as a function of *n*, *c* and *q*, where *n* is the number of opponents, *c* is the number of seen cards and *q* is the probability of one opponent holding a higher formation than yours. This procedure returns a function P(n, c, q) whose expression is relatively simple.

The probability *q* can be computed through a simple (F3) formula, by using the partial results from the tables for one opponent (the $n = 1$ column).

(F7) $P(A \cap B) = P(A) \cdot P(B)$, if events *A* and *B* are independent (the definition of independent events).

We used this definition to work out the expression of P(n, c, q), namely, the overall probability of at least one opponent of *n* holding a formation higher than yours.

Specifically, it was used several times to figure out the expressions of the probabilities of two or more specific opponents simultaneously holding a formation higher than yours.

<u>The scheme of nonreturned ball</u>

An urn has the following contents: a_1 balls of color c_1, a_2 balls of color c_2, ..., a_s balls of color c_s.

n draws are performed without putting back the extracted ball (or just one draw of *n* balls).

The probability of occurrence of exactly α_k balls of color c_k, $k = 1, ..., s$ in the group of *n* extracted balls, where $\alpha = (\alpha_1, ..., \alpha_s)$, $0 \le \alpha_k \le a_k$, $\sum_{k=1}^{s} \alpha_k = n$, is:

$$\text{(F8)} \quad P(n; \alpha_1, ..., \alpha_s) = \frac{C_{a_1}^{\alpha_1} C_{a_2}^{\alpha_2} ... C_{a_s}^{\alpha_s}}{C_{a_1+...a_s}^{n}}.$$

This formula was an important tool in finding the probabilities of at least one opponent holding the various valuable types of formations.

For a given type of formation, if *F* is the number of favorable combinations for one opponent to hold that type of formation and $F' = C_{52-c}^{5} - F$, then we have the following correspondences: the balls are equivalent to the 5-size combinations of cards; drawing a ball is equivalent to dealing such card combination to one opponent; the first color is equivalent to the favorable combinations (numbering *F*) and the second color is equivalent to the rest of combinations (numbering *F'*).

To find the expression of the probability of at least one opponent of *n* holding a given type of formation (event denoted by E_n), we must apply in succession (F8) for *n* experiments, two colors and all 2-size partitions of *n*, then add the partial results:

P(n; 1, n – 1) + P(n; 2, n – 2) + ... + P(n; n, 0).

The final result is the following formula:

$$P(E_n) = \frac{C_F^1 C_{F'}^{n-1} + C_F^2 C_{F'}^{n-2} + ... + C_F^n C_{F'}^0}{C_{C_{52-c}^5}^n}$$, which returns the probability of the measured event as function of *c*, *n* and *F*.

Of course, *F* is a function of the card distribution (the seen cards).

This formula is used in every section that lists numerical probabilities of more opponents holding the various types of formations.

The goal of this section is to underline the theoretical results and formulas used in our applications. Their detailed application is presented in the sections in which they occur.

For readers who want to delve deeper into probability theory and its applications, we recommend the beginner's guide *Understanding and Calculating the Odds*.

Initial Probabilities of the First Card Distribution for Your Own Hand

In this section we calculate the probabilities of receiving various valuable formations or parts of them from the first five cards dealt.

There are no seen cards and 52 unseen cards.

The total number of possible combinations for each hand dealt is $C_{52}^{5} = 2598960$.

One pair

One certain pair

Let us denote by *D* the paired card (as value).

We will calculate the probability of being dealt exactly one pair (DD) (no two pair, three of a kind, full house or four of a kind).

The favorable combinations are (DDxyz), with $x, y, z \neq D$, *x*, *y* and *z* mutually distinct.

They number $C_4^2(C_{48}^3 - 12 \cdot 44C_4^2 - 12C_4^3) = 84480$.

The probability is P = 84480/2598960 = 0.0325053.

The probability of being initially dealt exactly one certain pair is P = 3.25053%**.**

Any pair

The sets of combinations (DDxyz), with $x, y, z \neq D$, *x*, *y* and *z* mutually distinct, are mutually exclusive for the different values of *D*.

There are 13 values for *D*, so the number of favorable combinations for the event *you are dealt exactly one pair (any value)* is 13 x 84480 = 1098240.

The probability is P = 1098240/2598960 = 0.4225689.

The probability of being initially dealt exactly one pair (any value) is P = 42.25689%**.**

At least one certain pair

This means you can also achieve two pair, three of a kind, a full house or four of a kind containing a certain pair (DD).

The favorable combinations are (PPxyz), *x*, *y*, *z* any, numbering $C_4^2 C_{50}^3 = 117600$.

The probability is P = 117600/2598960 = 0.0452488.

The probability of being initially dealt at least one certain pair is P = 4.52488%**.**

At least one pair (any value)

The sets of favorable combinations are (22xyz), (33xyz), (44xyz), ..., (AAxyz), *x*, *y*, *z* any value.

There are 13 sets of $C_4^2 C_{50}^3 = 117600$ elements each.

Observe that the intersections of three or more sets from them are empty (a common combination of the sets (DDxyz), (EExyz) and (FFxyz), with *D*, *E* and *F* mutually distinct, should contain DDEEFF, or six elements, which is impossible).

An intersection of two sets is $(\text{DDxyz}) \cap (\text{EExyz}) = (\text{DDEEx})$, *x* any value, and has $C_4^2 C_4^2 (52-4)$ elements.

There are C_{13}^2 such intersections.

We can now apply the inclusion-exclusion principle and find that the probability of the union of all above 13 sets (the event to be measured) is $P = 13 \cdot \frac{C_4^2 C_{50}^3}{C_{52}^5} - C_{13}^2 \cdot \frac{C_4^2 C_4^2 \cdot 48}{C_{52}^5} = 0.5363745$.

The probability of being initially dealt at least one pair (any value) is P = 53.63745%**.**

Two pair

Two certain pairs

Let *D* and *E* be two distinct values. The favorable combinations are (DDEEx), with $x \neq D, E$, numbering $C_4^2 C_4^2 (52-4-4) = 1584$.

The probability is P = 1584/2598960 = 0.0006094.

The probability of being initially dealt two certain pairs is P = 0.06094%**.**

Two pair (any value)

There are C_{13}^2 ways to choose *D* and *E*.

The sets (DDEEx), $x \neq D, E$, are mutually exclusive for the different values of *D* and *E*, so the number of favorable combinations for the event *you are dealt two pair (any value)* is $C_{13}^2 \cdot 1584 = 123552$.

The probability is P = 123552/2598960 = 0.0475390.

The probability of being initially dealt two pair is P = 4.75390%**.**

.........................missing part..........................

Prediction Probabilities After the First Card Distribution and Before the Second for Your Own Hand

These are the probabilities of receiving various card combinations at the second distribution, after you have been dealt the first five cards and you have discarded (or not).

The calculations take into account your own first five cards as seen cards.

The way you discard does not affect the calculated probabilities, but the calculated results may become the basis for the decision as to which cards to hold and which to discard.

The possible types of dealt combinations that require making a decision to keep or discard cards the most are the following:

1) *one pair + high card* – example: (7729K);

2) *one pair + three from a straight* (the straight does not contain a paired card) – example: (88234);

3) *one pair + three from a straight* (the straight contains one of the paired cards) – example: (88257);

4) *one pair + four from a straight* (obviously, the straight contains one of the paired cards) – example: (55234);

5) *one pair + three suited* (the suit does not contain any paired card) – example: (7♥ 7♠ 8♣ J♣ K♣);

6) *one pair + three suited* (the suit contains one of the paired cards) – example: (7♥ 7♠ 8♥ J♥ K♣);

7) *one pair + four suited* (obviously, the suit contains one of the paired cards) – example: (7♥ 7♠ 8♥ J♥ K♥);

8) *two pairs + high card* – example: (3355K);

9) *two pairs + three from a straight* – example: (33556);

10) *two pairs + three suited* – example: (3♣ 3♦ 5♣ 5♥ 6♣);

11) *three of a kind + three from a straight* – example: (7779J);

12) *three of a kind + three suited* – example: (7♥ 7♠ 7♦ 8♥ K♥);

13) *three from a straight* (not all three suited) + *three suited* – example: (5♦ 6♥ 8♦ J♠ K♦);

14) *three from a straight + four suited* – example: (3♥ 5♥ 7♣ J♥ Q♥);

15) *four from a straight + three suited* – example: (3♦ 4♠ 5♦ 7♣ K♦);

16) *four from a straight + four suited* – example: (3♥ 4♣ 5♥ 7♥ K♥);

17) *two high cards* – example: (357JQ);

Simply stated, when calculating the probabilities of predicted events, we will use as a hypothesis the examples attached to these cases rather than more general denotations. The probability is the same for any other similar hand.

The recommendations in the conclusions at the end of each case take into account only the comparison between probabilities in order to achieve the more valuable formation as possible final hand.

Other strategies that include psychological actions (with the goal of confusing opponents) may contradict these recommendations.

1) One pair + high card (7729K)

The player has three playing options to achieve a valuable formation: holding only the pair and discarding the rest, holding only the high card and discarding the rest, or holding the pair and the high card and discarding the rest.

A) Held: (77) Discarded: (29K) Goal: two pair, three of a kind, full house or four of a kind

The number of all possible combinations for the second distribution is $C_{47}^{3} = 16215$.

Two pair

The favorable combinations of the cards to come on the second distribution to achieve exactly two pair are: (xxy), with $x \neq y,\ x, y \neq 7$, numbering

$9C_4^2(47-2-2-2) + 3C_3^2(47-2-1-2) = 2592$.

The probability of achieving two pair is

P = 2592/16215 = 0.1598519 = 15.98519%.

Three of a kind

Favorable combinations: (7xy), with $x \neq y,\ x, y \neq 7$, numbering

$2(C_{45}^2 - 9C_4^2 - 3C_3^2) = 1854$.

The probability of achieving three of a kind is

P = 1854/16215 = 0.1143385 = 11.43385%.

Full house

Favorable combinations:

(7xx), with $x \neq 7$, numbering $2(9C_4^2 + 3C_3^2) = 126$ and

(xxx), with $x \neq 7$, numbering $9C_4^3 + 3C_3^3 = 39$.

In total, there are 165 favorable combinations for a full house.

The probability of achieving a full house is

P = 165/16215 = 0.0101757 = 1.01757%.

Four of a kind

Favorable combinations: (77x), with $x \neq 7$, numbering 1(47 – 4 – 3) = 40. The probability of achieving four of a kind is P = 40/16215 = 0.0024668 = 0.24668%.

If we add up all the probabilities calculated above, we find that the probability of achieving two pair, three of a kind, a full house or four of a kind (two pair or better) is P = 28.68329%.

B) Held: (K) Discarded: (7729) Goal: one pair, two pair, three of a kind, full house or four of a kind

The number of all possible combinations for the second distribution is $C_{47}^{4} = 178365$.

One pair

Favorable combinations:

(Kxyz), with $x, y, z \neq K$ and x, y, z mutually distinct, numbering

$$3\{C_{47-3}^{3} - C_{3}^{3} - C_{3}^{3} - 9C_{4}^{3} - [C_{2}^{2}(47-3-2) + 2C_{3}^{2}(47-3-3) +$$
$$+ 9C_{4}^{2}(47-3-4)]\} = 32274$$ and

(xxyz), with $x, y, z \neq K$ and x, y, z mutually distinct, numbering

$$C_{2}^{2}(C_{47-5}^{2} - C_{3}^{3} - C_{3}^{3} - 9C_{4}^{2}) + 2C_{3}^{2}(C_{47-6}^{2} - C_{3}^{3} - C_{2}^{2} - 9C_{4}^{2}) +$$
$$+ 9C_{4}^{2}(C_{47-7}^{2} - C_{2}^{2} - 2C_{3}^{2} - 8C_{4}^{2}) = 44523.$$

In total, there are 76797 favorable combinations. The probability of achieving exactly one pair is P = 6797/178365 = 0.4305609 = 43.05609%.

Two pair

Favorable combinations:

(Kxxy), with $x, y \neq K$, $x \neq y$, numbering

$$3[C_{2}^{2}(47-3-2) + 2C_{3}^{2}(47-3-3) + 9C_{4}^{2}(47-3-4)] = 7584$$ and

(xxyy), with $x, y \neq K$, $x \neq y$, numbering

$$C_{2}^{2}(C_{3}^{2} + C_{3}^{2} + 9C_{4}^{2}) + 2C_{3}^{2}(C_{2}^{2} + C_{3}^{2} + 9C_{4}^{2}) + 9C_{4}^{2}(C_{2}^{2} + 2C_{3}^{2} + 8C_{4}^{2}) =$$
$$= 2970.$$

In total, there are 10554 favorable combinations, so the probability of achieving two pair is
P = 10554/178365 = 0.0591708 = 5.91708%.

Three of a kind

Favorable combinations:

(KKxy), with $x, y \neq K$, $x \neq y$, numbering $C_3^2(C_{47-3}^2 - C_2^2 - 2C_3^2 - 9C_4^2) = 2655$ and

(xxxy), with $x, y \neq K$, $x \neq y$, numbering $2C_3^3(47-3-3) + 9C_4^3(47-3-3-1) = 1522$.

In total, there are 4177 favorable combinations, so the probability of achieving three of a kind is
P = 4177/178365 = 0.0234182 = 2.34182%.

Full house

Favorable combinations:

(KKxx), with $x \neq K$, numbering $C_3^2(C_2^2 + 2C_3^2 + 9C_4^2) = 183$ and

(Kxxx), with $x \neq K$, numbering $3(2C_3^3 + 9C_4^3) = 114$.

In total, there are 297 favorable combinations and the probability of achieving a full house is
P = 297/178365 = 0.0016651 = 0.16651%.

Four of a kind

Favorable combinations:

(KKKx), with $x \neq K$, numbering $C_3^3(47-3) = 44$ and

(xxxx), with $x \neq K$, numbering $9C_4^4 = 9$.

In total, there are 53 favorable combinations and the probability of achieving four of a kind is
P = 53/178365 = 0.0002971 = 0.02971%.

If we make a partial total of these calculated probabilities, we find that the probability of achieving two pair, three of a kind, a full house or four of a kind (two pair or better) is P = 8.45512%.

C) Held: (77K) Discarded: (29) Goal: two pair, three of a kind, full house or four of a kind

The number of all possible combinations for the second distribution is $C_{47}^2 = 1081$.

Two pair

Favorable combinations:

(Kx), with $x \neq 7, K$, numbering 3(47 – 2 – 3) = 126 and

(xx), with $x \neq 7, K$, numbering $2C_3^2 + 9C_4^2 = 60$.

In total, there are 186 favorable combinations, so the probability of achieving two pair is P = 186/1081 = 0.1720629 = 17.20629%.

Three of a kind

The favorable combinations are (7x), with $x \neq 7, K$, numbering 2(47 – 2 – 3) = 84, so the probability of achieving three of a kind is P = 84/1081 = 0.0777058 = 7.77058%.

Full house

Favorable combinations:

(7K), numbering 2 x 3 = 6 and

(KK), numbering $C_3^2 = 3$.

In total, there are nine favorable combinations, so the probability of achieving a full house is P = 9/1081 = 0.0083256 = 0.83256%.

Four of a kind

The favorable combinations are (77), numbering 1, so the probability of achieving three of a kind is
P = 1/1081 = 0.000925 = 0.0925%.

By totaling all these probabilities, we find that the probability of achieving two pair, three of a kind, a full house or four of a kind (two pair or better) is P = 25.90193%.

Conclusions:

– The probabilities of achieving *two pair, three of a kind, a full house or four of a kind* are not much different in subcases *A* and *C,* and each is significantly higher than the probability of the same event in subcase *B*;

– The probability of achieving *three of a kind* is higher in subcase *A* than in subcase *C*.

Therefore, holding the pair and discarding the rest (subcase *A*) is recommended.

2) One pair + three from a straight (straight does not contain a paired card) (88234)

The player has two playing options to achieve a valuable formation: holding only the pair and discarding the rest, or holding only the three cards from the straight and discarding the rest.

A) Held: (88) Discarded: (234) Goal: two pair, three of a kind, full house or four of a kind

The probabilities are identical with those from the situation 1) A):

two pair: 15.98519%
three of a kind: 11.43385%
full house: 1.01757%
four of a kind: 0.24668%, totaling 28.68329%.

B) Held: the three straight cards Discarded: the paired cards (88) Goal: straight

The number of all possible combinations for the second distribution is $C_{47}^{2} = 1081$.

There are three subcases for the three straight cards, depending on the number of types of combinations that may complete respective straight:

– One type of combination to complete the straight – example: (88JKA), waiting for (10 Q);

– Two types of combinations to complete the straight – example: (88234), waiting for (A5) or (56);

– Three types of combinations to complete the straight – example: (88345), waiting for (A2), (26) or (67).

B1) (88JKA) Held: (JKA) Discarded: (88) Goal: straight

The favorable combinations are (10 Q), numbering 4 x 4 = 16, so the probability of achieving a straight is
P = 16/1081 = 0.0148011 = 1.48011%.

B2) (88234) Held: (234) Discarded: (88) Goal: straight

The favorable combinations are (A5) and (56), numbering 2 x 4 x 4 = 32, so the probability of achieving a straight is
P = 32/1081 = 0.0296022 = 2.96022%.

B3) (88345) Held: (345) Discarded: (88) Goal: straight

The favorable combinations are (A2), (26) and (67), numbering 3 x 4 x 4 = 48, so the probability of achieving a straight is P = 48/1081 = 0.0444033 = 4.44033%.

Let us also calculate the probability of achieving three of a kind in subcase *B*:

The favorable combinations are (xx), *x* being any of the held cards, numbering $3C_3^2 = 9$. The probability of achieving three of a kind is P = 9/1081 = 0.0083256 = 0.83256%.

Conclusions:

– The probability of achieving a straight in subcase *B* is much lower than the probability of achieving three of a kind in subcase *A*.

– The probability of achieving three of a kind is much lower in subcase *B* than in subcase *A*.

Therefore, holding the pair and discarding the rest (subcase *A*) is recommended.

3) One pair + three from a straight (the straight contains one of the paired cards) (88257)

The player has two playing options to achieve a valuable formation: holding only the pair and discarding the rest, or holding only the three cards from the straight and discarding the rest.

A) Held: (88) Discarded: (257) Goal: two pair, three of a kind, full house or four of a kind

The probabilities are identical with those from the situation 1) A):

two pair: 15.98519%
three of a kind: 11.43385%
full house: 1.01757%
four of a kind: 0.24668%,

totaling 28.68329%.

B) Held: the three straight cards Discarded: the paired card left and the unpaired card not belonging to respective straight Goal: straight

The number of all possible combinations for the second distribution is $C_{47}^{2} = 1081$.

As in the previous case, there are also three subcases for the three straight cards:

– One type of combination to complete the straight – example: (8846K), waiting for (57);

– Two types of combinations to complete the straight – example: (88257), waiting for (46) or (69);

– Three types of combinations to complete the straight – example: (8867J), waiting for (45), (59) or (9 10).

The probabilities of achieving a straight are the same as in case 2); that is, 1.48011%, 2.96022% and 4.44033%.

Therefore, we must compare the same probabilities as in case 2), so the conclusions are the same: holding the pair and discarding the rest is recommended.

4) One pair + four from a straight (obviously, the straight contains one of the paired cards) (55234)

The player has two playing options to achieve a valuable formation: holding only the pair and discarding the rest, or holding the four cards from the straight and discarding the paired card left.

A) Held: (55) Discarded: (234) Goal: two pair, three of a kind, full house or four of a kind

The probabilities are identical with those in situation 1) A):

two pair: 15.98519%
three of a kind: 11.43385%
full house: 1.01757%
four of a kind: 0.24668%,

totaling 28.68329%.

B) Held: the four straight cards Discarded: the paired card left Goal: straight

The number of all possible cards to come on second distribution is 47.

There are two subcases for the four straight cards:

– One card (as value) to complete the straight – example: (55246), waiting for a 3;

– Two cards (as value) to complete the straight – example: (55234), waiting for A or 6;

B1) (55246) Held: (2456) Discarded: (5) Goal: straight

The favorable cards are 3's, numbering 4, so the probability of achieving a straight is $P = 4/47 = 0.0851063 = 8.51063\%$.

B2) (55234) Held: (2345) Discarded: (5) Goal: straight

The favorable cards are A's and 6's, numbering $4 + 4 = 8$, so the probability of achieving a straight is

$P = 8/47 = 0.1702127 = 17.02127\%$.

Conclusions:

– The probability of achieving a straight in subcase *B2* is higher than the probability of achieving three of a kind or better in subcase *A*.

– The probability of achieving a straight in subcase *B1* is lower than the probability of achieving three of a kind or better in subcase *A*.

Therefore, if you have a hand as in subcase *B2*, playing for straight is recommended, while if you have a hand as in subcase *B1*, holding the pair and discarding the rest is recommended.

5) One pair + three suited (the suit does not contain any paired card) (7♥ 7♠ 8♣ J♣ K♣)

The player has two playing options to achieve a valuable formation: holding only the pair and discarding the suited cards, or holding the suited cards and discarding the paired cards.

A) Held: (77) Discarded: (8♣ J♣ K♣) Goal: two pair, three of a kind, full house or four of a kind

The probabilities are identical with those in situation 1) A):

two pair: 15.98519%
three of a kind: 11.43385%
full house: 1.01757%
four of a kind: 0.24668%,

totaling 28.68329%.

B) Held: (8♣ J♣ K♣) Discarded: (77) Goal: flush

The number of all possible combinations for the second distribution is $C_{47}^{2} = 1081$.

The favorable combinations are (♣♣), numbering $C_{13-3}^{2} = 45$, so the probability of achieving a flush is
P = 45/1081 = 0.0416281 = 4.16281%.

Let us also calculate the probability of achieving three of a kind in subcase *B*:

The favorable combinations are (xx), *x* being any of the held cards, numbering $3C_{3}^{2} = 9$. The probability of achieving three of a kind is P = 9/1081 = 0.0083256 = 0.83256%.

Conclusions:

– The probability of achieving a flush in subcase *B* is lower than the probability of achieving three of a kind or better in subcase *A* and has a low value itself.

– The probability of achieving three of a kind in subcase *B* is lower than the probability of achieving three of a kind in subcase *A*.

Therefore, holding the pair and discarding the suited cards is recommended.

........................missing part.........................

These calculations and results are for the probabilities of the gaming events specific for your own hand.

Of course, they do not entirely cover all possible distributions, nor do they cover the situations requiring decisions about holding and discarding cards.

These 17 cases are the most frequently met and are also the situations in which the balance of probability figures is not obviously biased one way or another, so a decision based on multiple criteria is required.

The recommendations at the end of each case are based solely on the difference between numerical probabilities, having as a goal the achievement of the most valuable formation possible.

On the basis of a player's subjective strategies and goals, these recommendations may or may not be followed.

A player's strategic criteria also contain personal elements such as the accepted limit of risk.

In addition, professional players can manage an ulterior dialogue, which finally is the gaming part that establishes the winner.

Therefore, an efficient dialogue can offset an eventual negative result of a high risk assumed in keeping and discarding cards.

Prediction Probabilities for Opponents' Hands

These are the probabilities for one or many of your opponents to hold various card formations as their final hand.

The information taken into account in calculations is your own seen cards.

The results are not affected by the way your opponents discard, because you don't see any of their cards.

Therefore, the probability of one or more of your opponents holding a full house is the same for both the first dealt hand and the final hand in the probability field that only uses the information given by your own cards.

Of course, the way they discard may give you additional information about what they hold, including here psychological aspects of the game. But this information is not taken into account in our calculations.

These probability results, along with your own final hand, stand for the base of your betting decisions.

For using the results, you have to keep in mind all cards you see in your hand, including those discarded and those received at the second distribution. In fact, you don't have always to memorize each of them as value and symbol, but in a whole, as a distribution with certain properties.

For example, if you are interested in probabilities of your opponents holding a three of a kind formation, you will have to retain the distribution of the values (from 2 to *A*) in your seen cards (how many 2's, 3's, ..., and so on to *A*'s).

If you are interested in probabilities of your opponents holding a flush, you will have to retain the distribution of the symbols (how many spades, clubs, hearts and diamonds).

The probabilities for opponents' hands are much difficult to calculate than those for your own hand. Moreover, the total number of possible cases may be huge in some situations.

For example, for a prediction on a card formation made of values (one pair, two pair, three of a kind, etc., except the flush) in opponents' hands, there are dozens of possible value distribution types to count for your own hand. Multiplying by other parameters as number of discarded cards (1, 2, 3, 4, 5) and number of opponents (1, 2, 3, 4) result in hundreds of cases to cover.

But there are also predictions that use possible distributions (of values and symbols) numbering in the millions (as in *straight flush* case).

Still, for most of the final formations in your opponents' hands, there are ways to group all possible distributions conveniently in a reasonable number of categories so that we can work out the probability formulas and list the numerical returns in tables covering all cases.

Where this grouping effort is not possible and the possible distributions number in the millions, we build simple calculus algorithms for the reader to work some partial calculations to find the number of favorable card combinations for the event to measure.

Then, the partial result is listed in the final table that holds the numerical returns for the last probability formula.

All formulas and results are worked out for one specific opponent and for more opponents ("at least one opponent holding..." events).

Odds of holding three of a kind

We calculate here the probabilities of one or more opponents holding *three of a kind* as a final (or initial) hand, as a function of the information given by your own seen cards (the first five dealt plus their replacements).

The set of parameters the formula depends on are:

n = number of opponents (*n* could be 1, 2, 3, or 4);

c = number of seen cards (*c* could be 5, 6, 7, 8, 9 or 10);

$c_1, c_2, ..., c_{13}$ – the distribution of the values in the seen cards; we have 13 card values (from 2 to *A*); c_i is the number of seen cards with the value *i*, $i = 1, ..., 13$ (this designation has the goal of numbering; the indexes *i* do not represent the values inscribed on the cards; so, c_1 is not necessarily the number of aces, c_2 is not the number of 2's, and the like).

Of course, we have $c_i \in \{0, 1, 2, 3, 4\}$, for any $i = 1, ..., 13$ and $\sum_{i=1}^{13} c_i = c$.

We are not interested in how many cards of each value in a distribution, but in a cumulative representation: how many values are represented by four cards, how many by three, and so on, how many by zero.

This is the hypothesis for any calculus of the probability of the occurrence of a card formation made of values in your opponents' hands.

For now, we call *value distribution* any finite sequence of numbers of values from the seen cards, in the cumulative form.

For example, if the seen cards are 2♣ 3♠ 3♥ 5♦ 5♣ 5♥ 8♦ J♠ Q♥, their value distribution is 3-2-1-1-1-1-0-0-0-0, meaning there are six represented values, in this way: three cards of one value (5), two cards of another value (3), one card of another value (2), one card of another value (8), one card of another value (J) and one card of another value (Q); the rest of the values (4, 6, 7, 9, 10, K, A) are not represented.

As a convention, we remove the zeroes from this denotation, so the above value distribution is denoted by 3-2-1-1-1-1.

If a maximum number of five replacements is allowed (*c* could be maximum 10), there are 82 possible value distributions:

For c = 5:	For c = 6:	For c = 7:	For c = 8:
4-1	4-2	4-3	4-4
3-2	4-1-1	4-2-1	4-3-1
3-1-1	3-3	4-1-1-1	4-2-2
2-2-1	3-2-1	3-3-1	4-2-1-1
2-1-1-1	3-1-1-1	3-2-2	4-1-1-1-1
1-1-1-1-1	2-2-2	3-2-1-1	3-3-2
	2-2-1-1	3-1-1-1-1	3-3-1-1
	2-1-1-1-1	2-2-2-1	3-2-2-1
	1-1-1-1-1-1	2-2-1-1-1	3-2-1-1-1
		2-1-1-1-1-1	3-1-1-1-1-1
		1-1-1-1-1-1-1	2-2-2-2
			2-2-2-1-1
			2-2-1-1-1-1
			2-1-1-1-1-1-1
			1-1-1-1-1-1-1-1

For c = 9:	For c = 10:
4-4-1	4-4-2
4-3-2	4-4-1-1
4-3-1-1	4-3-3
4-2-2-1	4-3-2-1
4-2-1-1-1	4-3-1-1-1
4-1-1-1-1-1	4-2-2-2
3-3-3	4-2-2-1-1
3-3-2-1	4-2-1-1-1-1
3-3-1-1-1	4-1-1-1-1-1-1
3-2-2-2	3-3-3-1
3-2-2-1-1	3-3-2-2
3-2-1-1-1-1	3-3-2-1-1
3-1-1-1-1-1-1	3-3-1-1-1-1
2-2-2-2-1	3-2-2-2-1
2-2-2-1-1-1	3-2-2-1-1-1
2-2-1-1-1-1-1	3-2-1-1-1-1-1
2-1-1-1-1-1-1-1	3-1-1-1-1-1-1-1
1-1-1-1-1-1-1-1-1	2-2-2-2-2
	2-2-2-2-1-1
	2-2-2-1-1-1-1
	2-2-1-1-1-1-1-1
	2-1-1-1-1-1-1-1-1
	1-1-1-1-1-1-1-1-1-1

Any distribution of the seen cards belongs to one of the previously enumerated 82 value distributions.

Let us come back to the problem of calculating the probability of one or more opponents holding exactly three of a kind.

Let us denote the events:

E – one specific opponent holds exactly three of a kind (the probability of *E* is the same for any opponent); E_2 – at least one from two opponents holds exactly three of a kind; E_3 – at least one from three opponents holds exactly three of a kind; E_4 – at least one from four opponents holds exactly three of a kind; generally, E_n – at least one from *n* opponents holds exactly three of a kind.

Here, “exactly three of a kind” means three cards of same kind, without four of a kind or full house.

We will calculate the probabilities of these events in the probability field determined by the information given by the seen cards.

We have *52 – c* unknown cards and C_{52-c}^5 possible hands for one opponent.

Now we have to calculate the number of favorable combinations for *E* to occur, for one specific opponent. These are the five-card combinations (xxxyz), with *x*, *y* and *z* mutually distinct.

We do not present here the entire process of counting these combinations, for space reasons.

Those interested in the detailed calculation should consult the book *Understanding and Calculating the Odds*, where are plenty of applications and detailed explanations of how to count specific card combinations.

The total number of favorable combinations for a specific opponent to hold exactly *three of a kind* is given by the formula:

$$F = \frac{1}{12}\sum_{i=1}^{13}(2-c_i)(3-c_i)(4-c_i)[(47-c+c_i)(48-c+c_i)-2S+(3-c_i)(4-c_i)]$$

, where $S = \frac{1}{2}\sum_{j=1}^{13}(3-c_j)(4-c_j)$.

The expression of *F* will always differ for the various formations whose probability we intend to calculate.

Observe that *S* is a constant for the sum indexed by *i*.

The probability of *E* is then $P(E) = \frac{F}{C_{52-c}^5}$.

Denote $F' = C_{52-c}^5 - F$.

For finding the probability of E_2 we will use *the scheme of nonreturned ball* for two experiments and two colors.

For applying it, let us observe the correspondences: the balls are equivalent to the 5-size combinations of cards in play; drawing a ball is equivalent to the occurrence of such card combination in one opponent's hand; the first color is equivalent to the favorable combinations (numbering *F*) and the second color is equivalent to the rest of combinations (numbering F').

The probability of exactly one opponent holding a favorable combination (for the event *E*; in fact, it may not be favorable for you!) is $P(2; 1, 1) = \frac{C_F^1 C_{F'}^1}{C_{C_{52-c}^5}^2} = \frac{F \cdot F'}{C_{C_{52-c}^5}^2}$.

The probability of exactly two opponents holding a favorable combination is $P(2; 2, 0) = \frac{C_F^2 C_{F'}^0}{C_{C_{52-c}^5}^2} = \frac{C_F^2}{C_{C_{52-c}^5}^2}$.

The probability of at least one opponent of two opponents holding a favorable combination (three of a kind) is obtained by summing the above probabilities:

$$P(E_2) = P(2; 1, 1) + P(2; 2, 0) = \frac{F \cdot F' + C_F^2}{C_{C_{52-c}^5}^2}.$$

For n = 3 opponents, we have similarly:

$$P(3; 1, 2) = \frac{C_F^1 C_{F'}^2}{C_{C_{52-c}^5}^3}$$

$$P(3; 2, 1) = \frac{C_F^2 C_{F'}^1}{C_{C_{52-c}^5}^3}$$

$$P(3; 3, 0) = \frac{C_F^3}{C_{C_{52-c}^5}^3} \quad \text{and}$$

$$P(E_3) = P(3; 1, 2) + P(3; 2, 1) + P(3; 3, 0) = \frac{F \cdot C_{F'}^2 + C_F^2 \cdot F' + C_F^3}{C_{C_{52-c}^5}^3}.$$

For n = 4 opponents, we have:

$$P(4; 1, 3) = \frac{C_F^1 C_{F'}^3}{C_{C_{52-c}^5}^4}$$

$$P(4;\,2,\,2) = \frac{C_F^2 C_{F'}^2}{C_{C_{52-c}^5}^4}$$

$$P(4;\,3,\,1) = \frac{C_F^3 C_{F'}^1}{C_{C_{52-c}^5}^4}$$

$$P(4;\,4,\,0) = \frac{C_F^4}{C_{C_{52-c}^5}^4} \quad \text{and}$$

$$P(E_4) = P(4;\,1,\,3) + P(4;\,2,\,2) + P(4;\,3,\,1) + P(4;\,4,\,0) =$$

$$= \frac{F \cdot C_{F'}^3 + C_F^2 \cdot C_{F'}^2 + C_F^3 \cdot F' + C_F^4}{C_{C_{52-c}^5}^4}.$$

In general, we see that the probability of at least one opponent of *n* opponents holding three of a kind is

$$P(E_n) = \frac{C_F^1 C_{F'}^{n-1} + C_F^2 C_{F'}^{n-2} + \ldots + C_F^n C_{F'}^0}{C_{C_{52-c}^5}^n}.$$

This is the final formula that returns the numerical values of the probabilities of the measured events (occurrences of a *three of a kind* formation in opponents' hands).

This formula is used not only in this case (three of a kind), but for every formation whose occurrence in opponent's hands we make predictions upon. Its variables differ from one formation to another.

For the current case, the variables are the number of opponents (*n*), the value distribution of the seen cards ($c_1, c_2, ..., c_{13}$) and the number of seen cards (*c*).

The variables ($c_1, c_2, ..., c_{13}$) and *c* are the variables of *F*.

Let us calculate the probability, given a specific distribution of the seen cards:

Assume you are playing against four opponents. You are initially dealt (2♠ 3♣ 7♥ 8♦ Q♦), you keep the queen (*Q*) and discard the rest, and the replacements are (2♣ 4♣ 4♥ Q♥). Your final hand is two pair (244QQ).

We want to calculate the probability of one or more of your opponents holding *three of a kind*.

The seen cards are (2♠ 2♣ 3♣ 4♣ 4♥ 7♥ 8♦ Q♦ Q♥).

We have c = 9 and their value distribution is two 2's, two 4's, two Q's, one 3, one 7, one 8 and zero of the rest of the values.

This means a 2-2-2-1-1-1 distribution.

We can write:

$$c_1 = 2, c_2 = 2, c_3 = 2, c_4 = 1, c_5 = 1, c_6 = 1, c_7 = c_8 = ... = c_{13} = 0.$$

By replacing these values in the formula of *S*, we find:

$$S = \frac{3}{2} \cdot 1 \cdot 2 + \frac{3}{2} \cdot 2 \cdot 3 + \frac{7}{2} \cdot 3 \cdot 4 = 54.$$

Then,

$$F = 0 + \frac{3}{12} \cdot 1 \cdot 2 \cdot 3 \cdot [(47 - 9 + 1)(48 - 9 + 1) - 108 + 2 \cdot 3] +$$

$$+ \frac{7}{12} \cdot 2 \cdot 3 \cdot 4 \cdot [(47 - 9)(48 - 9) - 108 + 3 \cdot 4] = 21591.$$

This is the number of favorable card combinations for one opponent to hold a three of a kind formation.

This is the first major calculation to be made.

We easily calculate $C_{52-9}^5 = C_{43}^5 = 962598$ as the total number of possible card combinations for one opponent's hand and F' = 962598 – 21591.

We now apply the final probability formula and find:

$P(E) = \frac{F}{C_{43}^5} = \frac{21591}{962598} = 2.24299\%$ (the probability for one specific opponent to hold three of a kind) and

$P(E_4) = \frac{F \cdot C_{F'}^3 + C_F^2 \cdot C_{F'}^2 + C_F^3 \cdot F' + C_F^4}{C_{C_{52-c}^5}^4} = 8.67461\%$ (the probability for at least one of the four opponents to hold three of a kind).

This is the second major calculation to be made and, obviously, the most difficult.

As we saw, there are only 82 possible value distributions for the seen cards in this case, so we can list them all in a table along with the corresponding probabilities for a *three of a kind* formation in opponents' hands.

The next table notes the probabilities of one and at least one of your opponents holding a three of a kind formation, corresponding to the value distribution of your own seen cards.

The existence of a dash in the column $n = 5$ indicates that such a case is impossible (if a five-card replacement is allowed – so c could be up to 10, the maximum number of players can be five, so you are playing against a maximum of four opponents).

Probabilities of opponents holding three of a kind

Distribution	n = 1	n = 2	n = 3	n = 4	n = 5
4-1	2.46685%	4.87285%	7.21950%	9.50826%	11.74056%
3-2	2.41522%	4.77211%	7.07207%	9.31649%	11.50670%
3-1-1	2.31339%	4.57326%	6.78086%	8.93739%	11.04403%
2-2-1	2.25850%	4.46599%	6.62363%	8.73254%	10.79382%
2-1-1-1	2.15621%	4.26594%	6.33017%	8.34989%	10.32607%
1-1-1-1-1	2.05367%	4.06516%	6.03534%	7.96507%	9.85517%
4-2	2.56793%	5.06992%	7.50766%	9.88280%	12.19696%
4-1-1	2.45996%	4.85941%	7.19983%	9.48268%	11.70938%
3-3	2.57114%	5.07617%	7.51680%	9.89468%	12.21142%
3-2-1	2.40466%	4.75150%	7.04191%	9.27724%	11.45882%
3-1-1-1	2.29604%	4.53936%	6.73117%	8.87266%	10.96498%
2-2-2	2.34615%	4.63727%	6.87463%	9.05949%	11.19311%
2-2-1-1	2.23731%	4.42456%	6.56288%	8.65337%	10.69708%
2-1-1-1-1	2.12817%	4.21105%	6.24961%	8.24478%	10.19750%
1-1-1-1-1-1	2.01874%	3.99673%	5.93480%	7.83374%	9.69434%
4-3	2.73704%	5.39916%	7.98843%	10.50682%	12.95629%
4-2-1	2.56024%	5.05494%	7.48577%	9.85436%	12.16232%
4-1-1-1	2.44492%	4.83006%	7.15689%	9.42683%	11.64128%
3-3-1	2.56360%	5.06148%	7.49533%	9.86678%	12.17744%
3-2-2	2.50131%	4.94006%	7.31781%	9.63609%	11.89638%
3-2-1-1	2.38558%	4.71425%	6.98736%	9.20626%	11.37222%
3-1-1-1-1	2.26951%	4.48752%	6.65520%	8.77368%	10.84408%
2-2-2-1	2.32288%	4.59180%	6.80803%	8.97277%	11.08723%
2-2-1-1-1	2.20657%	4.36446%	6.47473%	8.53844%	10.55661%
2-1-1-1-1-1	2.08994%	4.13620%	6.13970%	8.10132%	10.02195%
1-1-1-1-1-1-1	1.97298%	3.90703%	5.80292%	7.66141%	9.48323%
4-4	2.91711%	5.74912%	8.49852%	11.16772%	13.75906%
4-3-1	2.73294%	5.39120%	7.97681%	10.49176%	12.93798%
4-2-2	2.66665%	5.26219%	7.78851%	10.24748%	12.64087%
4-2-1-1	2.54363%	5.02256%	7.43843%	9.79286%	12.08740%
4-1-1-1-1	2.42024%	4.78191%	7.08642%	9.33516%	11.52947%
3-3-2	2.67033%	5.26936%	7.79898%	10.26106%	12.65740%
3-3-1-1	2.54713%	5.02938%	7.44840%	9.80582%	12.10319%
3-2-2-1	2.48028%	4.89904%	7.25781%	9.55808%	11.80130%
3-2-1-1-1	2.35643%	4.65733%	6.90402%	9.09776%	11.23982%
3-1-1-1-1-1	2.23221%	4.41460%	6.54827%	8.63432%	10.67380%
2-2-2-2	2.41324%	4.76825%	7.06643%	9.30914%	11.49774%
2-2-2-1-1	2.28912%	4.52584%	6.71136%	8.84685%	10.93346%
2-2-1-1-1-1	2.16462%	4.28240%	6.35433%	8.38141%	10.36462%
2-1-1-1-1-1-1	2.03976%	4.03792%	5.99533%	7.91281%	9.79117%
1-1-1-1-1-1-1-1	1.91453%	3.79242%	5.63435%	7.44102%	9.21310%
4-4-1	2.91711%	5.74912%	8.49852%	11.16773%	-
4-3-2	2.85062%	5.61998%	8.31040%	10.92413%	-
4-3-1-1	2.71952%	5.36508%	7.93869%	10.44232%	-

4-2-2-1	2.64825%	5.22637%	7.73622%	10.17960%	-
4-2-1-1-1	2.51642%	4.96952%	7.36089%	9.69208%	-
4-1-1-1-1-1	2.38417%	4.71151%	6.98335%	9.20104%	-
3-3-3	2.85477%	5.62805%	8.32217%	10.93937%	-
3-3-2-1	2.65209%	5.23385%	7.74715%	10.19379%	-
3-3-1-1-1	2.52006%	4.97661%	7.37125%	9.70556%	-
3-2-2-2	2.58052%	5.09444%	7.54350%	9.92937%	-
3-2-2-1-1	2.44817%	4.83640%	7.16617%	9.43890%	-
3-2-1-1-1-1	2.31540%	4.57719%	6.78662%	8.94489%	-
3-1-1-1-1-1-1	2.18222%	4.31682%	6.40484%	8.44730%	-
2-2-2-2-1	2.37607%	4.69568%	6.96019%	9.17088%	-
2-2-2-1-1-1	2.24299%	4.43568%	6.57918%	8.67461%	-
2-2-1-1-1-1-1	2.10950%	4.17450%	6.19594%	8.17475%	-
2-1-1-1-1-1-1-1	1.97559%	3.91215%	5.81046%	7.67127%	-
1-1-1-1-1-1-1-1-1	1.84127%	3.64863%	5.42272%	7.16415%	-
4-4-2	3.04702%	6.00119%	8.86536%	11.64226%	-
4-4-1-1	2.90736%	5.73020%	8.47097%	11.13206%	-
4-3-3	3.05172%	6.01031%	8.87862%	11.65940%	-
4-3-2-1	2.83565%	5.59090%	8.26802%	10.86923%	-
4-3-1-1-1	2.69494%	5.31726%	7.86891%	10.35180%	-
4-2-2-2	2.75924%	5.44236%	8.05144%	10.58853%	-
4-2-2-1-1	2.61818%	5.16781%	7.65069%	10.06857%	-
4-2-1-1-1-1	2.47664%	4.89195%	7.24744%	9.54460%	-
4-1-1-1-1-1-1	2.33464%	4.61477%	6.84167%	9.01659%	-
3-3-3-1	2.84000%	5.59935%	8.28034%	10.88519%	-
3-3-2-2	2.76348%	5.45059%	8.06344%	10.60409%	-
3-3-2-1-1	2.62217%	5.17559%	7.66206%	10.08333%	-
3-3-1-1-1-1	2.48040%	4.89929%	7.25817%	9.55855%	-
3-2-2-2-1	2.54506%	5.02535%	7.44251%	9.79816%	-
3-2-2-1-1-1	2.40294%	4.74813%	7.03698%	9.27083%	-
3-2-1-1-1-1-1	2.26034%	4.46959%	6.62891%	8.73942%	-
3-1-1-1-1-1-1-1	2.11728%	4.18973%	6.21830%	8.20393%	-
2-2-2-2-2	2.46771%	4.87452%	7.22195%	9.51145%	-
2-2-2-2-1-1	2.32523%	4.59640%	6.81476%	8.98154%	-
2-2-2-1-1-1-1	2.18228%	4.31695%	6.40503%	8.44755%	-
2-2-1-1-1-1-1-1	2.03887%	4.03617%	5.99275%	7.90944%	-
2-1-1-1-1-1-1-1-1	1.89498%	3.75406%	5.57790%	7.36719%	-
1-1-1-1-1-1-1-1-1-1	1.75062%	3.47060%	5.16047%	6.82076%	-

Let us take an isolated example to see how to use this table.

Example:

Assume you are playing against three opponents. You are dealt (77JJK), you hold (77JJ), discard (*K*) and replace it with (3).

Your final hand is two pair: (77JJ3).

You want to find the probability for at least one opponent to hold three of a kind.

You have c = 6 seen cards having the value distribution 2-2-1-1.
By looking in the table at the intersection of row *2-2-1-1* with column $n = 3$, you find the probability 6.56288%.

Of course, this is not the probability that one or more opponents hold a higher formation than yours (there are also the straights, flushes, full houses, four of a kind and also the higher two pairs to count, but these can be also calculated, as you will see further).

Odds of holding four of a kind

The *four of a kind* formation is still a formation made of values, like *three of a kind*, so the variables of the probability formulas are the same as in previous case:

n = number of opponents (n could be 1, 2, 3, 4 or 5);

c = number of seen cards (c could be 5, 6, 7, 8, 9 or 10);

$c_1, c_2, ..., c_{13}$ – the value distribution of the seen cards.

The initial conditions are: $c_i \in \{0, 1, 2, 3, 4\}$, for any i = 1, ..., 13 and $\sum_{i=1}^{13} c_i = c$.

We make a similar denotation of the events to be measured:

E – one specific opponent holds four of a kind (the probability of E is the same for any opponent); E_2 – at least one from two opponents holds four of a kind; E_3 – at least one from three opponents holds four of a kind; E_4 – at least one from four opponents holds four of a kind;

Generally, E_n – at least one from n opponents holds four of a kind.

We use this denotation in every section, for all formations.

For one specific opponent, the favorable card combinations for E to occur are of the (xxxxy) type, numbering

$$F = \frac{1}{24}\sum_{i=1}^{13}(1-c_i)(2-c_i)(3-c_i)(4-c_i)(48-c+c_i).$$

$$F' = C_{52-c}^{5} - F$$

The denotations F and F' are also used in all sections for all formations, even if the corresponding expressions are different.

The next table notes the probabilities of one and at least one of your opponents holding a four of a kind formation, corresponding to the value distribution of your own seen cards.

Probabilities of opponents holding four of a kind

Distribution	n = 1	n = 2	n = 3	n = 4	n = 5
4-1	0.03084%	0.06166%	0.09248%	0.12329%	0.15408%
3-2	0.03084%	0.06166%	0.09248%	0.12329%	0.15408%
3-1-1	0.02803%	0.05606%	0.08407%	0.11208%	0.14008%
2-2-1	0.02803%	0.05606%	0.08407%	0.11208%	0.14008%
2-1-1-1	0.02523%	0.05045%	0.07567%	0.10088%	0.12608%
1-1-1-1-1	0.02243%	0.04485%	0.06726%	0.08967%	0.11208%
4-2	0.03370%	0.06740%	0.10108%	0.13475%	0.16841%
4-1-1	0.03064%	0.06127%	0.09189%	0.12250%	0.15311%
3-3	0.03370%	0.06740%	0.10108%	0.13475%	0.16841%
3-2-1	0.03064%	0.06127%	0.09189%	0.12250%	0.15311%
3-1-1-1	0.02758%	0.05514%	0.08271%	0.11026%	0.13780%
2-2-2	0.03064%	0.06127%	0.09189%	0.12250%	0.15311%
2-2-1-1	0.02758%	0.05514%	0.08271%	0.11026%	0.13780%
2-1-1-1-1	0.02451%	0.04902%	0.07352%	0.09801%	0.12250%
1-1-1-1-1-1	0.02145%	0.04289%	0.06433%	0.08576%	0.10719%
4-3	0.03691%	0.07381%	0.11070%	0.14757%	0.18443%
4-2-1	0.03356%	0.06711%	0.10064%	0.13417%	0.16768%
4-1-1-1	0.03020%	0.06040%	0.09058%	0.12075%	0.15092%
3-3-1	0.03356%	0.06711%	0.10064%	0.13417%	0.16768%
3-2-2	0.03356%	0.06711%	0.10064%	0.13417%	0.16768%
3-2-1-1	0.03020%	0.06040%	0.09058%	0.12075%	0.15092%
3-1-1-1-1	0.02685%	0.05369%	0.08052%	0.10734%	0.13416%
2-2-2-1	0.03020%	0.06040%	0.09058%	0.12075%	0.15092%
2-2-1-1-1	0.02685%	0.05369%	0.08052%	0.10734%	0.13416%
2-1-1-1-1-1	0.02349%	0.04698%	0.07046%	0.09393%	0.11740%
1-1-1-1-1-1-1	0.02013%	0.04027%	0.06039%	0.08052%	0.10063%
4-4	0.04052%	0.08101%	0.12150%	0.16196%	0.20241%
4-3-1	0.03683%	0.07365%	0.11046%	0.14725%	0.18403%
4-2-2	0.03683%	0.07365%	0.11046%	0.14725%	0.18403%
4-2-1-1	0.03315%	0.06629%	0.09941%	0.13253%	0.16564%
4-1-1-1-1	0.02947%	0.05892%	0.08837%	0.11781%	0.14724%
3-3-2	0.03683%	0.07365%	0.11046%	0.14725%	0.18403%
3-3-1-1	0.03315%	0.06629%	0.09941%	0.13253%	0.16564%
3-2-2-1	0.03315%	0.06629%	0.09941%	0.13253%	0.16564%
3-2-1-1-1	0.02947%	0.05892%	0.08837%	0.11781%	0.14724%
3-1-1-1-1-1	0.02578%	0.05156%	0.07733%	0.10309%	0.12885%
2-2-2-2	0.03315%	0.06629%	0.09941%	0.13253%	0.16564%
2-2-2-1-1	0.02947%	0.05892%	0.08837%	0.11781%	0.14724%
2-2-1-1-1-1	0.02578%	0.05156%	0.07733%	0.10309%	0.12885%
2-1-1-1-1-1-1	0.02210%	0.04419%	0.06628%	0.08837%	0.11045%
1-1-1-1-1-1-1-1	0.01842%	0.03683%	0.05524%	0.07364%	0.09205%
4-4-1	0.04052%	0.08101%	0.12150%	0.16196%	-
4-3-2	0.04052%	0.08101%	0.12150%	0.16196%	-
4-3-1-1	0.03646%	0.07291%	0.10935%	0.14578%	-
4-2-2-1	0.03646%	0.07291%	0.10935%	0.14578%	-
4-2-1-1-1	0.03241%	0.06481%	0.09721%	0.12959%	-
4-1-1-1-1-1	0.02836%	0.05671%	0.08506%	0.11339%	-
3-3-3	0.04052%	0.08101%	0.12150%	0.16196%	-
3-3-2-1	0.03646%	0.07291%	0.10935%	0.14578%	-
3-3-1-1-1	0.03241%	0.06481%	0.09721%	0.12959%	-
3-2-2-2	0.03646%	0.07291%	0.10935%	0.14578%	-
3-2-2-1-1	0.03241%	0.06481%	0.09721%	0.12959%	-
3-2-1-1-1-1	0.02836%	0.05671%	0.08506%	0.11339%	-

3-1-1-1-1-1-1	0.02431%	0.04861%	0.07291%	0.09720%	-
2-2-2-2-1	0.03241%	0.06481%	0.09721%	0.12959%	-
2-2-2-1-1-1	0.02836%	0.05671%	0.08506%	0.11339%	-
2-2-1-1-1-1-1	0.02431%	0.04861%	0.07291%	0.09720%	-
2-1-1-1-1-1-1-1	0.02026%	0.04051%	0.06076%	0.08101%	-
1-1-1-1-1-1-1-1-1	0.01621%	0.03241%	0.04861%	0.06481%	-
4-4-2	0.04467%	0.08932%	0.13395%	0.17856%	-
4-4-1-1	0.04020%	0.08039%	0.12056%	0.16072%	-
4-3-3	0.04467%	0.08932%	0.13395%	0.17856%	-
4-3-2-1	0.04020%	0.08039%	0.12056%	0.16072%	-
4-3-1-1-1	0.03574%	0.07146%	0.10717%	0.14287%	-
4-2-2-2	0.04020%	0.08039%	0.12056%	0.16072%	-
4-2-2-1-1	0.03574%	0.07146%	0.10717%	0.14287%	-
4-2-1-1-1-1	0.03127%	0.06253%	0.09378%	0.12502%	-
4-1-1-1-1-1-1	0.02680%	0.05360%	0.08039%	0.10717%	-
3-3-3-1	0.04020%	0.08039%	0.12056%	0.16072%	-
3-3-2-2	0.04020%	0.08039%	0.12056%	0.16072%	-
3-3-2-1-1	0.03574%	0.07146%	0.10717%	0.14287%	-
3-3-1-1-1-1	0.03127%	0.06253%	0.09378%	0.12502%	-
3-2-2-2-1	0.03574%	0.07146%	0.10717%	0.14287%	-
3-2-2-1-1-1	0.03127%	0.06253%	0.09378%	0.12502%	-
3-2-1-1-1-1-1	0.02680%	0.05360%	0.08039%	0.10717%	-
3-1-1-1-1-1-1-1	0.02234%	0.04467%	0.06699%	0.08931%	-
2-2-2-2-2	0.03574%	0.07146%	0.10717%	0.14287%	-
2-2-2-2-1-1	0.03127%	0.06253%	0.09378%	0.12502%	-
2-2-2-1-1-1-1	0.02680%	0.05360%	0.08039%	0.10717%	-
2-2-1-1-1-1-1-1	0.02234%	0.04467%	0.06699%	0.08931%	-
2-1-1-1-1-1-1-1-1	0.01787%	0.03573%	0.05360%	0.07145%	-
1-1-1-1-1-1-1-1-1-1	0.01340%	0.02680%	0.04020%	0.05359%	-

Example:

You are playing against five opponents. You are initially dealt (4588J), you keep the pair (88), discard (45J) and the replacements are (822). You hold a full house: (88822). You want to find the probability of at least one of your opponents holding four of a kind.

Your seen cards have the values (8882245J). c = 8.

Their value distribution is then 3-2-1-1-1.

By looking in the table at the intersection of row *3-2-1-1-1* with column $n = 5$, you find 0.14724% as the probability for at least one of your opponents to hold four of a kind.

Observe that all probabilities from this table are very low (less than 0.2%).

.........................missing part..........................

Example:

You are playing against several opponents, you are initially dealt (3459J), you keep (345), discard the rest (9J) and the replacements are (8K). You hold no valuable formation: (3458K) (assuming they are not suited).

You want to find the probability of one specific opponent holding one pair.

Your seen cards have the values (3459J8K). c = 7.

Their value distribution is then 1-1-1-1-1-1-1.

By looking in the table at the intersection of row *1-1-1-1-1-1-1* with column *n = 1*, you find 41.81799% as the probability for at least one of your opponents to hold one pair.

Observe that for one pair, the probabilities are the highest (they may reach 46% for one opponent and 95% for five opponents!).

.........................missing part..........................

Odds of holding a straight

The straight is also a formation made of values, but the probability calculus for events involving straights uses the exact distributions of values, not the cumulated distributions used thus far for *three of a kind*, *four of a kind*, *two pair* and *one pair*.

We call *exact distribution of values* a 13-size vector ($c_A, c_2, c_3, ..., c_{10}, c_J, c_Q, c_K$), where c_A is the number of aces, c_2 is the number of 2's, and so on, c_{10} is the number of 10's, c_J is the number of jacks, c_Q is the number of queens and c_K is the number of kings, from the seen cards.

We are interested in how each value is represented in the seen cards: how many 2's, 3's, etc.

The initial conditions of the exact distribution are $c_x \in \{0,1,2,3,4\}$ for any value *x* from 2 to *A* and $\sum_x c_x = c$.

Even in these conditions, the number of possible exact distributions number in the millions, so we cannot list them all in a table.

The favorable combinations for your opponents to hold a straight are the following types: (A2345), (23456), (34567), (45678), (56789), (6789 10), (789 10 J), (89 10 JQ), (9 10 JQK) and (10 JQKA). They number in total:

$$
\begin{aligned}
F = &(4-c_A)(4-c_2)(4-c_3)(4-c_4)(4-c_5) + \\
&+(4-c_2)(4-c_3)(4-c_4)(4-c_5)(4-c_6) + \\
&+(4-c_3)(4-c_4)(4-c_5)(4-c_6)(4-c_7) + \\
&+(4-c_4)(4-c_5)(4-c_6)(4-c_7)(4-c_8) + \\
&+(4-c_5)(4-c_6)(4-c_7)(4-c_8)(4-c_9) + \\
&+(4-c_6)(4-c_7)(4-c_8)(4-c_9)(4-c_{10}) + \\
&+(4-c_7)(4-c_8)(4-c_9)(4-c_{10})(4-c_J) + \\
&+(4-c_8)(4-c_9)(4-c_{10})(4-c_J)(4-c_Q) + \\
&+(4-c_9)(4-c_{10})(4-c_J)(4-c_Q)(4-c_K) + \\
&+(4-c_{10})(4-c_J)(4-c_Q)(4-c_K)(4-c_A).
\end{aligned}
$$

This number also includes the number of favorable combinations for a *straight flush*.

For now, we define a *straight* as any formation of five consecutive values, whether suited or not.

Although the formula of *F* is long and seems complicated, it is just a repeated sum of similar five-factor products.

While the exact distributions of values number in the millions, *F* may take a limited number of values because of the initial conditions on c_x.

Namely, *F* may take 8964 values.

As we said earlier, there is no way to list millions of distributions in a table. To still provide a table with the numerical returns of the probabilities $P(E_n)$ for the *straight* case, we elaborate on a simple descriptive algorithm for the reader to calculate *F* for any exact distribution of values.

Then, we list all possible values of *F*, along with the corresponding probabilities $P(E_n)$.

After calculating *F*, the reader must search in the table for the respective value, where the *c* corresponds to the considered distribution and on the column corresponding to the number of opponents *n*.

At the intersection of a respective row and column, the reader can find the probability of at least one from *n* opponents holding a straight.

The algorithm for calculating *F* is relatively easy. It consists of ten multiplications and one sum.

For a given exact distribution $(c_A, c_2, c_3, ..., c_{10}, c_J, c_Q, c_K)$, whose elements c_x are from 0 to 4, we consider the numbers $4 - c_x$, which may still have values from 0 to 4. These are the numbers we should operate with to compute *F*.

To avoid computing errors, such as missing some factors, we recommend using the following manner of calculation:

Draw a table with 12 rows and 15 columns. In the top row note the variables $4 - c_x$, for *x* from 2 to *A*, one per column, running from *A* to *K*, then repeat the first for *A*. For simplification purposes, you may note the values *A*, 2, 3, ..., *K*, *A* instead of $4 - c_A$, $4 - c_2$, etc.

Note *P* (from *product*) in the last cell of this row, in the last column.

In each row except the first, highlight one sequence of five adjacent cells (using bold, underline, color, border, or fill), in this way:

– in the second row, the cells corresponding to the values $4 - c_x$, from A to 5;

– in the third row, the cells corresponding to the values $4 - c_x$, from 2 to 6;

– in the fourth row, the cells corresponding to the values $4 - c_x$, from 3 to 7;

... and so on,

– in the tenth row, the cells corresponding to the values $4 - c_x$, from 9 to K;

– in the eleventh row, the cells corresponding to the values $4 - c_x$, from 10 to A.

In the last row, note *F* in the last but one cell, near the *P* column.

The table should look like this:

A	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**J**	**Q**	**K**	**A**	***P***
													F:	

This table must be filled with the numerical values according to the given distribution for which we calculate the probability of opponents having a straight.

This must be done using the following procedure:

Step 1: Note the numerical value of $4-c_A$ in every bold cell in the two columns labeled *A* (only one cell in each of the two columns). Then note the numerical value of $4-c_2$ in every bold cell in the column labeled *2*; and so on, to the column named *K*.

Step 2: Do the product of the numerical values in the five adjacent bold cells in each row and note the result in the corresponding cell in column *P*.

Step 3: Do the sum of all numerical values obtained in column *P* and note it in the last cell from column *P*. This is the numerical value of *F*.

Let us take a concrete example and perform the calculation. Assume we have the exact distribution:

$c_A = 1$		$4-c_A = 3$
$c_2 = 0$		$4-c_2 = 4$
$c_3 = 0$		$4-c_3 = 4$
$c_4 = 2$	We have c = 8.	$4-c_4 = 2$
$c_5 = 1$	The corresponding	$4-c_5 = 3$
$c_6 = 0$	values of $4-c_x$	$4-c_6 = 4$
$c_7 = 2$	are as seen in the right side. These	$4-c_7 = 2$
$c_8 = 1$	values must be put	$4-c_8 = 3$
$c_9 = 0$	in the table.	$4-c_9 = 4$
$c_{10} = 0$		$4-c_{10} = 4$
$c_J = 0$		$4-c_J = 4$
$c_Q = 0$		$4-c_Q = 4$
$c_K = 1$		$4-c_K = 3$

The table should be filled as follows:

A	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**J**	**Q**	**K**	**A**	***P***
3	4	4	2	3										288
	4	4	2	3	4									384
		4	2	3	4	2								192
			2	3	4	2	3							144
				3	4	2	3	4						288
					4	2	3	4	4					384
						2	3	4	4	4				384
							3	4	4	4	4			768
								4	4	4	4	3		768
									4	4	4	3	3	576
													F:	**4176**

F = 4176

The calculus algorithm returns 4176 as the numerical value of *F* (the number of favorable combinations for one opponent to hold a straight).

So, for a given exact distribution, we can manually calculate *F*.

Observe that if we have 0 in a cell, the respective partial product (of the five adjacent values) is still 0.

Therefore, if we have 0 in the columns 5 and 10, all partial products will be 0, and thus the final result F will still be 0.

This means that the probability of one or more of your opponents holding a straight is 0.

This only happens if the seen cards contain all four 5's and all four 10's.

If you encounter this situation, you should know without requiring calculation that no opponent holds or will hold a straight.

With *c* and *F* we can go to the next table to find the probabilities $P(E_n)$.

The complete table is very long, holding all possible 8964 values of *F*, for all values of *c*.

Because the probabilities $P(E_n)$ in that table do not vary too much within intervals for several values of *F*, we can compress the table into a short partial table so that the difference between two neighboring probabilities corresponding to two values of *F* – one listed and one missing – in the same column not to exceed a certain absolute value.

We choose that absolute value as 0.1%.

We use the following compression rules to create the partial table.

In the original table, we left the first row as is, but took the last column (n = 5).

From the second row in descending order, we removed all rows in which the value of the probability (in the last column) has the same integer part and the same first digit after the decimal point with the upper value.

After this removal was completed, we left the next row as is, then continued the same removal step down to the last row but one and left the last row as is.

When column $n = 5$ was exhausted, we continued the procedure for column $n = 4$.

This compression rule is applied separately for each section (sub-table) from $c = 5$ to $c = 10$.

In this way, we transformed a table containing more than 8000 rows into a two-page table with the following property: for any value of *F* from the original table that is not listed in the partial table, we find an approximation with less than 0.1% in absolute value for the probabilities $P(E_n)$.

This happens because the values of the probabilities increase from column $n = 1$ to column $n = 5$ on the same row and increase in the same column as *F* goes up.

In fact, this approximation may go down to 0.02% or even less for the probabilities from column $n = 1$, which are the lowest.

Of course, this approximation does not influence any decision based on odds.

It was done with the purpose of including a table of reasonable length in the book.

In practice, the use of the partial table works in this way:

Let us say you calculated F = 3720 for c = 7 and you want to find $P(E_3)$. This value of *F* is not listed in the partial table.

We go to section *c = 7* and search in the column of *F* for the values immediately lower and higher than 3720.

We find 3688 and 3936. The corresponding probabilities from column *n = 3* are 0.90285% and 0.96337%.

This means that $P(E_3)$ will be somewhere between these numbers.

You may choose one or the other, you may choose any number between (for a higher accuracy of the approximation), or you may take the integer part and the first decimal digit (0.9%) as the approximation every time the real probability is approximated with less than 0.1% in absolute value.

In this case, the error range will be less than 0.07%.

The original table of this section would fill more than 100 pages.

Other following tables would fill up to over 800 pages.

In each section holding such a long table, we present a partial table obtained through the compression procedure.

Each of the partial tables offers a very reasonable approximation for the unlisted probabilities that does not affect in any way a gaming (betting) decision.

Still, for readers interested in having the original tables, we have provided a download link in the heading of each partial table.

Probabilities of opponents holding a straight – partial table (you can download the complete table from http://probability.infarom.ro/opstraight.zip)

F	n = 1	n = 2	n = 3	n = 4	n = 5
c=5					
3840	0.25034%	0.50005%	0.74913%	0.99759%	1.24543%
4096	0.26702%	0.53334%	0.79894%	1.06383%	1.32802%
4352	0.28371%	0.56662%	0.84873%	1.13004%	1.41055%
4672	0.30458%	0.60822%	0.91095%	1.21275%	1.51363%
4960	0.32335%	0.64566%	0.96692%	1.28714%	1.60633%
5256	0.34265%	0.68412%	1.02442%	1.36356%	1.70154%
5568	0.36299%	0.72466%	1.08501%	1.44406%	1.80181%
5880	0.38333%	0.76518%	1.14558%	1.52452%	1.90200%
6192	0.40367%	0.80570%	1.20612%	1.60492%	2.00211%
6504	0.42401%	0.84622%	1.26663%	1.68527%	2.10213%
6816	0.44435%	0.88672%	1.32712%	1.76558%	2.20208%
7136	0.46521%	0.92825%	1.38914%	1.84789%	2.30450%
7440	0.48503%	0.96770%	1.44803%	1.92604%	2.40172%
7760	0.50589%	1.00922%	1.51000%	2.00825%	2.50398%
8064	0.52571%	1.04865%	1.56884%	2.08630%	2.60104%
c=6					
2560	0.18676%	0.37317%	0.55923%	0.74494%	0.93031%
3072	0.22411%	0.44772%	0.67083%	0.89343%	1.11554%
3328	0.24279%	0.48498%	0.72659%	0.96761%	1.20805%
3584	0.26146%	0.52224%	0.78234%	1.04175%	1.30049%
3872	0.28247%	0.56415%	0.84503%	1.12511%	1.40441%
4144	0.30232%	0.60372%	0.90421%	1.20379%	1.50247%

4416	0.32216%	0.64328%	0.96337%	1.28242%	1.60045%
4696	0.34259%	0.68400%	1.02424%	1.36332%	1.70123%
4972	0.36272%	0.72412%	1.08422%	1.44301%	1.80049%
5256	0.38344%	0.76541%	1.14591%	1.52496%	1.90255%
5528	0.40328%	0.80494%	1.20497%	1.60340%	2.00021%
5807	0.42364%	0.84548%	1.26553%	1.68381%	2.10031%
6096	0.44472%	0.88746%	1.32823%	1.76705%	2.20391%
6368	0.46456%	0.92697%	1.38722%	1.84534%	2.30133%
6644	0.48470%	0.96704%	1.44705%	1.92474%	2.40011%
6944	0.50658%	1.01060%	1.51206%	2.01099%	2.50738%
7216	0.52643%	1.05008%	1.57098%	2.08914%	2.60456%
7484	0.54598%	1.08897%	1.62901%	2.16609%	2.70024%
7744	0.56494%	1.12670%	1.68528%	2.24070%	2.79299%
c=7					
1280	0.10477%	0.20942%	0.31397%	0.41841%	0.52274%
2048	0.16763%	0.33497%	0.50204%	0.66883%	0.83533%
2240	0.18334%	0.36635%	0.54902%	0.73136%	0.91336%
2560	0.20953%	0.41863%	0.62729%	0.83551%	1.04329%
2736	0.22394%	0.44738%	0.67032%	0.89275%	1.11470%
2976	0.24358%	0.48657%	0.72897%	0.97078%	1.21200%
3200	0.26192%	0.52315%	0.78370%	1.04356%	1.30275%
3444	0.28189%	0.56298%	0.84329%	1.12280%	1.40152%
3688	0.30186%	0.60281%	0.90285%	1.20198%	1.50022%
3936	0.32216%	0.64328%	0.96337%	1.28242%	1.60045%
4184	0.34246%	0.68374%	1.02386%	1.36281%	1.70060%
4431	0.36267%	0.72403%	1.08408%	1.44282%	1.80027%
4680	0.38305%	0.76464%	1.14477%	1.52344%	1.90066%
4928	0.40335%	0.80508%	1.20519%	1.60368%	2.00056%
5176	0.42365%	0.84551%	1.26558%	1.68387%	2.10039%
5424	0.44395%	0.88593%	1.32595%	1.76401%	2.20013%
5676	0.46458%	0.92699%	1.38726%	1.84540%	2.30140%
5924	0.48487%	0.96740%	1.44758%	1.92544%	2.40098%
6172	0.50517%	1.00779%	1.50788%	2.00544%	2.50048%
6426	0.52596%	1.04916%	1.56961%	2.08731%	2.60230%
6672	0.54610%	1.08921%	1.62936%	2.16657%	2.70083%
6928	0.56705%	1.13089%	1.69153%	2.24899%	2.80329%
7192	0.58866%	1.17385%	1.75560%	2.33393%	2.90885%
7424	0.60765%	1.21161%	1.81189%	2.40853%	3.00155%
c=8					
0	0.00000%	0.00000%	0.00000%	0.00000%	0.00000%
1024	0.09429%	0.18849%	0.28260%	0.37663%	0.47056%
1152	0.10608%	0.21204%	0.31789%	0.42363%	0.52926%
1216	0.11197%	0.22381%	0.33553%	0.44713%	0.55860%
1280	0.11786%	0.23559%	0.35317%	0.47062%	0.58793%
1536	0.14144%	0.28267%	0.42371%	0.56454%	0.70518%
1760	0.16206%	0.32386%	0.48540%	0.64667%	0.80769%
1968	0.18121%	0.36210%	0.54266%	0.72289%	0.90279%
2192	0.20184%	0.40327%	0.60430%	0.80492%	1.00514%
2400	0.22099%	0.44150%	0.66152%	0.88105%	1.10009%
2624	0.24162%	0.48265%	0.72311%	0.96298%	1.20227%
2840	0.26151%	0.52233%	0.78248%	1.04194%	1.30072%
3060	0.28177%	0.56274%	0.84292%	1.12231%	1.40092%
3280	0.30202%	0.60314%	0.90334%	1.20263%	1.50103%
3498	0.32210%	0.64316%	0.96318%	1.28218%	1.60015%
3720	0.34254%	0.68390%	1.02410%	1.36313%	1.70100%
3940	0.36280%	0.72428%	1.08445%	1.44331%	1.80087%
4160	0.38305%	0.76464%	1.14477%	1.52344%	1.90066%
4380	0.40331%	0.80500%	1.20506%	1.60352%	2.00036%
4602	0.42375%	0.84571%	1.26588%	1.68427%	2.10089%
4823	0.44410%	0.88624%	1.32640%	1.76462%	2.20089%
5044	0.46445%	0.92675%	1.38690%	1.84491%	2.30080%
5264	0.48471%	0.96707%	1.44710%	1.92480%	2.40018%
5488	0.50534%	1.00812%	1.50836%	2.00608%	2.50128%
5708	0.52559%	1.04843%	1.56851%	2.08586%	2.60050%
5932	0.54622%	1.08946%	1.62973%	2.16705%	2.70144%
6152	0.56648%	1.12975%	1.68983%	2.24674%	2.80049%
6376	0.58710%	1.17076%	1.75099%	2.32782%	2.90126%
6608	0.60847%	1.21323%	1.81432%	2.41175%	3.00554%
6832	0.62909%	1.25423%	1.87543%	2.49273%	3.10614%
7072	0.65119%	1.29814%	1.94088%	2.57944%	3.21384%
7168	0.66003%	1.31571%	1.96706%	2.61411%	3.25689%
c=9					
0	0.00000%	0.00000%	0.00000%	0.00000%	-
768	0.07978%	0.15950%	0.23916%	0.31876%	-
976	0.10139%	0.20268%	0.30387%	0.40495%	-
1216	0.12632%	0.25249%	0.37850%	0.50434%	-
1456	0.15126%	0.30229%	0.45309%	0.60366%	-
1696	0.17619%	0.35207%	0.52764%	0.70290%	-
1936	0.20112%	0.40184%	0.60216%	0.80207%	-
2176	0.22605%	0.45160%	0.67663%	0.90116%	-
2416	0.25099%	0.50135%	0.75107%	1.00018%	-
2660	0.27634%	0.55191%	0.82672%	1.10077%	-
2904	0.30168%	0.60246%	0.90232%	1.20129%	-
3144	0.32662%	0.65217%	0.97665%	1.30008%	-

3387	0.35186%	0.70248%	1.05187%	1.40003%	-
3631	0.37721%	0.75299%	1.12736%	1.50032%	-
3874	0.40245%	0.80329%	1.20251%	1.60012%	-
4118	0.42780%	0.85377%	1.27792%	1.70026%	-
4362	0.45315%	0.90424%	1.35330%	1.80031%	-
4606	0.47850%	0.95470%	1.42863%	1.90030%	-
4850	0.50384%	1.00515%	1.50393%	2.00020%	-
5096	0.52940%	1.05600%	1.57981%	2.10085%	-
5340	0.55475%	1.10642%	1.65503%	2.20060%	-
5584	0.58010%	1.15683%	1.73022%	2.30028%	-
5832	0.60586%	1.20805%	1.80659%	2.40151%	-
6078	0.63142%	1.25885%	1.88232%	2.50185%	-
6320	0.65656%	1.30880%	1.95677%	2.60048%	-
6580	0.68357%	1.36246%	2.03672%	2.70636%	-
6848	0.71141%	1.41776%	2.11908%	2.81541%	-
6912	0.71806%	1.43096%	2.13874%	2.84144%	-
c=10					
0	0.00000%	0.00000%	0.00000%	0.00000%	-
512	0.06019%	0.12034%	0.18046%	0.24054%	-
640	0.07523%	0.15041%	0.22554%	0.30060%	-
864	0.10157%	0.20303%	0.30439%	0.40565%	-
1068	0.12555%	0.25094%	0.37617%	0.50125%	-
1280	0.15047%	0.30071%	0.45073%	0.60052%	-
1500	0.17633%	0.35235%	0.52806%	0.70347%	-
1708	0.20078%	0.40116%	0.60114%	0.80072%	-
1924	0.22618%	0.45184%	0.67699%	0.90164%	-
2136	0.25110%	0.50156%	0.75140%	1.00061%	-
2350	0.27625%	0.55174%	0.82647%	1.10045%	-
2564	0.30141%	0.60191%	0.90151%	1.20020%	-
2779	0.32668%	0.65230%	0.97686%	1.30035%	-
2994	0.35196%	0.70268%	1.05217%	1.40042%	-
3209	0.37723%	0.75304%	1.12744%	1.50042%	-
3424	0.40251%	0.80339%	1.20267%	1.60034%	-
3639	0.42778%	0.85373%	1.27786%	1.70018%	-
3855	0.45317%	0.90429%	1.35337%	1.80041%	-
4070	0.47845%	0.95461%	1.42849%	1.90010%	-
4286	0.50384%	1.00514%	1.50392%	2.00018%	-
4503	0.52935%	1.05590%	1.57966%	2.10065%	-
4718	0.55462%	1.10617%	1.65466%	2.20011%	-
4936	0.58025%	1.15713%	1.73067%	2.30088%	-
5152	0.60564%	1.20762%	1.80595%	2.40065%	-
5368	0.63103%	1.25809%	1.88118%	2.50035%	-
5588	0.65690%	1.30948%	1.95777%	2.60181%	-
5808	0.68276%	1.36085%	2.03432%	2.70319%	-
6024	0.70815%	1.41128%	2.10944%	2.80266%	-
6236	0.73307%	1.46077%	2.18313%	2.90020%	-
6464	0.75987%	1.51397%	2.26234%	3.00503%	-
6656	0.78244%	1.55877%	2.32902%	3.09324%	-

Example:

You are playing against five opponents, you are initially dealt (335JK), you discard (5J) and the replacements are (37). You hold three of a kind: (3337K).

You want to find the probability of at least one opponent holding a straight (including straight flush).

Your seen cards are (33357JK), c = 7.

Their exact distribution is

$(c_A, c_2, c_3, ..., c_{10}, c_J, c_Q, c_K) = (0, 0, 3, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1)$.

The vector with the values $4 - c_x$ is then

(4, 4, 1, 4, 3, 4, 3, 4, 4, 4, 3, 4, 3).

Filling them in the algorithm table for calculating *F* result in:

A	2	3	4	5	6	7	8	9	10	J	Q	K	A	*P*
4	4	1	4	3										192
	4	1	4	3	4									192
		1	4	3	4	3								144
			4	3	4	3	4							576
				3	4	3	4	4						576
					4	3	4	4	4					768
						3	4	4	4	3				576
							4	4	4	3	4			768
								4	4	3	4	3		576
									4	3	4	3	4	576
													F:	**4424**

We look now in the table with probabilities in the area of *c = 7*, at the intersection of row *F = 4424* with column *n = 5* and find 1.79744% as the probability for at least one opponent to hold a straight (including a straight flush).

The result is taken from the complete table. If using the corresponding partial table, we find 4184 and 4431 as neighboring values of F = 4424, with the corresponding probabilities 1.70060% and 1.80027% in the column *n = 5*. You may consider any approximation between these two numbers. If you want to be more rigorous, you may take a number closer to 1.80027% because 4424 is closer to 4431 than it is to 4184.

If you want the probability of your opponents holding only a common straight (no straight flush), we must subtract the probability of opponents holding a straight flush. This is the subject of the next section.

.........................missing part..........................

In previous sections we covered all types of valuable formations with respect to the probability of opponents achieving them: one pair, two pair, three of a kind, straight (no straight flush), flush (no straight flush), full house, four of a kind and straight flush.

Because the opponents holding each type of above enumerated formations are incompatible events (the occurrence of one of them excludes the occurrence of another – for example, if one opponent holds a straight, he or she could not hold a full house or one pair), the results of this chapter can be used cumulatively.

This means that, for a given distribution of your seen cards, we can calculate the probability of one of your opponents holding a type of formation of a higher rank.

This calculation can be effectively made by searching the tables from each section (for each type of formation with a value higher than yours), then making the sum of all numerical results.

For example, if your final hand is two pair, you may be interested in the event *one of your opponents holds three of a kind or straight or flush or full house or four of a kind or straight flush*. This event is measurable and its probability is the sum of the probabilities of the partial events, because these partial events are incompatible.

But this final result (the mentioned sum) is not exactly the probability of one of your opponents holding a higher formation than yours, because the formation *higher two pair* has not been counted and it is also a higher formation. This is a formation of same type with your final formation.

If we add the probability of your opponent holding a higher two pair formation, we finally have the probability of your opponent holding a formation higher than yours and this is more valuable information to take into account when making the betting decisions.

With a symbolic denotation, for our example in which you hold two pair,

P(higher formation) = P(three of a kind) + P(straight) + P(flush) + P(full house) + P(four of a kind) + P(straight flush) +

P(higher two pair), where *P* means probability for one of your opponents to hold respective formations.

For calculating the probability of at least one of your opponents holding a higher formation than yours, the above way of summing is not allowed any more, because the events “at least one of your opponents holds *three of a kind (straight, flush, full house, four of a kind* and *straight flush)* are not incompatible any more.

This probability, whose importance is capital when evaluating your chance of winning the hand against your opponents and driving the betting dialogue, can be calculated by using a formula than will be described in a future chapter.

The next sections deal with the probabilities of your opponents holding formations of same type with yours, but higher.

These probabilities will be also factored in when estimating the probability of your opponents holding a higher formation than yours.

Three of a kind against higher three of a kind

This section discusses the probabilities of your opponents holding a higher three of a kind formation, assuming you hold three of a kind after the second card distribution.

The value of the triple card is the one that makes the ranking between two *three of a kind* formations. (The rest of the cards, the two unpaired cards, do not count in the hierarchy because we have only four cards of the same value. They do count as parameters in the final formula of *F* and implicitly of $P(E_n)$.)

We have an additional variable in comparison with the three of a kind case from the previous group of formulas; namely, the number of values that are higher than the value of the triple card from your own three of a kind formation.

Therefore, the variables are represented by double value distributions (double vectors) $(c_1, c_2, , ..., c_m)$ $(c_{m+1}, ..., c_{13})$, where:

m = the number of values higher than the value of own triple card (for example, if you achieved a (KKKxy) three of a kind formation, there is only one value that is higher than *K* (king), and that is *A* (ace), so we have m = 1; if you achieved (555xy), there are nine values higher than 5, namely 6, 7, 8, 9, 10, J, Q, K, A, so m = 9, and so on);

$(c_1, c_2, , ..., c_m)$ is the value distribution (the cumulated distribution of values) of the values that are higher than the value of the triple card from the three of a kind formation achieved (c_1 is the number of cards of one value, c_2 is the number of cards of another value, and so on, from the *m* values);

$(c_{m+1}, ..., c_{13})$ is the value distribution (the cumulated distribution of values) of the rest of the values up to the thirteenth.

The initial conditions on these variables are:

$m \in \{1, 2, ..., 12\}$; the case m = 0 (when there are three aces) is trivial because we have F = 0 and, consequently, probabilities $P(E_n)$ are 0; the case m = 13 is not counted because we hypothetically already hold three of a kind (a triple of a certain value), so we cannot have 13 higher values (all values);

$c_i \in \{0, 1, 2, 3, 4\}$ for any $i = 1, ..., 13$ and $\sum_{i=1}^{13} c_i = c$;

$\sum_{i=1}^{m} c_i \le c - 3$ (because three cards from all seen *c* cards are those making the triple).

When we calculate the probability of opponents having a higher three of a kind formation, we assume that you already hold such a formation.

Therefore, the formula is not applied to all distributions described earlier, but only to those that are specific for a three of a kind formation. These are the distributions from which we can extract a partial distribution (from the entire sequence seen as one vector) of 3-1-1 type.

This means that we keep those distributions holding three elements *a*, *b* and *c* (regardless of their position within the vector), so that $a \ge 3, b \ge 1, c \ge 1$.

For example, the distribution (20)(30000000000) (from the category m = 2, c = 5) does not meet this condition because we cannot extract from it a 3-1-1 partial distribution.

The distribution (110)(3110000000) (from the category m = 3, c = 7) meets that condition.

The formula of *F* for the case *higher three of a kind* is almost identical to the one from the case *three of a kind*, but the sum is from 1 to *m* instead 1 to 13:

$$F = \frac{1}{12}\sum_{i=1}^{m}(2-c_i)(3-c_i)(4-c_i)[(47-c+c_i)(48-c+c_i)-2S+(3-c_i)(4-c_i)]$$

, where $S = \frac{1}{2}\sum_{j=1}^{13}(3-c_j)(4-c_j)$

S is constant with respect to the sum indexed by *i*.

Of course, this formula stands for the case $m \ne 0$.

If m = 0 (we have triple aces), then F = 0.

The number of all possible distributions fitting the hypothetical conditions also number in the millions here, so we can only list the values of *F* along with the corresponding probabilities:

Probabilities of opponents holding a higher three of a kind

F	n = 1	n = 2	n = 3	n = 4	n = 5
c=5					
0	0.00000%	0.00000%	0.00000%	0.00000%	0.00000%
883	0.05756%	0.11510%	0.17259%	0.23006%	0.28749%
1766	0.11513%	0.23012%	0.34499%	0.45972%	0.57432%
3372	0.21983%	0.43917%	0.65803%	0.87641%	1.09431%
4255	0.27739%	0.55401%	0.82987%	1.10495%	1.37928%
5138	0.33495%	0.66879%	1.00150%	1.33310%	1.66359%
6744	0.43965%	0.87737%	1.31317%	1.74705%	2.17902%
7627	0.49722%	0.99196%	1.48425%	1.97408%	2.46149%
8510	0.55478%	1.10648%	1.65513%	2.20073%	2.74330%
10116	0.65948%	1.31461%	1.96542%	2.61194%	3.25419%
10999	0.71704%	1.42894%	2.13574%	2.83747%	3.53417%
11882	0.77461%	1.54321%	2.30587%	3.06262%	3.81350%
13488	0.87930%	1.75088%	2.61479%	3.47110%	4.31989%
14371	0.93687%	1.86496%	2.78436%	3.69514%	4.59740%
15254	0.99443%	1.97898%	2.95373%	3.91880%	4.87426%
16860	1.09913%	2.18618%	3.26129%	4.32457%	5.37617%
17743	1.15670%	2.30001%	3.43010%	4.54713%	5.65123%
18626	1.21426%	2.41378%	3.59873%	4.76929%	5.92564%
20232	1.31896%	2.62052%	3.90491%	5.17237%	6.42311%
21115	1.37652%	2.73410%	4.07298%	5.39344%	6.69573%
21998	1.43409%	2.84761%	4.24086%	5.61413%	6.96771%
23604	1.53878%	3.05389%	4.54568%	6.01452%	7.46076%
24487	1.59635%	3.16721%	4.71300%	6.23412%	7.73095%
25370	1.65391%	3.28047%	4.88013%	6.45333%	8.00051%
26976	1.75861%	3.48629%	5.18360%	6.85105%	8.48918%
27859	1.81617%	3.59936%	5.35017%	7.06918%	8.75697%
28742	1.87374%	3.71237%	5.51655%	7.28692%	9.02413%
30348	1.97844%	3.91773%	5.81866%	7.68198%	9.50844%
31231	2.03600%	4.03055%	5.98449%	7.89865%	9.77384%
32114	2.09356%	4.14330%	6.15012%	8.11494%	10.03861%
33720	2.19826%	4.34820%	6.45088%	8.50734%	10.51859%
34603	2.25583%	4.46077%	6.61597%	8.72255%	10.78162%
35486	2.31339%	4.57326%	6.78086%	8.93739%	11.04403%
c=6					
0	0.00000%	0.00000%	0.00000%	0.00000%	0.00000%
840	0.06128%	0.12252%	0.18373%	0.24490%	0.30603%
842	0.06143%	0.12281%	0.18417%	0.24548%	0.30675%
843	0.06150%	0.12296%	0.18438%	0.24577%	0.30712%
1680	0.12256%	0.24497%	0.36723%	0.48934%	0.61130%
1686	0.12300%	0.24584%	0.36854%	0.49109%	0.61348%
2529	0.18450%	0.36865%	0.55247%	0.73595%	0.91909%
3204	0.23374%	0.46693%	0.69958%	0.93169%	1.16325%
3212	0.23432%	0.46810%	0.70133%	0.93401%	1.16614%
3216	0.23462%	0.46868%	0.70220%	0.93517%	1.16759%
4044	0.29502%	0.58917%	0.88245%	1.17487%	1.46642%
4054	0.29575%	0.59062%	0.88463%	1.17776%	1.47003%
4059	0.29611%	0.59135%	0.88572%	1.17921%	1.47183%
4884	0.35630%	0.71133%	1.06510%	1.41760%	1.76885%
4902	0.35761%	0.71395%	1.06901%	1.42280%	1.77533%
5745	0.41911%	0.83647%	1.25208%	1.66594%	2.07807%

6408	0.46748%	0.93277%	1.39589%	1.85685%	2.31565%
6424	0.46865%	0.93510%	1.39936%	1.86145%	2.32138%
6432	0.46923%	0.93626%	1.40110%	1.86376%	2.32424%
7248	0.52876%	1.05472%	1.57791%	2.09833%	2.61599%
7266	0.53007%	1.05734%	1.58181%	2.10350%	2.62242%
7275	0.53073%	1.05864%	1.58376%	2.10608%	2.62563%
8088	0.59004%	1.17660%	1.75970%	2.33936%	2.91560%
8118	0.59223%	1.18095%	1.76619%	2.34796%	2.92628%
8961	0.65373%	1.30318%	1.94839%	2.58938%	3.22619%
9612	0.70122%	1.39752%	2.08894%	2.77552%	3.45728%
9636	0.70297%	1.40100%	2.09412%	2.78237%	3.46579%
9648	0.70385%	1.40274%	2.09671%	2.78580%	3.47004%
10452	0.76250%	1.51919%	2.27010%	3.01530%	3.75481%
10478	0.76440%	1.52295%	2.27571%	3.02271%	3.76400%
10491	0.76535%	1.52483%	2.27851%	3.02642%	3.76860%
11292	0.82378%	1.64077%	2.45104%	3.25463%	4.05160%
11334	0.82684%	1.64685%	2.46008%	3.26659%	4.06642%
12177	0.88834%	1.76880%	2.64143%	3.50631%	4.36350%
12816	0.93496%	1.86118%	2.77874%	3.68772%	4.58820%
12848	0.93729%	1.86580%	2.78561%	3.69680%	4.59945%
12864	0.93846%	1.86812%	2.78905%	3.70134%	4.60507%
13656	0.99624%	1.98256%	2.95905%	3.92581%	4.88294%
13690	0.99872%	1.98747%	2.96634%	3.93544%	4.89486%
13707	0.99996%	1.98992%	2.96999%	3.94025%	4.90081%
14496	1.05752%	2.10386%	3.13913%	4.16346%	5.17695%
14550	1.06146%	2.11165%	3.15070%	4.17872%	5.19583%
15393	1.12296%	2.23331%	3.33119%	4.41674%	5.49011%
16020	1.16870%	2.32374%	3.46529%	4.59349%	5.70851%
16060	1.17162%	2.32951%	3.47384%	4.60476%	5.72243%
16080	1.17308%	2.33239%	3.47811%	4.61039%	5.72939%
16860	1.22998%	2.44483%	3.64474%	4.82990%	6.00047%
16902	1.23304%	2.45088%	3.65371%	4.84170%	6.01505%
16923	1.23458%	2.45391%	3.65819%	4.84761%	6.02234%
17700	1.29126%	2.56585%	3.82398%	5.06586%	6.29171%
17766	1.29608%	2.57535%	3.83805%	5.08438%	6.31457%
18609	1.35757%	2.69672%	4.01768%	5.32072%	6.60606%
19224	1.40244%	2.78521%	4.14859%	5.49285%	6.81826%
19272	1.40594%	2.79212%	4.15881%	5.50628%	6.83481%
19296	1.40769%	2.79557%	4.16391%	5.51299%	6.84308%
20064	1.46372%	2.90602%	4.32720%	5.72759%	7.10748%
20114	1.46737%	2.91320%	4.33783%	5.74155%	7.12467%
20139	1.46919%	2.91680%	4.34314%	5.74852%	7.13326%
20904	1.52500%	3.02675%	4.50559%	5.96188%	7.39597%
20982	1.53069%	3.03795%	4.52214%	5.98362%	7.42272%
21825	1.59219%	3.15903%	4.70092%	6.21827%	7.71146%
22428	1.63618%	3.24559%	4.82867%	6.38585%	7.91755%
22484	1.64027%	3.25363%	4.84053%	6.40140%	7.93667%
22512	1.64231%	3.25765%	4.84645%	6.40917%	7.94623%
23268	1.69746%	3.36611%	5.00643%	6.61891%	8.20402%
23326	1.70169%	3.37443%	5.01870%	6.63499%	8.22378%
23355	1.70381%	3.37859%	5.02483%	6.64303%	8.23365%
24108	1.75874%	3.48655%	5.18397%	6.85154%	8.48979%
24198	1.76531%	3.49945%	5.20298%	6.87644%	8.52036%
25041	1.82680%	3.62024%	5.38091%	7.10942%	8.80636%
25632	1.86992%	3.70487%	5.50552%	7.27249%	9.00643%
25696	1.87459%	3.71404%	5.51901%	7.29014%	9.02807%
25728	1.87692%	3.71862%	5.52575%	7.29896%	9.03889%
26472	1.93120%	3.82511%	5.68244%	7.50390%	9.29019%
26538	1.93601%	3.83455%	5.69633%	7.52207%	9.31246%
26571	1.93842%	3.83927%	5.70327%	7.53115%	9.32359%
27312	1.99248%	3.94526%	5.85914%	7.73488%	9.57325%
27414	1.99992%	3.95985%	5.88058%	7.76290%	9.60757%
28257	2.06142%	4.08035%	6.05766%	7.99421%	9.89084%

28836	2.10366%	4.16307%	6.17915%	8.15283%	10.08499%
28908	2.10891%	4.17335%	6.19425%	8.17254%	10.10911%
28944	2.11154%	4.17849%	6.20180%	8.18239%	10.12116%
29676	2.16494%	4.28301%	6.35523%	8.38259%	10.36606%
29750	2.17034%	4.29357%	6.37073%	8.40281%	10.39078%
29787	2.17304%	4.29886%	6.37848%	8.41292%	10.40314%
30516	2.22622%	4.40288%	6.53109%	8.61191%	10.64642%
30630	2.23454%	4.41914%	6.55494%	8.64300%	10.68442%
31473	2.29604%	4.53936%	6.73117%	8.87266%	10.96498%
32040	2.33740%	4.62017%	6.84958%	9.02688%	11.15329%
32120	2.34324%	4.63157%	6.86628%	9.04862%	11.17984%
32880	2.39868%	4.73982%	7.02481%	9.25500%	11.43169%
32962	2.40466%	4.75150%	7.04191%	9.27724%	11.45882%
33720	2.45996%	4.85941%	7.19983%	9.48268%	11.70938%
c=7					
0	0.00000%	0.00000%	0.00000%	0.00000%	0.00000%
800	0.06548%	0.13092%	0.19631%	0.26166%	0.32697%
801	0.06556%	0.13108%	0.19655%	0.26199%	0.32738%
803	0.06572%	0.13141%	0.19705%	0.26264%	0.32819%
804	0.06581%	0.13157%	0.19729%	0.26297%	0.32860%
1602	0.13112%	0.26207%	0.39285%	0.52346%	0.65390%
1606	0.13145%	0.26273%	0.39383%	0.52476%	0.65552%
1608	0.13161%	0.26305%	0.39432%	0.52542%	0.65634%
2403	0.19668%	0.39298%	0.58889%	0.78442%	0.97956%
2412	0.19742%	0.39445%	0.59109%	0.78735%	0.98321%
3048	0.24948%	0.49833%	0.74656%	0.99418%	1.24118%
3052	0.24980%	0.49898%	0.74754%	0.99548%	1.24280%
3056	0.25013%	0.49964%	0.74852%	0.99678%	1.24442%
3060	0.25046%	0.50029%	0.74950%	0.99808%	1.24604%
3064	0.25079%	0.50094%	0.75047%	0.99938%	1.24766%
3216	0.26323%	0.52576%	0.78760%	1.04876%	1.30923%
3848	0.31496%	0.62892%	0.94190%	1.25389%	1.56489%
3853	0.31536%	0.62974%	0.94312%	1.25551%	1.56691%
3863	0.31618%	0.63137%	0.94556%	1.25875%	1.57095%
3868	0.31659%	0.63218%	0.94678%	1.26037%	1.57297%
4654	0.38093%	0.76040%	1.13843%	1.51502%	1.89018%
4666	0.38191%	0.76236%	1.14136%	1.51891%	1.89502%
4672	0.38240%	0.76334%	1.14282%	1.52085%	1.89743%
5455	0.44649%	0.89098%	1.33349%	1.77403%	2.21259%
5476	0.44821%	0.89440%	1.33860%	1.78081%	2.22104%
6096	0.49895%	0.99542%	1.48940%	1.98093%	2.47000%
6104	0.49961%	0.99672%	1.49135%	1.98351%	2.47321%
6112	0.50026%	0.99802%	1.49329%	1.98609%	2.47641%
6120	0.50092%	0.99933%	1.49524%	1.98867%	2.47962%
6128	0.50157%	1.00063%	1.49718%	1.99125%	2.48283%
6280	0.51401%	1.02538%	1.53413%	2.04026%	2.54378%
6896	0.56443%	1.12568%	1.68376%	2.23869%	2.79049%
6905	0.56517%	1.12714%	1.68594%	2.24158%	2.79409%
6923	0.56664%	1.13007%	1.69031%	2.24738%	2.80129%
6932	0.56738%	1.13154%	1.69250%	2.25028%	2.80489%
7706	0.63073%	1.25748%	1.88028%	2.49915%	3.11412%
7726	0.63237%	1.26074%	1.88513%	2.50558%	3.12210%
7736	0.63319%	1.26236%	1.88756%	2.50879%	3.12609%
8507	0.69629%	1.38773%	2.07436%	2.75621%	3.43332%
8540	0.69899%	1.39310%	2.08235%	2.76679%	3.44645%
9144	0.74843%	1.49126%	2.22853%	2.96028%	3.68655%
9156	0.74941%	1.49321%	2.23143%	2.96412%	3.69132%
9168	0.75039%	1.49516%	2.23433%	2.96796%	3.69609%
9180	0.75138%	1.49711%	2.23723%	2.97180%	3.70085%
9192	0.75236%	1.49906%	2.24014%	2.97564%	3.70562%
9344	0.76480%	1.52375%	2.27690%	3.02428%	3.76595%
9944	0.81391%	1.62119%	2.42191%	3.21611%	4.00384%

9957	0.81497%	1.62330%	2.42505%	3.22026%	4.00899%
9983	0.81710%	1.62753%	2.43133%	3.22856%	4.01929%
9996	0.81816%	1.62964%	2.43447%	3.23272%	4.02444%
10758	0.88053%	1.75331%	2.61841%	3.47589%	4.32582%
10786	0.88283%	1.75786%	2.62517%	3.48482%	4.33688%
10800	0.88397%	1.76013%	2.62854%	3.48928%	4.34241%
11559	0.94609%	1.88324%	2.81152%	3.73102%	4.64182%
11604	0.94978%	1.89054%	2.82236%	3.74533%	4.65954%
12192	0.99791%	1.98585%	2.96394%	3.93227%	4.89094%
12208	0.99922%	1.98845%	2.96779%	3.93736%	4.89723%
12224	1.00052%	1.99104%	2.97165%	3.94244%	4.90352%
12240	1.00183%	1.99363%	2.97550%	3.94752%	4.90981%
12256	1.00314%	1.99623%	2.97935%	3.95261%	4.91610%
12408	1.01558%	2.02086%	3.01592%	4.00088%	4.97583%
12992	1.06338%	2.11546%	3.15635%	4.18618%	5.20505%
13009	1.06478%	2.11822%	3.16044%	4.19157%	5.21172%
13043	1.06756%	2.12372%	3.16861%	4.20235%	5.22505%
13060	1.06895%	2.12648%	3.17270%	4.20774%	5.23171%
13810	1.13034%	2.24790%	3.35283%	4.44527%	5.52537%
13846	1.13328%	2.25373%	3.36147%	4.45666%	5.53944%
13864	1.13476%	2.25664%	3.36579%	4.46236%	5.54648%
14611	1.19590%	2.37750%	3.54496%	4.69847%	5.83819%
14668	1.20056%	2.38672%	3.55863%	4.71647%	5.86041%
15240	1.24738%	2.47921%	3.69566%	4.89695%	6.08325%
15260	1.24902%	2.48244%	3.70045%	4.90326%	6.09104%
15280	1.25066%	2.48567%	3.70524%	4.90956%	6.09882%
15300	1.25229%	2.48890%	3.71003%	4.91587%	6.10660%
15320	1.25393%	2.49214%	3.71482%	4.92217%	6.11438%
15472	1.26637%	2.51671%	3.75121%	4.97008%	6.17351%
16040	1.31286%	2.60849%	3.88710%	5.14894%	6.39420%
16061	1.31458%	2.61188%	3.89213%	5.15554%	6.40236%
16103	1.31802%	2.61866%	3.90217%	5.16876%	6.41866%
16124	1.31974%	2.62206%	3.90719%	5.17537%	6.42681%
16862	1.38014%	2.74124%	4.08355%	5.40733%	6.71285%
16906	1.38374%	2.74834%	4.09405%	5.42115%	6.72988%
16928	1.38554%	2.75189%	4.09931%	5.42806%	6.73840%
17663	1.44570%	2.87051%	4.27471%	5.65862%	7.02252%
17732	1.45135%	2.88164%	4.29117%	5.68024%	7.04915%
18288	1.49686%	2.97131%	4.42370%	5.85434%	7.26357%
18312	1.49882%	2.97518%	4.42941%	5.86185%	7.27282%
18336	1.50079%	2.97905%	4.43513%	5.86936%	7.28207%
18360	1.50275%	2.98292%	4.44085%	5.87687%	7.29131%
18384	1.50472%	2.98679%	4.44657%	5.88438%	7.30055%
18536	1.51716%	3.01130%	4.48277%	5.93192%	7.35909%
19088	1.56234%	3.10027%	4.61417%	6.10442%	7.57139%
19113	1.56438%	3.10430%	4.62012%	6.11223%	7.58100%
19163	1.56848%	3.11235%	4.63201%	6.12784%	7.60021%
19188	1.57052%	3.11638%	4.63796%	6.13565%	7.60981%
19914	1.62995%	3.23332%	4.81057%	6.36211%	7.88836%
19966	1.63420%	3.24170%	4.82293%	6.37831%	7.90829%
19992	1.63633%	3.24588%	4.82910%	6.38642%	7.91825%
20715	1.69551%	3.36227%	5.00077%	6.61149%	8.19490%
20796	1.70214%	3.37530%	5.01999%	6.63668%	8.22586%
21336	1.74633%	3.46217%	5.14805%	6.80449%	8.43200%
21364	1.74863%	3.46668%	5.15469%	6.81318%	8.44268%
21392	1.75092%	3.47118%	5.16132%	6.82188%	8.45335%
21420	1.75321%	3.47568%	5.16796%	6.83057%	8.46403%
21448	1.75550%	3.48019%	5.17460%	6.83926%	8.47471%
21600	1.76794%	3.50463%	5.21062%	6.88644%	8.53264%
22136	1.81181%	3.59080%	5.33756%	7.05267%	8.73671%
22165	1.81419%	3.59546%	5.34443%	7.06166%	8.74774%
22223	1.81893%	3.60479%	5.35815%	7.07963%	8.76980%
22252	1.82131%	3.60945%	5.36502%	7.08862%	8.78083%

22966	1.87975%	3.72416%	5.53391%	7.30964%	9.05199%
23026	1.88466%	3.73380%	5.54809%	7.32820%	9.07475%
23056	1.88712%	3.73862%	5.55519%	7.33747%	9.08613%
23767	1.94531%	3.85278%	5.72314%	7.55713%	9.35543%
23860	1.95292%	3.86771%	5.74510%	7.58583%	9.39061%
24384	1.99581%	3.95179%	5.86873%	7.74742%	9.58861%
24416	1.99843%	3.95692%	5.87628%	7.75728%	9.60070%
24448	2.00105%	3.96206%	5.88383%	7.76714%	9.61277%
24480	2.00367%	3.96719%	5.89137%	7.77700%	9.62485%
24512	2.00629%	3.97233%	5.89892%	7.78686%	9.63693%
24664	2.01873%	3.99671%	5.93476%	7.83368%	9.69428%
25184	2.06129%	4.08009%	6.05728%	7.99372%	9.89024%
25217	2.06399%	4.08538%	6.06506%	8.00387%	9.90267%
25283	2.06939%	4.09596%	6.08060%	8.02417%	9.92751%
25316	2.07209%	4.10125%	6.08837%	8.03431%	9.93994%
26018	2.12955%	4.21376%	6.25358%	8.24996%	10.20383%
26086	2.13512%	4.22465%	6.26957%	8.27083%	10.22936%
26120	2.13790%	4.23010%	6.27757%	8.28126%	10.24213%
26819	2.19511%	4.34204%	6.44185%	8.49556%	10.50419%
26924	2.20371%	4.35885%	6.46651%	8.52772%	10.54351%
27432	2.24529%	4.44016%	6.58576%	8.68318%	10.73351%
27468	2.24823%	4.44592%	6.59421%	8.69419%	10.74697%
27504	2.25118%	4.45168%	6.60265%	8.70520%	10.76042%
27540	2.25413%	4.45744%	6.61110%	8.71621%	10.77387%
27728	2.26951%	4.48752%	6.65520%	8.77368%	10.84408%
28232	2.31077%	4.56814%	6.77335%	8.92761%	11.03208%
28269	2.31380%	4.57406%	6.78202%	8.93890%	11.04587%
28343	2.31985%	4.58589%	6.79936%	8.96148%	11.07345%
29070	2.37936%	4.70210%	6.96958%	9.18311%	11.34398%
29146	2.38558%	4.71425%	6.98736%	9.20626%	11.37222%
29871	2.44492%	4.83006%	7.15689%	9.42683%	11.64128%
30480	2.49476%	4.92729%	7.29913%	9.61181%	11.86679%
30520	2.49804%	4.93368%	7.30847%	9.62395%	11.88158%
30560	2.50131%	4.94006%	7.31781%	9.63609%	11.89638%
31280	2.56024%	5.05494%	7.48577%	9.85436%	12.16232%
31321	2.56360%	5.06148%	7.49533%	9.86678%	12.17744%
c=8					
0	0.00000%	0.00000%	0.00000%	0.00000%	0.00000%
760	0.06998%	0.13991%	0.20980%	0.27963%	0.34942%
762	0.07017%	0.14028%	0.21035%	0.28037%	0.35033%
763	0.07026%	0.14047%	0.21062%	0.28073%	0.35079%
764	0.07035%	0.14065%	0.21090%	0.28110%	0.35125%
765	0.07044%	0.14083%	0.21118%	0.28147%	0.35171%
766	0.07053%	0.14102%	0.21145%	0.28184%	0.35217%
1524	0.14033%	0.28046%	0.42040%	0.56014%	0.69969%
1526	0.14051%	0.28083%	0.42095%	0.56088%	0.70060%
1530	0.14088%	0.28157%	0.42205%	0.56234%	0.70243%
1532	0.14107%	0.28194%	0.42260%	0.56308%	0.70335%
2289	0.21077%	0.42110%	0.63098%	0.84043%	1.04943%
2295	0.21132%	0.42220%	0.63264%	0.84262%	1.05217%
2298	0.21160%	0.42275%	0.63346%	0.84372%	1.05354%
2892	0.26630%	0.53188%	0.79676%	1.06094%	1.32441%
2896	0.26666%	0.53262%	0.79786%	1.06240%	1.32623%
2900	0.26703%	0.53335%	0.79896%	1.06386%	1.32806%
2904	0.26740%	0.53409%	0.80006%	1.06532%	1.32988%
2908	0.26777%	0.53482%	0.80116%	1.06679%	1.33170%
2912	0.26814%	0.53556%	0.80226%	1.06825%	1.33352%
2916	0.26851%	0.53629%	0.80336%	1.06971%	1.33534%
3052	0.28103%	0.56127%	0.84072%	1.11939%	1.39727%
3064	0.28213%	0.56347%	0.84402%	1.12377%	1.40274%
3652	0.33628%	0.67142%	1.00544%	1.33834%	1.67012%
3662	0.33720%	0.67326%	1.00819%	1.34199%	1.67466%

3667	0.33766%	0.67418%	1.00956%	1.34381%	1.67693%
3672	0.33812%	0.67510%	1.01093%	1.34563%	1.67920%
3677	0.33858%	0.67601%	1.01230%	1.34746%	1.68148%
3682	0.33904%	0.67693%	1.01368%	1.34928%	1.68375%
3830	0.35267%	0.70409%	1.05428%	1.40323%	1.75095%
4424	0.40736%	0.81307%	1.21712%	1.61953%	2.02029%
4430	0.40792%	0.81417%	1.21876%	1.62171%	2.02301%
4442	0.40902%	0.81637%	1.22205%	1.62608%	2.02845%
4448	0.40957%	0.81747%	1.22370%	1.62826%	2.03116%
5193	0.47817%	0.95406%	1.42767%	1.89902%	2.36811%
5207	0.47946%	0.95663%	1.43150%	1.90410%	2.37444%
5214	0.48011%	0.95791%	1.43342%	1.90664%	2.37760%
5784	0.53259%	1.06235%	1.58929%	2.11341%	2.63475%
5792	0.53333%	1.06381%	1.59147%	2.11631%	2.63836%
5800	0.53407%	1.06528%	1.59366%	2.11921%	2.64196%
5808	0.53480%	1.06675%	1.59584%	2.12211%	2.64557%
5816	0.53554%	1.06821%	1.59803%	2.12501%	2.64917%
5824	0.53628%	1.06968%	1.60022%	2.12791%	2.65278%
5832	0.53701%	1.07114%	1.60240%	2.13081%	2.65638%
5956	0.54843%	1.09385%	1.63629%	2.17574%	2.71224%
5980	0.55064%	1.09825%	1.64284%	2.18444%	2.72305%
6544	0.60257%	1.20152%	1.79685%	2.38860%	2.97678%
6562	0.60423%	1.20481%	1.80176%	2.39511%	2.98487%
6571	0.60506%	1.20646%	1.80422%	2.39837%	2.98892%
6580	0.60589%	1.20811%	1.80668%	2.40162%	2.99296%
6589	0.60672%	1.20975%	1.80913%	2.40488%	2.99700%
6598	0.60755%	1.21140%	1.81159%	2.40813%	3.00105%
6746	0.62117%	1.23849%	1.85197%	2.46164%	3.06753%
7324	0.67440%	1.34425%	2.00958%	2.67042%	3.32681%
7334	0.67532%	1.34607%	2.01230%	2.67403%	3.33129%
7354	0.67716%	1.34973%	2.01775%	2.68125%	3.34026%
7364	0.67808%	1.35156%	2.02048%	2.68486%	3.34474%
8097	0.74557%	1.48559%	2.22009%	2.94912%	3.67270%
8119	0.74760%	1.48961%	2.22608%	2.95704%	3.68253%
8130	0.74861%	1.49162%	2.22907%	2.96100%	3.68745%
8676	0.79889%	1.59140%	2.37757%	3.15747%	3.93114%
8688	0.79999%	1.59359%	2.38084%	3.16179%	3.93649%
8700	0.80110%	1.59578%	2.38410%	3.16610%	3.94184%
8712	0.80220%	1.59797%	2.38736%	3.17041%	3.94719%
8724	0.80331%	1.60017%	2.39062%	3.17473%	3.95254%
8736	0.80441%	1.60236%	2.39388%	3.17904%	3.95789%
8748	0.80552%	1.60455%	2.39715%	3.18336%	3.96324%
8860	0.81583%	1.62501%	2.42758%	3.22361%	4.01315%
8896	0.81915%	1.63158%	2.43737%	3.23655%	4.02919%
9436	0.86887%	1.73019%	2.58403%	3.43045%	4.26952%
9462	0.87126%	1.73494%	2.59109%	3.43978%	4.28108%
9475	0.87246%	1.73731%	2.59462%	3.44444%	4.28686%
9488	0.87366%	1.73968%	2.59815%	3.44911%	4.29264%
9501	0.87486%	1.74206%	2.60167%	3.45377%	4.29841%
9514	0.87605%	1.74443%	2.60520%	3.45843%	4.30419%
9662	0.88968%	1.77145%	2.64537%	3.51152%	4.36996%
10224	0.94143%	1.87400%	2.79779%	3.71288%	4.61936%
10238	0.94272%	1.87655%	2.80158%	3.71789%	4.62556%
10266	0.94530%	1.88166%	2.80917%	3.72791%	4.63797%
10280	0.94659%	1.88421%	2.81296%	3.73293%	4.64418%
11001	1.01298%	2.01569%	3.00825%	3.99076%	4.96331%
11031	1.01574%	2.02116%	3.01637%	4.00147%	4.97657%
11046	1.01712%	2.02389%	3.02043%	4.00683%	4.98320%
11568	1.06519%	2.11903%	3.16164%	4.19315%	5.21368%
11584	1.06666%	2.12194%	3.16597%	4.19886%	5.22073%
11600	1.06813%	2.12486%	3.17029%	4.20457%	5.22779%
11616	1.06961%	2.12777%	3.17462%	4.21027%	5.23485%
11632	1.07108%	2.13069%	3.17895%	4.21598%	5.24190%

11648	1.07255%	2.13360%	3.18327%	4.22168%	5.24896%
11664	1.07403%	2.13652%	3.18760%	4.22739%	5.25601%
11764	1.08323%	2.15473%	3.21463%	4.26304%	5.30010%
11812	1.08765%	2.16348%	3.22760%	4.28015%	5.32126%
12328	1.13517%	2.25745%	3.36699%	4.46394%	5.54844%
12362	1.13830%	2.26364%	3.37617%	4.47604%	5.56339%
12379	1.13986%	2.26673%	3.38076%	4.48209%	5.57087%
12396	1.14143%	2.26983%	3.38535%	4.48814%	5.57834%
12413	1.14299%	2.27292%	3.38994%	4.49419%	5.58582%
12430	1.14456%	2.27602%	3.39453%	4.50024%	5.59329%
12578	1.15819%	2.30296%	3.43448%	4.55289%	5.65835%
13124	1.20846%	2.40232%	3.58176%	4.74694%	5.89804%
13142	1.21012%	2.40560%	3.58661%	4.75333%	5.90593%
13178	1.21343%	2.41215%	3.59631%	4.76611%	5.92172%
13196	1.21509%	2.41542%	3.60117%	4.77250%	5.92961%
13905	1.28038%	2.54436%	3.79216%	5.02399%	6.24005%
13943	1.28388%	2.55127%	3.80239%	5.03746%	6.25666%
13962	1.28563%	2.55472%	3.80751%	5.04419%	6.26497%
14460	1.33148%	2.64524%	3.94150%	5.22051%	6.48248%
14480	1.33332%	2.64887%	3.94688%	5.22758%	6.49121%
14500	1.33517%	2.65250%	3.95226%	5.23466%	6.49993%
14520	1.33701%	2.65614%	3.95764%	5.24173%	6.50866%
14540	1.33885%	2.65977%	3.96301%	5.24881%	6.51739%
14560	1.34069%	2.66341%	3.96839%	5.25588%	6.52611%
14580	1.34253%	2.66704%	3.97377%	5.26295%	6.53483%
14668	1.35063%	2.68303%	3.99743%	5.29408%	6.57321%
14728	1.35616%	2.69393%	4.01356%	5.31529%	6.59937%
15220	1.40146%	2.78329%	4.14574%	5.48911%	6.81365%
15262	1.40533%	2.79091%	4.15702%	5.50394%	6.83192%
15283	1.40726%	2.79473%	4.16266%	5.51135%	6.84106%
15304	1.40920%	2.79854%	4.16830%	5.51876%	6.85020%
15325	1.41113%	2.80235%	4.17394%	5.52618%	6.85933%
15346	1.41307%	2.80616%	4.17958%	5.53359%	6.86846%
15494	1.42669%	2.83303%	4.21931%	5.58581%	6.93282%
16024	1.47550%	2.92922%	4.36150%	5.77265%	7.16297%
16046	1.47752%	2.93321%	4.36740%	5.78039%	7.17251%
16090	1.48157%	2.94120%	4.37920%	5.79589%	7.19160%
16112	1.48360%	2.94519%	4.38509%	5.80364%	7.20114%
16809	1.54778%	3.07160%	4.57184%	6.04886%	7.50302%
16855	1.55201%	3.07994%	4.58416%	6.06503%	7.52292%
16878	1.55413%	3.08411%	4.59032%	6.07311%	7.53287%
17352	1.59778%	3.17003%	4.71716%	6.23957%	7.73766%
17376	1.59999%	3.17438%	4.72358%	6.24800%	7.74802%
17400	1.60220%	3.17873%	4.73000%	6.25642%	7.75838%
17424	1.60441%	3.18308%	4.73642%	6.26484%	7.76874%
17448	1.60662%	3.18743%	4.74284%	6.27326%	7.77910%
17472	1.60883%	3.19177%	4.74925%	6.28168%	7.78945%
17496	1.61104%	3.19612%	4.75567%	6.29010%	7.79981%
17572	1.61804%	3.20989%	4.77599%	6.31676%	7.83259%
17644	1.62467%	3.22294%	4.79524%	6.34201%	7.86364%
18112	1.66776%	3.30771%	4.92030%	6.50601%	8.06527%
18162	1.67236%	3.31676%	4.93366%	6.52352%	8.08679%
18187	1.67467%	3.32129%	4.94034%	6.53227%	8.09755%
18212	1.67697%	3.32581%	4.94701%	6.54102%	8.10831%
18237	1.67927%	3.33034%	4.95369%	6.54978%	8.11906%
18262	1.68157%	3.33487%	4.96036%	6.55853%	8.12982%
18410	1.69520%	3.36166%	4.99988%	6.61032%	8.19347%
18924	1.74253%	3.45469%	5.13703%	6.79005%	8.41426%
18950	1.74492%	3.45940%	5.14396%	6.79913%	8.42542%
19002	1.74971%	3.46881%	5.15783%	6.81730%	8.44773%
19028	1.75210%	3.47351%	5.16476%	6.82638%	8.45888%
19713	1.81518%	3.59741%	5.34730%	7.06542%	8.75235%
19767	1.82015%	3.60718%	5.36168%	7.08424%	8.77546%

19794	1.82264%	3.61206%	5.36887%	7.09365%	8.78701%
20244	1.86407%	3.69340%	5.48863%	7.25040%	8.97933%
20272	1.86665%	3.69846%	5.49608%	7.26015%	8.99128%
20300	1.86923%	3.70352%	5.50353%	7.26989%	9.00324%
20328	1.87181%	3.70858%	5.51098%	7.27964%	9.01519%
20356	1.87439%	3.71364%	5.51843%	7.28938%	9.02715%
20384	1.87697%	3.71870%	5.52587%	7.29913%	9.03910%
20412	1.87954%	3.72376%	5.53332%	7.30887%	9.05105%
20476	1.88544%	3.73533%	5.55034%	7.33113%	9.07835%
20560	1.89317%	3.75050%	5.57268%	7.36035%	9.11419%
21004	1.93406%	3.83071%	5.69068%	7.51468%	9.30340%
21062	1.93940%	3.84118%	5.70609%	7.53482%	9.32810%
21091	1.94207%	3.84642%	5.71379%	7.54490%	9.34044%
21120	1.94474%	3.85166%	5.72149%	7.55497%	9.35279%
21149	1.94741%	3.85689%	5.72919%	7.56504%	9.36513%
21178	1.95008%	3.86213%	5.73690%	7.57510%	9.37747%
21326	1.96371%	3.88885%	5.77620%	7.62648%	9.44043%
21824	2.00956%	3.97874%	5.90835%	7.79919%	9.65203%
21854	2.01232%	3.98416%	5.91631%	7.80958%	9.66476%
21914	2.01785%	3.99498%	5.93222%	7.83037%	9.69022%
21944	2.02061%	4.00040%	5.94018%	7.84077%	9.70295%
22617	2.08258%	4.12179%	6.11854%	8.07370%	9.98815%
22679	2.08829%	4.13297%	6.13496%	8.09514%	10.01439%
22710	2.09114%	4.13856%	6.14317%	8.10586%	10.02750%
23136	2.13037%	4.21536%	6.25593%	8.25303%	10.20759%
23168	2.13332%	4.22113%	6.26440%	8.26408%	10.22111%
23200	2.13626%	4.22689%	6.27286%	8.27513%	10.23462%
23232	2.13921%	4.23266%	6.28133%	8.28618%	10.24813%
23264	2.14216%	4.23843%	6.28980%	8.29722%	10.26165%
23296	2.14510%	4.24420%	6.29826%	8.30827%	10.27516%
23380	2.15284%	4.25933%	6.32048%	8.33725%	10.31061%
23476	2.16168%	4.27663%	6.34587%	8.37037%	10.35112%
23896	2.20035%	4.35229%	6.45688%	8.51516%	10.52816%
23962	2.20643%	4.36418%	6.47432%	8.53790%	10.55596%
23995	2.20947%	4.37012%	6.48304%	8.54927%	10.56985%
24028	2.21251%	4.37606%	6.49175%	8.56064%	10.58374%
24061	2.21555%	4.38201%	6.50047%	8.57200%	10.59764%
24242	2.23221%	4.41460%	6.54827%	8.63432%	10.67380%
24724	2.27659%	4.50136%	6.67548%	8.80011%	10.87637%
24758	2.27973%	4.50748%	6.68445%	8.81180%	10.89064%
24826	2.28599%	4.51972%	6.70239%	8.83517%	10.91919%
25521	2.34998%	4.64474%	6.88558%	9.07376%	11.21052%
25591	2.35643%	4.65733%	6.90402%	9.09776%	11.23982%
26028	2.39667%	4.73590%	7.01906%	9.24751%	11.42256%
26064	2.39998%	4.74237%	7.02854%	9.25984%	11.43760%
26100	2.40330%	4.74884%	7.03801%	9.27217%	11.45264%
26136	2.40661%	4.75531%	7.04748%	9.28450%	11.46767%
26172	2.40993%	4.76178%	7.05695%	9.29682%	11.48271%
26284	2.42024%	4.78191%	7.08642%	9.33516%	11.52947%
26788	2.46665%	4.87246%	7.21892%	9.50751%	11.73965%
26862	2.47346%	4.88575%	7.23837%	9.53280%	11.77048%
26899	2.47687%	4.89239%	7.24809%	9.54544%	11.78589%
26936	2.48028%	4.89904%	7.25781%	9.55808%	11.80130%
27624	2.54363%	5.02256%	7.43843%	9.79286%	12.08740%
27662	2.54713%	5.02938%	7.44840%	9.80582%	12.10319%
28920	2.66296%	5.25502%	7.77805%	10.23389%	12.62434%
28960	2.66665%	5.26219%	7.78851%	10.24748%	12.64087%
29000	2.67033%	5.26936%	7.79898%	10.26106%	12.65740%
29680	2.73294%	5.39120%	7.97681%	10.49176%	12.93798%
c=9					
0	0.00000%	0.00000%	0.00000%	0.00000%	-
720	0.07480%	0.14954%	0.22423%	0.29886%	-

723	0.07511%	0.15016%	0.22516%	0.30010%	-
724	0.07521%	0.15037%	0.22547%	0.30051%	-
725	0.07532%	0.15058%	0.22578%	0.30093%	-
726	0.07542%	0.15078%	0.22609%	0.30134%	-
727	0.07552%	0.15099%	0.22640%	0.30176%	-
728	0.07563%	0.15120%	0.22671%	0.30217%	-
729	0.07573%	0.15141%	0.22703%	0.30259%	-
1446	0.15022%	0.30021%	0.44998%	0.59952%	-
1450	0.15063%	0.30104%	0.45122%	0.60118%	-
1452	0.15084%	0.30146%	0.45184%	0.60200%	-
1454	0.15105%	0.30187%	0.45247%	0.60283%	-
1456	0.15126%	0.30229%	0.45309%	0.60366%	-
1458	0.15147%	0.30270%	0.45371%	0.60449%	-
2175	0.22595%	0.45139%	0.67632%	0.90075%	-
2178	0.22626%	0.45201%	0.67725%	0.90199%	-
2184	0.22689%	0.45326%	0.67912%	0.90446%	-
2187	0.22720%	0.45388%	0.68005%	0.90570%	-
2736	0.28423%	0.56765%	0.85027%	1.13209%	-
2744	0.28506%	0.56931%	0.85275%	1.13538%	-
2748	0.28548%	0.57014%	0.85399%	1.13703%	-
2752	0.28589%	0.57097%	0.85523%	1.13868%	-
2756	0.28631%	0.57180%	0.85647%	1.14033%	-
2760	0.28672%	0.57263%	0.85771%	1.14197%	-
2764	0.28714%	0.57345%	0.85895%	1.14362%	-
2768	0.28756%	0.57428%	0.86019%	1.14527%	-
2772	0.28797%	0.57511%	0.86143%	1.14692%	-
2904	0.30168%	0.60246%	0.90232%	1.20129%	-
2912	0.30251%	0.60411%	0.90480%	1.20458%	-
2916	0.30293%	0.60494%	0.90604%	1.20623%	-
3456	0.35903%	0.71677%	1.07322%	1.42840%	-
3471	0.36059%	0.71987%	1.07787%	1.43457%	-
3476	0.36111%	0.72091%	1.07941%	1.43662%	-
3481	0.36163%	0.72194%	1.08096%	1.43868%	-
3486	0.36214%	0.72298%	1.08251%	1.44073%	-
3491	0.36266%	0.72401%	1.08405%	1.44279%	-
3496	0.36318%	0.72505%	1.08560%	1.44484%	-
3501	0.36370%	0.72608%	1.08715%	1.44690%	-
3630	0.37710%	0.75279%	1.12705%	1.49991%	-
3645	0.37866%	0.75589%	1.13169%	1.50607%	-
4194	0.43570%	0.86949%	1.30140%	1.73143%	-
4206	0.43694%	0.87198%	1.30511%	1.73635%	-
4212	0.43757%	0.87322%	1.30696%	1.73881%	-
4218	0.43819%	0.87446%	1.30882%	1.74127%	-
4224	0.43881%	0.87570%	1.31067%	1.74373%	-
4230	0.43944%	0.87694%	1.31252%	1.74619%	-
4374	0.45440%	0.90673%	1.35700%	1.80523%	-
4931	0.51226%	1.02190%	1.52892%	2.03335%	-
4938	0.51299%	1.02334%	1.53108%	2.03621%	-
4952	0.51444%	1.02624%	1.53540%	2.04194%	-
4959	0.51517%	1.02768%	1.53756%	2.04481%	-
5472	0.56846%	1.13369%	1.69571%	2.25453%	-
5488	0.57012%	1.13700%	1.70064%	2.26107%	-
5496	0.57095%	1.13865%	1.70311%	2.26434%	-
5504	0.57179%	1.14030%	1.70557%	2.26761%	-
5512	0.57262%	1.14196%	1.70803%	2.27087%	-
5520	0.57345%	1.14361%	1.71050%	2.27414%	-
5528	0.57428%	1.14526%	1.71296%	2.27741%	-
5536	0.57511%	1.14691%	1.71543%	2.28068%	-
5544	0.57594%	1.14857%	1.71789%	2.28394%	-
5664	0.58841%	1.17335%	1.75486%	2.33294%	-
5680	0.59007%	1.17666%	1.75979%	2.33947%	-
5688	0.59090%	1.17831%	1.76225%	2.34274%	-
6192	0.64326%	1.28238%	1.91739%	2.54832%	-

6219	0.64606%	1.28795%	1.92570%	2.55932%	-
6228	0.64700%	1.28981%	1.92847%	2.56299%	-
6237	0.64793%	1.29167%	1.93124%	2.56666%	-
6246	0.64887%	1.29353%	1.93401%	2.57033%	-
6255	0.64980%	1.29539%	1.93677%	2.57399%	-
6264	0.65074%	1.29724%	1.93954%	2.57766%	-
6273	0.65167%	1.29910%	1.94231%	2.58133%	-
6390	0.66383%	1.32325%	1.97830%	2.62899%	-
6417	0.66663%	1.32882%	1.98660%	2.63999%	-
6942	0.72117%	1.43715%	2.14796%	2.85364%	-
6962	0.72325%	1.44127%	2.15410%	2.86177%	-
6972	0.72429%	1.44333%	2.15717%	2.86584%	-
6982	0.72533%	1.44540%	2.16024%	2.86991%	-
6992	0.72637%	1.44746%	2.16332%	2.87397%	-
7002	0.72741%	1.44952%	2.16639%	2.87804%	-
7146	0.74237%	1.47922%	2.21061%	2.93657%	-
7687	0.79857%	1.59076%	2.37663%	3.15622%	-
7698	0.79971%	1.59303%	2.38000%	3.16068%	-
7720	0.80200%	1.59756%	2.38675%	3.16960%	-
7731	0.80314%	1.59983%	2.39012%	3.17407%	-
8208	0.85269%	1.69811%	2.53633%	3.36740%	-
8232	0.85519%	1.70306%	2.54368%	3.37712%	-
8244	0.85643%	1.70553%	2.54736%	3.38198%	-
8256	0.85768%	1.70800%	2.55103%	3.38684%	-
8268	0.85893%	1.71047%	2.55471%	3.39170%	-
8280	0.86017%	1.71295%	2.55839%	3.39655%	-
8292	0.86142%	1.71542%	2.56206%	3.40141%	-
8304	0.86267%	1.71789%	2.56574%	3.40627%	-
8316	0.86391%	1.72036%	2.56941%	3.41113%	-
8424	0.87513%	1.74261%	2.60249%	3.45485%	-
8448	0.87762%	1.74755%	2.60984%	3.46456%	-
8460	0.87887%	1.75002%	2.61351%	3.46942%	-
8928	0.92749%	1.84638%	2.75675%	3.65867%	-
8967	0.93154%	1.85441%	2.76868%	3.67443%	-
8980	0.93289%	1.85708%	2.77265%	3.67968%	-
8993	0.93424%	1.85976%	2.77663%	3.68493%	-
9006	0.93559%	1.86243%	2.78060%	3.69018%	-
9019	0.93694%	1.86511%	2.78458%	3.69544%	-
9032	0.93829%	1.86779%	2.78856%	3.70069%	-
9045	0.93964%	1.87046%	2.79253%	3.70594%	-
9150	0.95055%	1.89207%	2.82464%	3.74835%	-
9189	0.95460%	1.90010%	2.83656%	3.76409%	-
9690	1.00665%	2.00317%	2.98966%	3.96622%	-
9718	1.00956%	2.00893%	2.99821%	3.97750%	-
9732	1.01101%	2.01181%	3.00248%	3.98315%	-
9746	1.01247%	2.01469%	3.00676%	3.98879%	-
9760	1.01392%	2.01757%	3.01103%	3.99443%	-
9774	1.01538%	2.02045%	3.01531%	4.00007%	-
9918	1.03034%	2.05006%	3.05927%	4.05809%	-
10443	1.08488%	2.15798%	3.21945%	4.26940%	-
10458	1.08643%	2.16107%	3.22403%	4.27544%	-
10488	1.08955%	2.16723%	3.23317%	4.28750%	-
10503	1.09111%	2.17032%	3.23775%	4.29353%	-
10944	1.13692%	2.26092%	3.37214%	4.47073%	-
10976	1.14025%	2.26749%	3.38189%	4.48358%	-
10992	1.14191%	2.27078%	3.38676%	4.49000%	-
11008	1.14357%	2.27407%	3.39164%	4.49643%	-
11024	1.14523%	2.27735%	3.39651%	4.50285%	-
11040	1.14690%	2.28064%	3.40138%	4.50927%	-
11056	1.14856%	2.28393%	3.40625%	4.51569%	-
11072	1.15022%	2.28721%	3.41113%	4.52212%	-
11088	1.15188%	2.29050%	3.41600%	4.52854%	-
11184	1.16186%	2.31021%	3.44523%	4.56706%	-

11216	1.16518%	2.31678%	3.45497%	4.57990%	-
11232	1.16684%	2.32007%	3.45984%	4.58632%	-
11664	1.21172%	2.40876%	3.59130%	4.75950%	-
11715	1.21702%	2.41923%	3.60681%	4.77993%	-
11732	1.21878%	2.42272%	3.61198%	4.78674%	-
11749	1.22055%	2.42621%	3.61715%	4.79355%	-
11766	1.22232%	2.42969%	3.62232%	4.80036%	-
11783	1.22408%	2.43318%	3.62749%	4.80717%	-
11800	1.22585%	2.43667%	3.63265%	4.81398%	-
11817	1.22762%	2.44016%	3.63782%	4.82078%	-
11910	1.23728%	2.45925%	3.66610%	4.85802%	-
11961	1.24257%	2.46971%	3.68160%	4.87843%	-
12438	1.29213%	2.56756%	3.82652%	5.06920%	-
12474	1.29587%	2.57494%	3.83745%	5.08359%	-
12492	1.29774%	2.57864%	3.84291%	5.09078%	-
12510	1.29961%	2.58233%	3.84838%	5.09798%	-
12528	1.30148%	2.58602%	3.85384%	5.10517%	-
12546	1.30335%	2.58971%	3.85931%	5.11236%	-
12690	1.31831%	2.61924%	3.90302%	5.16987%	-
13199	1.37119%	2.72357%	4.05741%	5.37297%	-
13218	1.37316%	2.72746%	4.06317%	5.38054%	-
13256	1.37711%	2.73525%	4.07469%	5.39569%	-
13275	1.37908%	2.73914%	4.08045%	5.40326%	-
13680	1.42115%	2.82211%	4.20316%	5.56459%	-
13720	1.42531%	2.83031%	4.21528%	5.58051%	-
13740	1.42739%	2.83440%	4.22133%	5.58847%	-
13760	1.42946%	2.83850%	4.22739%	5.59643%	-
13780	1.43154%	2.84259%	4.23345%	5.60439%	-
13800	1.43362%	2.84669%	4.23950%	5.61235%	-
13820	1.43570%	2.85079%	4.24556%	5.62031%	-
13840	1.43778%	2.85488%	4.25161%	5.62826%	-
13860	1.43985%	2.85898%	4.25767%	5.63622%	-
13944	1.44858%	2.87618%	4.28310%	5.66964%	-
13984	1.45274%	2.88437%	4.29520%	5.68555%	-
14004	1.45481%	2.88846%	4.30126%	5.69350%	-
14400	1.49595%	2.96953%	4.42106%	5.85088%	-
14463	1.50250%	2.98242%	4.44011%	5.87590%	-
14484	1.50468%	2.98672%	4.44646%	5.88424%	-
14505	1.50686%	2.99101%	4.45281%	5.89257%	-
14526	1.50904%	2.99531%	4.45916%	5.90091%	-
14547	1.51122%	2.99961%	4.46550%	5.90925%	-
14568	1.51340%	3.00391%	4.47185%	5.91758%	-
14589	1.51559%	3.00820%	4.47820%	5.92592%	-
14670	1.52400%	3.02478%	4.50268%	5.95807%	-
14733	1.53055%	3.03767%	4.52172%	5.98307%	-
15186	1.57761%	3.13032%	4.65855%	6.16267%	-
15230	1.58218%	3.13932%	4.67183%	6.18010%	-
15252	1.58446%	3.14382%	4.67847%	6.18881%	-
15274	1.58675%	3.14832%	4.68511%	6.19753%	-
15296	1.58903%	3.15282%	4.69175%	6.20624%	-
15318	1.59132%	3.15732%	4.69839%	6.21495%	-
15462	1.60628%	3.18676%	4.74185%	6.27196%	-
15955	1.65749%	3.28752%	4.89052%	6.46696%	-
15978	1.65988%	3.29222%	4.89745%	6.47605%	-
16024	1.66466%	3.30161%	4.91132%	6.49423%	-
16047	1.66705%	3.30631%	4.91825%	6.50332%	-
16416	1.70538%	3.38169%	5.02941%	6.64902%	-
16464	1.71037%	3.39149%	5.04386%	6.66797%	-
16488	1.71286%	3.39639%	5.05108%	6.67744%	-
16512	1.71536%	3.40129%	5.05831%	6.68690%	-
16536	1.71785%	3.40619%	5.06553%	6.69637%	-
16560	1.72034%	3.41109%	5.07276%	6.70584%	-
16584	1.72284%	3.41600%	5.07998%	6.71531%	-

16608	1.72533%	3.42090%	5.08721%	6.72477%	-
16632	1.72782%	3.42580%	5.09443%	6.73424%	-
16704	1.73530%	3.44050%	5.11610%	6.76263%	-
16752	1.74029%	3.45030%	5.13055%	6.78155%	-
16776	1.74278%	3.45520%	5.13777%	6.79102%	-
17136	1.78018%	3.52868%	5.24605%	6.93284%	-
17211	1.78797%	3.54398%	5.26859%	6.96237%	-
17236	1.79057%	3.54908%	5.27611%	6.97221%	-
17261	1.79317%	3.55418%	5.28362%	6.98205%	-
17286	1.79577%	3.55928%	5.29114%	6.99189%	-
17311	1.79836%	3.56439%	5.29865%	7.00173%	-
17336	1.80096%	3.56949%	5.30616%	7.01157%	-
17361	1.80356%	3.57459%	5.31368%	7.02140%	-
17430	1.81072%	3.58866%	5.33441%	7.04855%	-
17505	1.81852%	3.60396%	5.35695%	7.07805%	-
17934	1.86308%	3.69146%	5.48577%	7.24665%	-
17986	1.86849%	3.70206%	5.50138%	7.26707%	-
18012	1.87119%	3.70736%	5.50918%	7.27728%	-
18038	1.87389%	3.71266%	5.51698%	7.28749%	-
18064	1.87659%	3.71796%	5.52478%	7.29770%	-
18090	1.87929%	3.72326%	5.53259%	7.30791%	-
18234	1.89425%	3.75262%	5.57579%	7.36442%	-
18711	1.94380%	3.84982%	5.71880%	7.55144%	-
18738	1.94661%	3.85532%	5.72689%	7.56202%	-
18792	1.95222%	3.86632%	5.74307%	7.58317%	-
18819	1.95502%	3.87182%	5.75115%	7.59375%	-
19152	1.98962%	3.93965%	5.85088%	7.72409%	-
19208	1.99543%	3.95105%	5.86765%	7.74600%	-
19236	1.99834%	3.95675%	5.87603%	7.75695%	-
19264	2.00125%	3.96245%	5.88441%	7.76790%	-
19292	2.00416%	3.96815%	5.89279%	7.77885%	-
19320	2.00707%	3.97386%	5.90117%	7.78980%	-
19348	2.00998%	3.97956%	5.90955%	7.80075%	-
19376	2.01289%	3.98526%	5.91793%	7.81170%	-
19464	2.02203%	4.00317%	5.94426%	7.84610%	-
19520	2.02785%	4.01457%	5.96101%	7.86798%	-
19548	2.03075%	4.02027%	5.96939%	7.87892%	-
19872	2.06441%	4.08621%	6.06627%	8.00546%	-
19959	2.07345%	4.10391%	6.09228%	8.03941%	-
19988	2.07646%	4.10981%	6.10094%	8.05073%	-
20017	2.07948%	4.11571%	6.10961%	8.06204%	-
20046	2.08249%	4.12161%	6.11827%	8.07336%	-
20075	2.08550%	4.12751%	6.12694%	8.08467%	-
20104	2.08851%	4.13341%	6.13560%	8.09598%	-
20190	2.09745%	4.15091%	6.16130%	8.12952%	-
20277	2.10649%	4.16860%	6.18728%	8.16344%	-
20682	2.14856%	4.25096%	6.30819%	8.32122%	-
20742	2.15479%	4.26316%	6.32609%	8.34458%	-
20772	2.15791%	4.26926%	6.33504%	8.35626%	-
20802	2.16103%	4.27536%	6.34399%	8.36793%	-
20832	2.16414%	4.28145%	6.35294%	8.37961%	-
21006	2.18222%	4.31682%	6.40484%	8.44730%	-
21467	2.23011%	4.41049%	6.54225%	8.62646%	-
21498	2.23333%	4.41679%	6.55148%	8.63850%	-
21560	2.23977%	4.42938%	6.56995%	8.66258%	-
21888	2.27385%	4.49599%	6.66761%	8.78985%	-
21952	2.28050%	4.50899%	6.68666%	8.81467%	-
21984	2.28382%	4.51548%	6.69618%	8.82708%	-
22016	2.28714%	4.52198%	6.70570%	8.83948%	-
22048	2.29047%	4.52848%	6.71523%	8.85189%	-
22080	2.29379%	4.53497%	6.72475%	8.86429%	-
22112	2.29712%	4.54147%	6.73427%	8.87670%	-
22224	2.30875%	4.56420%	6.76758%	8.92010%	-

22288	2.31540%	4.57719%	6.78662%	8.94489%	-
22608	2.34864%	4.64213%	6.88175%	9.06877%	-
22707	2.35893%	4.66221%	6.91117%	9.10708%	-
22740	2.36236%	4.66891%	6.92097%	9.11984%	-
22773	2.36579%	4.67560%	6.93078%	9.13260%	-
22806	2.36921%	4.68230%	6.94058%	9.14536%	-
22839	2.37264%	4.68899%	6.95038%	9.15813%	-
22950	2.38417%	4.71151%	6.98335%	9.20104%	-
23430	2.43404%	4.80883%	7.12583%	9.38643%	-
23498	2.44110%	4.82262%	7.14600%	9.41267%	-
23532	2.44463%	4.82951%	7.15608%	9.42578%	-
23566	2.44817%	4.83640%	7.16617%	9.43890%	-
24223	2.51642%	4.96952%	7.36089%	9.69208%	-
24258	2.52006%	4.97661%	7.37125%	9.70556%	-
24624	2.55808%	5.05072%	7.47960%	9.84635%	-
24696	2.56556%	5.06530%	7.50090%	9.87403%	-
24732	2.56930%	5.07258%	7.51156%	9.88787%	-
24768	2.57304%	5.07987%	7.52221%	9.90170%	-
24804	2.57678%	5.08716%	7.53286%	9.91553%	-
24840	2.58052%	5.09444%	7.54350%	9.92937%	-
25344	2.63287%	5.19643%	7.69250%	10.12284%	-
25455	2.64441%	5.21889%	7.72529%	10.16541%	-
25492	2.64825%	5.22637%	7.73622%	10.17960%	-
25529	2.65209%	5.23385%	7.74715%	10.19379%	-
26178	2.71952%	5.36508%	7.93869%	10.44232%	-
27360	2.84231%	5.60383%	8.28687%	10.89364%	-
27440	2.85062%	5.61998%	8.31040%	10.92413%	-
27480	2.85477%	5.62805%	8.32217%	10.93937%	-
28080	2.91711%	5.74912%	8.49852%	11.16773%	-
c=10					
0	0.00000%	0.00000%	0.00000%	0.00000%	-
684	0.08041%	0.16075%	0.24103%	0.32124%	-
686	0.08064%	0.16122%	0.24173%	0.32218%	-
687	0.08076%	0.16145%	0.24208%	0.32265%	-
688	0.08088%	0.16169%	0.24244%	0.32312%	-
689	0.08100%	0.16192%	0.24279%	0.32359%	-
690	0.08111%	0.16216%	0.24314%	0.32406%	-
691	0.08123%	0.16239%	0.24349%	0.32453%	-
692	0.08135%	0.16263%	0.24385%	0.32500%	-
693	0.08147%	0.16286%	0.24420%	0.32546%	-
1368	0.16081%	0.32137%	0.48167%	0.64171%	-
1374	0.16152%	0.32278%	0.48378%	0.64452%	-
1376	0.16176%	0.32325%	0.48448%	0.64545%	-
1378	0.16199%	0.32372%	0.48518%	0.64639%	-
1380	0.16223%	0.32419%	0.48589%	0.64733%	-
1382	0.16246%	0.32466%	0.48659%	0.64826%	-
1384	0.16270%	0.32513%	0.48729%	0.64920%	-
1386	0.16293%	0.32560%	0.48800%	0.65013%	-
2061	0.24228%	0.48397%	0.72508%	0.96561%	-
2067	0.24299%	0.48538%	0.72719%	0.96841%	-
2070	0.24334%	0.48608%	0.72824%	0.96981%	-
2073	0.24369%	0.48679%	0.72929%	0.97121%	-
2076	0.24404%	0.48749%	0.73035%	0.97261%	-
2079	0.24440%	0.48820%	0.73140%	0.97401%	-
2592	0.30470%	0.60848%	0.91132%	1.21325%	-
2596	0.30517%	0.60941%	0.91273%	1.21511%	-
2604	0.30611%	0.61129%	0.91553%	1.21884%	-
2608	0.30658%	0.61223%	0.91693%	1.22070%	-
2612	0.30705%	0.61316%	0.91833%	1.22257%	-
2616	0.30752%	0.61410%	0.91974%	1.22443%	-
2620	0.30799%	0.61504%	0.92114%	1.22630%	-
2624	0.30846%	0.61598%	0.92254%	1.22816%	-

2628	0.30893%	0.61691%	0.92394%	1.23002%	-
2632	0.30940%	0.61785%	0.92534%	1.23189%	-
2756	0.32398%	0.64691%	0.96880%	1.28964%	-
2760	0.32445%	0.64785%	0.97020%	1.29150%	-
2768	0.32539%	0.64972%	0.97300%	1.29523%	-
2772	0.32586%	0.65066%	0.97440%	1.29709%	-
3280	0.38558%	0.76967%	1.15228%	1.53342%	-
3290	0.38675%	0.77201%	1.15578%	1.53807%	-
3295	0.38734%	0.77319%	1.15753%	1.54039%	-
3300	0.38793%	0.77436%	1.15928%	1.54272%	-
3305	0.38852%	0.77553%	1.16103%	1.54504%	-
3310	0.38911%	0.77670%	1.16278%	1.54737%	-
3315	0.38969%	0.77787%	1.16453%	1.54969%	-
3320	0.39028%	0.77904%	1.16628%	1.55201%	-
3325	0.39087%	0.78021%	1.16803%	1.55434%	-
3450	0.40556%	0.80948%	1.21176%	1.61242%	-
3460	0.40674%	0.81182%	1.21526%	1.61706%	-
3465	0.40733%	0.81300%	1.21701%	1.61938%	-
3964	0.46599%	0.92980%	1.39146%	1.85096%	-
3982	0.46810%	0.93401%	1.39775%	1.85931%	-
3988	0.46881%	0.93542%	1.39984%	1.86209%	-
3994	0.46951%	0.93682%	1.40194%	1.86487%	-
4000	0.47022%	0.93823%	1.40403%	1.86765%	-
4006	0.47092%	0.93963%	1.40613%	1.87043%	-
4012	0.47163%	0.94103%	1.40823%	1.87322%	-
4018	0.47233%	0.94244%	1.41032%	1.87600%	-
4140	0.48668%	0.97098%	1.45294%	1.93254%	-
4158	0.48879%	0.97520%	1.45922%	1.94088%	-
4669	0.54886%	1.09471%	1.63757%	2.17745%	-
4683	0.55051%	1.09799%	1.64245%	2.18392%	-
4690	0.55133%	1.09962%	1.64489%	2.18716%	-
4697	0.55215%	1.10126%	1.64734%	2.19040%	-
4704	0.55298%	1.10290%	1.64978%	2.19363%	-
4711	0.55380%	1.10453%	1.65222%	2.19687%	-
4851	0.57026%	1.13726%	1.70104%	2.26160%	-
5184	0.60940%	1.21509%	1.81709%	2.41543%	-
5192	0.61034%	1.21696%	1.81988%	2.41912%	-
5208	0.61222%	1.22070%	1.82545%	2.42651%	-
5216	0.61317%	1.22257%	1.82824%	2.43020%	-
5224	0.61411%	1.22444%	1.83103%	2.43389%	-
5232	0.61505%	1.22631%	1.83382%	2.43758%	-
5240	0.61599%	1.22818%	1.83660%	2.44128%	-
5248	0.61693%	1.23005%	1.83939%	2.44497%	-
5256	0.61787%	1.23192%	1.84218%	2.44866%	-
5264	0.61881%	1.23379%	1.84496%	2.45235%	-
5372	0.63150%	1.25902%	1.88257%	2.50219%	-
5380	0.63244%	1.26089%	1.88536%	2.50588%	-
5396	0.63433%	1.26463%	1.89093%	2.51326%	-
5404	0.63527%	1.26650%	1.89372%	2.51695%	-
5876	0.69075%	1.37673%	2.05798%	2.73451%	-
5894	0.69287%	1.38093%	2.06424%	2.74280%	-
5903	0.69393%	1.38304%	2.06737%	2.74695%	-
5912	0.69498%	1.38514%	2.07050%	2.75109%	-
5921	0.69604%	1.38724%	2.07363%	2.75524%	-
5930	0.69710%	1.38934%	2.07676%	2.75938%	-
5939	0.69816%	1.39144%	2.07989%	2.76352%	-
5948	0.69922%	1.39354%	2.08302%	2.76767%	-
5957	0.70027%	1.39564%	2.08614%	2.77181%	-
6070	0.71356%	1.42202%	2.12543%	2.82383%	-
6088	0.71567%	1.42622%	2.13169%	2.83211%	-
6097	0.71673%	1.42833%	2.13482%	2.83625%	-
6560	0.77116%	1.53637%	2.29568%	3.04914%	-
6590	0.77469%	1.54337%	2.30610%	3.06292%	-

6600	0.77586%	1.54570%	2.30957%	3.06752%	-
6610	0.77704%	1.54804%	2.31305%	3.07211%	-
6620	0.77821%	1.55037%	2.31652%	3.07670%	-
6630	0.77939%	1.55270%	2.31999%	3.08130%	-
6640	0.78056%	1.55503%	2.32346%	3.08589%	-
6650	0.78174%	1.55737%	2.32693%	3.09048%	-
6760	0.79467%	1.58303%	2.36512%	3.14099%	-
6790	0.79820%	1.59002%	2.37553%	3.15477%	-
7277	0.85545%	1.70357%	2.54445%	3.37813%	-
7299	0.85803%	1.70870%	2.55207%	3.38821%	-
7310	0.85932%	1.71127%	2.55589%	3.39325%	-
7321	0.86062%	1.71383%	2.55970%	3.39829%	-
7332	0.86191%	1.71639%	2.56351%	3.40333%	-
7343	0.86320%	1.71896%	2.56733%	3.40837%	-
7483	0.87966%	1.75159%	2.61584%	3.47250%	-
7776	0.91411%	1.81986%	2.71733%	3.60660%	-
7788	0.91552%	1.82265%	2.72148%	3.61209%	-
7812	0.91834%	1.82824%	2.72979%	3.62306%	-
7824	0.91975%	1.83104%	2.73395%	3.62855%	-
7836	0.92116%	1.83383%	2.73810%	3.63404%	-
7848	0.92257%	1.83663%	2.74225%	3.63953%	-
7860	0.92398%	1.83942%	2.74641%	3.64502%	-
7872	0.92539%	1.84222%	2.75056%	3.65050%	-
7884	0.92680%	1.84501%	2.75472%	3.65599%	-
7896	0.92821%	1.84781%	2.75887%	3.66148%	-
7988	0.93903%	1.86924%	2.79071%	3.70354%	-
8000	0.94044%	1.87203%	2.79487%	3.70902%	-
8024	0.94326%	1.87762%	2.80317%	3.71999%	-
8036	0.94467%	1.88042%	2.80732%	3.72548%	-
8472	0.99592%	1.98193%	2.95812%	3.92458%	-
8498	0.99898%	1.98798%	2.96710%	3.93645%	-
8511	1.00051%	1.99101%	2.97160%	3.94238%	-
8524	1.00204%	1.99403%	2.97609%	3.94831%	-
8537	1.00356%	1.99706%	2.98058%	3.95424%	-
8550	1.00509%	2.00008%	2.98508%	3.96017%	-
8563	1.00662%	2.00311%	2.98957%	3.96610%	-
8576	1.00815%	2.00614%	2.99406%	3.97203%	-
8589	1.00968%	2.00916%	2.99855%	3.97796%	-
8690	1.02155%	2.03267%	3.03345%	4.02402%	-
8716	1.02461%	2.03872%	3.04244%	4.03587%	-
8729	1.02613%	2.04174%	3.04693%	4.04180%	-
9156	1.07633%	2.14108%	3.19437%	4.23632%	-
9198	1.08127%	2.15085%	3.20886%	4.25543%	-
9212	1.08291%	2.15410%	3.21369%	4.26181%	-
9226	1.08456%	2.15736%	3.21852%	4.26818%	-
9240	1.08621%	2.16061%	3.22335%	4.27455%	-
9254	1.08785%	2.16387%	3.22818%	4.28092%	-
9268	1.08950%	2.16712%	3.23301%	4.28729%	-
9282	1.09114%	2.17038%	3.23784%	4.29366%	-
9380	1.10266%	2.19317%	3.27165%	4.33824%	-
9422	1.10760%	2.20293%	3.28614%	4.35734%	-
9885	1.16203%	2.31055%	3.44574%	4.56773%	-
9915	1.16555%	2.31753%	3.45607%	4.58135%	-
9930	1.16732%	2.32101%	3.46124%	4.58816%	-
9945	1.16908%	2.32450%	3.46641%	4.59497%	-
9960	1.17084%	2.32798%	3.47157%	4.60177%	-
9975	1.17261%	2.33147%	3.47674%	4.60858%	-
10115	1.18907%	2.36399%	3.52495%	4.67211%	-
10368	1.21881%	2.42276%	3.61204%	4.78683%	-
10384	1.22069%	2.42648%	3.61755%	4.79408%	-
10416	1.22445%	2.43391%	3.62856%	4.80858%	-
10432	1.22633%	2.43762%	3.63406%	4.81583%	-
10448	1.22821%	2.44134%	3.63957%	4.82308%	-

10464	1.23009%	2.44505%	3.64507%	4.83033%	-
10480	1.23197%	2.44877%	3.65058%	4.83758%	-
10496	1.23385%	2.45249%	3.65608%	4.84483%	-
10512	1.23573%	2.45620%	3.66159%	4.85208%	-
10528	1.23762%	2.45992%	3.66709%	4.85933%	-
10604	1.24655%	2.47756%	3.69323%	4.89375%	-
10620	1.24843%	2.48128%	3.69873%	4.90099%	-
10652	1.25219%	2.48871%	3.70974%	4.91548%	-
10668	1.25407%	2.49242%	3.71524%	4.92273%	-
11068	1.30110%	2.58526%	3.85272%	5.10370%	-
11102	1.30509%	2.59315%	3.86440%	5.11907%	-
11119	1.30709%	2.59710%	3.87024%	5.12675%	-
11136	1.30909%	2.60104%	3.87608%	5.13444%	-
11153	1.31109%	2.60499%	3.88192%	5.14212%	-
11170	1.31309%	2.60893%	3.88776%	5.14980%	-
11187	1.31508%	2.61288%	3.89360%	5.15749%	-
11204	1.31708%	2.61682%	3.89944%	5.16517%	-
11221	1.31908%	2.62076%	3.90528%	5.17285%	-
11310	1.32954%	2.64141%	3.93584%	5.21306%	-
11344	1.33354%	2.64930%	3.94751%	5.22842%	-
11361	1.33554%	2.65324%	3.95335%	5.23609%	-
11752	1.38150%	2.74392%	4.08752%	5.41256%	-
11806	1.38785%	2.75644%	4.10604%	5.43691%	-
11824	1.38997%	2.76061%	4.11221%	5.44503%	-
11842	1.39208%	2.76479%	4.11839%	5.45314%	-
11860	1.39420%	2.76896%	4.12456%	5.46126%	-
11878	1.39631%	2.77313%	4.13073%	5.46937%	-
11896	1.39843%	2.77731%	4.13690%	5.47748%	-
11914	1.40055%	2.78148%	4.14307%	5.48560%	-
12000	1.41066%	2.80141%	4.17256%	5.52436%	-
12054	1.41700%	2.81393%	4.19106%	5.54869%	-
12493	1.46861%	2.91565%	4.34145%	5.74631%	-
12531	1.47308%	2.92446%	4.35446%	5.76340%	-
12550	1.47531%	2.92886%	4.36096%	5.77194%	-
12569	1.47754%	2.93326%	4.36747%	5.78049%	-
12588	1.47978%	2.93766%	4.37397%	5.78903%	-
12607	1.48201%	2.94206%	4.38048%	5.79757%	-
12747	1.49847%	2.97449%	4.42839%	5.86050%	-
12960	1.52351%	3.02381%	4.50125%	5.95619%	-
12980	1.52586%	3.02844%	4.50809%	5.96517%	-
13020	1.53056%	3.03770%	4.52177%	5.98313%	-
13040	1.53291%	3.04233%	4.52861%	5.99211%	-
13060	1.53526%	3.04696%	4.53545%	6.00109%	-
13080	1.53762%	3.05159%	4.54229%	6.01006%	-
13100	1.53997%	3.05622%	4.54912%	6.01904%	-
13120	1.54232%	3.06085%	4.55596%	6.02802%	-
13140	1.54467%	3.06548%	4.56280%	6.03699%	-
13160	1.54702%	3.07011%	4.56964%	6.04597%	-
13220	1.55407%	3.08400%	4.59014%	6.07289%	-
13240	1.55642%	3.08863%	4.59698%	6.08186%	-
13280	1.56113%	3.09788%	4.61065%	6.09980%	-
13300	1.56348%	3.10251%	4.61749%	6.10877%	-
13664	1.60627%	3.18674%	4.74182%	6.27192%	-
13706	1.61120%	3.19645%	4.75616%	6.29074%	-
13727	1.61367%	3.20131%	4.76333%	6.30014%	-
13748	1.61614%	3.20617%	4.77050%	6.30954%	-
13769	1.61861%	3.21102%	4.77766%	6.31895%	-
13790	1.62108%	3.21588%	4.78483%	6.32835%	-
13811	1.62355%	3.22074%	4.79200%	6.33775%	-
13832	1.62602%	3.22560%	4.79917%	6.34715%	-
13853	1.62848%	3.23045%	4.80633%	6.35655%	-
13930	1.63754%	3.24826%	4.83261%	6.39102%	-
13972	1.64247%	3.25797%	4.84694%	6.40981%	-

13993	1.64494%	3.26283%	4.85410%	6.41920%	-
14348	1.68667%	3.34490%	4.97516%	6.57793%	-
14414	1.69443%	3.36016%	4.99766%	6.60742%	-
14436	1.69702%	3.36524%	5.00516%	6.61724%	-
14458	1.69961%	3.37033%	5.01265%	6.62707%	-
14480	1.70219%	3.37541%	5.02015%	6.63690%	-
14502	1.70478%	3.38050%	5.02765%	6.64672%	-
14524	1.70736%	3.38558%	5.03514%	6.65654%	-
14546	1.70995%	3.39066%	5.04264%	6.66637%	-
14620	1.71865%	3.40776%	5.06785%	6.69941%	-
14686	1.72641%	3.42301%	5.09033%	6.72886%	-
15101	1.77519%	3.51888%	5.23161%	6.91393%	-
15147	1.78060%	3.52950%	5.24726%	6.93443%	-
15170	1.78330%	3.53481%	5.25508%	6.94468%	-
15193	1.78601%	3.54012%	5.26291%	6.95492%	-
15216	1.78871%	3.54543%	5.27073%	6.96517%	-
15239	1.79142%	3.55074%	5.27855%	6.97541%	-
15379	1.80787%	3.58306%	5.32616%	7.03775%	-
15552	1.82821%	3.62300%	5.38498%	7.11475%	-
15576	1.83103%	3.62854%	5.39313%	7.12542%	-
15624	1.83667%	3.63962%	5.40945%	7.14677%	-
15648	1.83950%	3.64516%	5.41760%	7.15745%	-
15672	1.84232%	3.65069%	5.42576%	7.16812%	-
15696	1.84514%	3.65623%	5.43391%	7.17879%	-
15720	1.84796%	3.66177%	5.44207%	7.18947%	-
15744	1.85078%	3.66731%	5.45022%	7.20014%	-
15768	1.85360%	3.67285%	5.45837%	7.21081%	-
15836	1.86160%	3.68854%	5.48147%	7.24103%	-
15860	1.86442%	3.69408%	5.48962%	7.25170%	-
15908	1.87006%	3.70515%	5.50593%	7.27303%	-
15932	1.87288%	3.71069%	5.51408%	7.28369%	-
16260	1.91144%	3.78634%	5.62541%	7.42933%	-
16310	1.91732%	3.79787%	5.64238%	7.45152%	-
16335	1.92026%	3.80364%	5.65086%	7.46261%	-
16360	1.92319%	3.80940%	5.65934%	7.47370%	-
16385	1.92613%	3.81517%	5.66782%	7.48479%	-
16410	1.92907%	3.82093%	5.67630%	7.49588%	-
16435	1.93201%	3.82670%	5.68478%	7.50697%	-
16460	1.93495%	3.83246%	5.69326%	7.51805%	-
16550	1.94553%	3.85321%	5.72378%	7.55796%	-
16600	1.95141%	3.86474%	5.74073%	7.58012%	-
16625	1.95435%	3.87050%	5.74921%	7.59120%	-
16944	1.99185%	3.94402%	5.85731%	7.73250%	-
17022	2.00102%	3.96199%	5.88373%	7.76702%	-
17048	2.00407%	3.96798%	5.89254%	7.77853%	-
17074	2.00713%	3.97397%	5.90134%	7.79003%	-
17100	2.01018%	3.97996%	5.91015%	7.80154%	-
17126	2.01324%	3.98595%	5.91895%	7.81304%	-
17152	2.01630%	3.99194%	5.92776%	7.82454%	-
17240	2.02664%	4.01221%	5.95755%	7.86346%	-
17318	2.03581%	4.03018%	5.98395%	7.89795%	-
17709	2.08178%	4.12022%	6.11622%	8.07068%	-
17763	2.08812%	4.13265%	6.13448%	8.09452%	-
17790	2.09130%	4.13886%	6.14361%	8.10643%	-
17817	2.09447%	4.14508%	6.15274%	8.11835%	-
17844	2.09765%	4.15129%	6.16186%	8.13026%	-
18011	2.11728%	4.18973%	6.21830%	8.20393%	-
18144	2.13291%	4.22033%	6.26323%	8.26256%	-
18172	2.13620%	4.22678%	6.27269%	8.27490%	-
18228	2.14279%	4.23966%	6.29160%	8.29958%	-
18256	2.14608%	4.24610%	6.30106%	8.31192%	-
18284	2.14937%	4.25254%	6.31052%	8.32426%	-
18312	2.15266%	4.25899%	6.31997%	8.33659%	-

18340	2.15595%	4.26543%	6.32942%	8.34892%	-
18368	2.15924%	4.27187%	6.33888%	8.36126%	-
18452	2.16912%	4.29119%	6.36723%	8.39825%	-
18480	2.17241%	4.29763%	6.37668%	8.41057%	-
18536	2.17899%	4.31051%	6.39558%	8.43522%	-
18856	2.21661%	4.38409%	6.50353%	8.57599%	-
18914	2.22343%	4.39742%	6.52308%	8.60149%	-
18943	2.22684%	4.40409%	6.53286%	8.61423%	-
18972	2.23025%	4.41076%	6.54264%	8.62698%	-
19001	2.23366%	4.41742%	6.55241%	8.63972%	-
19030	2.23707%	4.42409%	6.56219%	8.65246%	-
19059	2.24047%	4.43075%	6.57196%	8.66520%	-
19170	2.25352%	4.45627%	6.60937%	8.71396%	-
19228	2.26034%	4.46959%	6.62891%	8.73942%	-
19540	2.29702%	4.54128%	6.73399%	8.87633%	-
19630	2.30760%	4.56195%	6.76428%	8.91579%	-
19660	2.31112%	4.56884%	6.77438%	8.92895%	-
19690	2.31465%	4.57573%	6.78447%	8.94210%	-
19720	2.31818%	4.58262%	6.79457%	8.95525%	-
19750	2.32170%	4.58951%	6.80466%	8.96839%	-
19860	2.33464%	4.61477%	6.84167%	9.01659%	-
20317	2.38836%	4.71968%	6.99532%	9.21661%	-
20379	2.39565%	4.73390%	7.01615%	9.24372%	-
20410	2.39929%	4.74102%	7.02656%	9.25728%	-
20441	2.40294%	4.74813%	7.03698%	9.27083%	-
20736	2.43761%	4.81581%	7.13604%	9.39971%	-
20768	2.44138%	4.82315%	7.14678%	9.41368%	-
20832	2.44890%	4.83783%	7.16826%	9.44162%	-
20864	2.45266%	4.84517%	7.17900%	9.45559%	-
20896	2.45642%	4.85251%	7.18974%	9.46956%	-
20928	2.46018%	4.85985%	7.20047%	9.48352%	-
20960	2.46395%	4.86718%	7.21121%	9.49748%	-
21068	2.47664%	4.89195%	7.24744%	9.54460%	-
21100	2.48040%	4.89929%	7.25817%	9.55855%	-
21452	2.52178%	4.97997%	7.37618%	9.71196%	-
21518	2.52954%	4.99510%	7.39829%	9.74070%	-
21551	2.53342%	5.00266%	7.40935%	9.75507%	-
21584	2.53730%	5.01022%	7.42041%	9.76944%	-
21617	2.54118%	5.01779%	7.43146%	9.78380%	-
21650	2.54506%	5.02535%	7.44251%	9.79816%	-
22136	2.60219%	5.13667%	7.60520%	10.00950%	-
22238	2.61418%	5.16003%	7.63932%	10.05380%	-
22272	2.61818%	5.16781%	7.65069%	10.06857%	-
22306	2.62217%	5.17559%	7.66206%	10.08333%	-
22925	2.69494%	5.31726%	7.86891%	10.35180%	-
23328	2.74232%	5.40943%	8.00341%	10.52625%	-
23364	2.74655%	5.41766%	8.01542%	10.54183%	-
23436	2.75501%	5.43412%	8.03943%	10.57296%	-
23472	2.75924%	5.44236%	8.05144%	10.58853%	-
23508	2.76348%	5.45059%	8.06344%	10.60409%	-
24048	2.82695%	5.57400%	8.24338%	10.83731%	-
24122	2.83565%	5.59090%	8.26802%	10.86923%	-
24159	2.84000%	5.59935%	8.28034%	10.88519%	-
24732	2.90736%	5.73020%	8.47097%	11.13206%	-
25920	3.04702%	6.00119%	8.86536%	11.64226%	-
25960	3.05172%	6.01031%	8.87862%	11.65940%	-

To use this table in a specific situation, you must calculate the value of *F* by its formula.

Example:

Assume you are playing against n = 4 opponents. You are initially dealt (88573), you discard (573) and replace with (85J). You hold three of a kind: (8885J). You want to find the probability of at least one of your opponents holding a higher three of a kind formation.

We have here c = 8, m = 6 and the distribution (001000)(0102013). First we calculate:

$$S = \frac{8}{2}\cdot(3-0)(4-0) + \frac{3}{2}\cdot(3-1)(4-1) + \frac{1}{2}\cdot(3-2)(4-2) + 0 = 58$$

Then, the formula of *F* gives us:

$$F = \frac{5}{12}\cdot 2\cdot 3\cdot 4\cdot\left[(47-8+0)(48-8+0) - 2\cdot 58 + 3\cdot 4\right] +$$

$$+\frac{1}{12}\cdot 1\cdot 2\cdot 3\cdot\left[(47-8+1)(48-8+1) - 2\cdot 58 + 2\cdot 3\right] = 15325\,.$$

By looking in the table with probabilities, in the area c = 8, at the intersection of row *F = 15325* with column *n = 4*, you find 5.52618% as the probability of at least one of your opponents holding a higher three of a kind formation.

.........................missing part..........................

Pair against higher pair

As with the previous sections, this section deals with *one pair* as the final formation containing *exactly one pair* (no three of a kind, no two pair, etc.).

Because pairs are ranked by the value of the paired card, then by the value of the high card, the variables are represented by multiple vectors of the type

$(c_0)(c_1, c_2, , ..., c_m)(c_{m+1}, ..., c_p)(d_0, ..., d_{12})$, where:

c_0 is the number of cards having the value of the paired card from the final formation (one pair);

$(c_1, c_2, , ..., c_m)$ is the cumulated distribution of the values that are higher than the value of the held pair;

$(c_{m+1}, ..., c_p)$ is the cumulated distribution of the values that are higher than the value of the high card and different from the value of the held pair;

$(d_0, ..., d_{12})$ is the cumulated distribution of all the values from 2 to *A*.

The parameters *m* and *p*, which represent the sizes of these vectors, are still variables for the formula of *F*.

The initial conditions of the variables are:

$c_0 \in \{2, 3, 4\}$; $c_i \in \{0, 1, 2, 3, 4\}$ for any $i = 0, ..., 13$;

$m \in \{0, 1, ..., 12\}$ (this is the size of the second vector; for $m = 0$, no vector exists);

$p - m \in \{0, 1, ..., 9\}$ (this is the size of the third vector; for $p = m$, no vector exists; the maximum value is 9 because the high card may take only values from 5 upward; these number 10, from which we must subtract one value – the value of the held pair); $\sum_{i=0}^{12} d_i = c$.

From all possible distributions (which number in the millions), we only keep those that are specific for a held pair.

These are the distributions from which we can extract a 2-1-1-1 partial distribution from the last vector $(d_0, ..., d_{12})$.

For example, (2211000000000) fits this condition, and (3221000000000) also fits, while (3210000000000) does not.

According to the rule of ranking the one pair formations, we found the following formula for the number of favorable combinations for one opponent to hold a higher pair, in case you hold one:

If $m \neq 0$ and $p \neq m$ (i.e., vectors 2 and 3 do exist), then:

........................missing part..........................

The next table notes the possible values of *F* for all possible distributions that are specific to a held one pair, along with the corresponding probabilities of your opponents holding a higher pair.

Probabilities of opponents holding a higher pair – partial table (you can download the complete table from http://probability.infarom.ro/ophigherpair.zip)

F	n = 1	n = 2	n = 3	n = 4	n = 5
c=5					
0	0.00000%	0.00000%	0.00000%	0.00000%	0.00000%
592	0.03859%	0.07717%	0.11574%	0.15428%	0.19282%
1648	0.10744%	0.21476%	0.32196%	0.42905%	0.53603%
3040	0.19818%	0.39597%	0.59337%	0.79038%	0.98699%
4640	0.30249%	0.60406%	0.90473%	1.20448%	1.50333%
6320	0.41201%	0.82233%	1.23095%	1.63789%	2.04315%
7952	0.51840%	1.03412%	1.54716%	2.05755%	2.56529%
9408	0.61332%	1.22288%	1.82871%	2.43082%	3.02923%
10560	0.68842%	1.37211%	2.05109%	2.72539%	3.39506%
32271	2.10380%	4.16334%	6.17955%	8.15335%	10.08563%
60174	3.92284%	7.69180%	11.31291%	14.79197%	18.13455%
60766	3.96144%	7.76594%	11.41974%	14.92880%	18.29885%
61822	4.03028%	7.89813%	11.61009%	15.17246%	18.59125%
63214	4.12102%	8.07222%	11.86059%	15.49285%	18.97542%
64542	4.20760%	8.23816%	12.09914%	15.79766%	19.34057%
64814	4.22533%	8.27213%	12.14794%	15.85999%	19.41519%
66494	4.33485%	8.48180%	12.44898%	16.24420%	19.87490%
68126	4.44125%	8.68525%	12.74077%	16.61617%	20.31946%
69582	4.53616%	8.86656%	13.00053%	16.94698%	20.71441%
70734	4.61127%	9.00990%	13.20570%	17.20802%	21.02579%
92445	6.02664%	11.69008%	17.01221%	22.01360%	26.71357%
96813	6.31140%	12.22446%	17.76433%	22.95456%	27.81722%
120940	7.88428%	15.14694%	21.83700%	27.99960%	33.67632%
121996	7.95312%	15.27372%	22.01211%	28.21460%	33.92379%
123388	8.04387%	15.44070%	22.24255%	28.49726%	34.24886%
124716	8.13044%	15.59985%	22.46196%	28.76616%	34.55780%
124988	8.14817%	15.63242%	22.50685%	28.82114%	34.62093%
126668	8.25769%	15.83350%	22.78372%	29.16002%	35.00978%
128300	8.36409%	16.02860%	23.05205%	29.48806%	35.38575%
129756	8.45901%	16.20247%	23.29092%	29.77976%	35.71971%
130908	8.53411%	16.33991%	23.47956%	30.00991%	35.98296%
152619	9.94948%	18.90905%	26.97719%	34.24260%	40.78514%
153211	9.98808%	18.97854%	27.07104%	34.35525%	40.91192%
156987	10.23424%	19.42109%	27.66774%	35.07041%	41.71548%
182170	11.87596%	22.34154%	31.56424%	39.69166%	46.85388%
183562	11.96671%	22.50140%	31.77544%	39.93969%	47.12695%
184890	12.05328%	22.65376%	31.97653%	40.17561%	47.38643%
185162	12.07101%	22.68494%	32.01767%	40.22384%	47.43945%
185482	12.09188%	22.72162%	32.06604%	40.28055%	47.50177%
186842	12.18054%	22.87743%	32.27138%	40.52111%	47.76597%
188474	12.28693%	23.06418%	32.51724%	40.80882%	48.08161%
189930	12.38185%	23.23060%	32.73608%	41.06462%	48.36193%
191082	12.45695%	23.36215%	32.90890%	41.26642%	48.58285%
212793	13.87232%	25.82024%	36.11071%	44.97366%	52.60711%
213385	13.91092%	25.88671%	36.19656%	45.07222%	52.71320%

........................missing part..........................

In this section, we covered all the events related to your opponents holding the various formations, including those higher than your held formation.

In the probability field in which we worked out the calculations (given by the information provided by the seen cards), the probability of one or more of your opponents holding a formation of a certain type is the same at the moment right after the first card distribution as well as after your opponents make their replacements.

We can refer to these probabilities as predictions for opponents' final hands.

The most valuable information for your own play is the chances for them to hold a higher formation than yours, of whatever type. We can use the previous results to reach that information.

The next chapter deals with the calculations that give us the overall probability of at least one opponent holding a higher formation than yours.

The Overall Probability of Someone Holding a Higher Formation Than Yours

One opponent

We define one opponent as a specific one. The probabilities of the events regarding the cards of one opponent are the same for any of them, irrespective of their number.

We have all the data (formulas and tables of values) regarding the probabilities of that opponent holding each type of formation – one pair, two pair, three of a kind, straight, flush, full house, four of a kind and straight flush – with the following meaning:

- *one pair* formation means a pair of cards of the same value, but no two pairs, three of a kind, or four of a kind;
- *two pair* formation means two different pairs, but no full house;
- *three of a kind* formation means a triple of cards of the same value, but not four of a kind;
- *straight* formation means five consecutive cards, even suited (including a straight flush);
- *flush* formation means five suited cards, even consecutive (including a straight flush);
- *full house* formation means a triple of cards of the same value and a pair of cards of another value;
- *four of a kind* formation means a quadruple of cards of the same value and
- *straight flush* formation means five suited and consecutive cards.

In this terminology, if we symbolically denote the event "your opponent holds one pair" by *one pair* and the like, we can state that any two events from *one pair*, *two pair, three of a kind*, *straight*, *flush*, *full house*, *four of a kind* and *straight flush* are mutually incompatible (mutually exclusive); that is, their intersection is empty or, in other words, they cannot occur simultaneously.

Now assume you hold a certain type of formation at a moment in the game. Let us say you hold three of a kind.

The event "your opponent holds a higher three of a kind formation" (denoted by *higher three of a kind*) is included in the event *three of a kind*.

By using the Boolean property $A \cap B = \phi, C \subset A \Rightarrow C \cap B = \phi$, we find that the events *higher three of a kind*, *straight*, *flush*, *full house*, *four of a kind* and *straight flush* are still incompatible.

This allows us to sum their probabilities in order to find the probability of the event *E* – your opponent holds a higher formation than yours:

P(E) = P(*higher three of a kind*) + P(*straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*) + P(*straight flush*)

This happens for any event of the type "higher", in the terminology described earlier, which is the term commonly used among poker players.

But our calculations used a slightly different term: when we worked out the formulas, the meaning of *straight* was *regular straight or straight flush* and the meaning of *flush* was *regular flush or straight flush* for mathematical development reasons.

In this terminology, we observe that the event *straight flush* is included in both *straight* and *flush,* so in fact it appears three times in the formula P(E) above.

With this new terminology, the formula becomes:

P(E) = P(*higher three of a kind*) + P(*straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*) – P(*straight flush*)

This is the formula to compute when using the previous probability results.

We now have the following formulas to determine the probability of one specific opponent holding a higher formation than yours for each of the various situations:

If you hold *one pair*:

P(E) = P(*higher one pair*) + P(*two pair*) + P(*three of a kind*) + P(*straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*) – P(*straight flush*).

If you hold *two pair*:

P(E) = P(*higher two pair*) + P(*three of a kind*) + P(*straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*) – P(*straight flush*).

If you hold *three of a kind*:

P(E) = P(*higher three of a kind*) + P(*straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*) – P(*straight flush*).

If you hold a *straight*:

P(E) = P(*higher straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*).

If you hold a *flush*:

P(E) = P(*higher flush*) + P(*full house*) + P(*four of a kind*) + P(*straight flush*).

If you hold a *full house*:

P(E) = P(*higher full house*) + P(*four of a kind*) + P(*straight flush*).

If you hold *four of a kind*:

P(E) = P(*higher four of a kind*) + P(*straight flush*).

Of course, for the case in which you hold no valuable formation (not even one pair), the formula is:

P(E) = P(*one pair*) + P(*two pair*) + P(*three of a kind*) + P(*straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*) – P(*straight flush*).

In the case where you hold a *straight flush*, the probability P(E) is given directly by the results from the section titled *Odds of holding a higher straight flush*:

P(E) = P(*higher straight flush*).

Some of these formulas contain the approximation discussed in the note from the end of the section titled *Odds of holding a higher straight flush.*

Now, for reaching the numerical result of P(E) for a given situation, we must simply replace the values of the probabilities from the right-hand member of these formulas and take the values directly from the odds tables of the previous sections.

For a better view and to aid memorization, you may want to use the following table, which describes the way to build these formulas.

In the cells, "+" means that respective value is added, "–" means that the respective value is subtracted and a blank cell means that the respective value is skipped.

What you hold	P(E) – probability of one opponent holding a higher formation															
	P(one pair)	P(two pair)	P(three of a kind)	P(straight)	P(flush)	P(full house)	P(four of a kind)	P(straight flush)	P(higher one pair)	P(higher two pair)	P(higher three of a kind)	P(higher straight)	P(higher flush)	P(higher full house)	P(higher four of a kind)	P(higher straight flush)
nothing	+	+	+	+	+	+	+	–								
one pair		+	+	+	+	+	+	–	+							
two pair			+	+	+	+	+	–		+						
three of a kind				+	+	+	+	–			+					
straight					+	+	+					+				
flush						+	+	+					+			
full house							+	+						+		
four of a kind								+							+	
straight flush																+

Now we can find the probability of a specific opponent holding a higher formation than yours at any moment in the game.

Example:
In the following example, we focus on the use of the table above and not on the detailed calculations for the values of *F*.

Assume you were dealt (2♠ 2♥ 5♦ 5♣ 7♦). Let us find the probability of one of your opponents holding a higher formation than two pair at this moment.

This probability is given by the formula:

P(E) = P(*higher two pair*) + P(*three of a kind*) + P(*straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*) – P(*straight flush*).

We check for the probability of holding *three of a kind* in the table in the section titled *Odds of holding three of a kind*. We have c = 5 and the distribution of values 2-2-1, n = 1; the table gives us the probability **2.258%**.

We check for the probability of holding a *straight* in the table in the section titled *Odds of holding a straight*. We find F = 6272, then **0.408%** is the probability.

We check for the probability of holding a *flush* in the table in the section titled *Odds of holding a flush*. We have the distribution of symbols 2-1-1-1; the table gives us **0.185%**.

The probability of holding a full house for the value distribution 2-2-1 and c = 5 is in the table in the section titled *Odds of holding a full house,* and is **0.157%**.

The probability of holding four of a kind for 2-2-1, c = 5 and n = 1 is **0.028%**, as given in the table in the section titled *Odds of holding four of a kind*.

For the probability of holding a straight flush, we calculate *F* and find it to be 24. Then the table in the section titled *Odds of holding a straight flush* gives us **0.001%** as the probability.

Finally, we have the probability for a higher two pair formation. This is found in the table in the section titled *Odds of holding a higher two pair formation,* and is **4.802%**.

We have now all partial terms from the formula of P(E) and we can compute it:

P(E) = 4.802% + 2.258% + 0.408% + 0.185% + 0.157% + 0.028% – 0.001% = **7.54%**.

This is the probability that one opponent (no matter which) holds a higher formation than yours (two pair) at this moment.

Now assume you discard the 7♦ and replace it with 2♣. You hold a full house (2♠ 2♥ 2♣ 5♦ 5♣).

At this moment, the event "one opponent holds a higher formation than yours" has a different probability because the information taken into account has changed.

We apply the formula:

P(E) = P(*higher full house*) + P(*four of a kind*) + P(*straight flush*).

We have now c = 6 and the seen cards are 2♠, 2♥, 2♣, 5♦, 5♣, 7♦.

Now let us find the partial probabilities for each type of formation:

Four of a kind: the distribution is 3-2-1 and the probability is **0.030%**;

Straight flush: we have F = 24 and the probability is **0.001%**;

Higher full house: the probability is **0.174%**.

Now we compute and find:

P(E) = 0.174% + 0.030% + 0.001% = **0.205%**.

At this final moment of card movement, you can evaluate your chances of holding the best formation with greater accuracy.

The probability we found for one opponent to hold a higher formation than yours is relevant information, but you can use it only if your play (including the betting dialogue) is focused against one person, or if only one opponent is left in play.

The probability that gives you more information about the strength of your hand is the overall probability of your opponents (that is *one or more*, or *at least one* of them) holding a higher formation than yours.

At least one opponent

In the previous section, we saw that the events *one pair*, *two pair, three of a kind*, *straight*, *flush*, *full house*, *four of a kind* and *straight flush* are mutually exclusive with respect to one opponent, and this allowed us to sum their probabilities in order to find the probability of their union.

Let us symbolically denote the event “at least one of your opponents holds one pair” by *one pair,* and the like.

For more than one opponent, that way of computing is no longer allowed because the events *three of a kind, straight, flush, full house, four of a kind* and *straight flush* are no longer mutually exclusive.

In other words, while one opponent could not simultaneously hold *one pair* and a *flush*, with more opponents at least one can hold *one pair* and at least one can hold a *flush*.

Therefore, we cannot use a formula similar to that of P(E) from the previous section.

How can we gather the data from all previous results about *one opponent* in order to find the probabilities of the various formations for *at least one opponent*?

We saw that all the tables contain probability results not only for n = 1, but also for up to n = 5 opponents.

As a consequence of what we said before, we cannot sum these results to find the overall probability of at least one opponent holding a higher formation than yours.

For each type of formation $\mathcal{F}$, these results were obtained by using the same formula:

$$P(E_n) = \frac{C_F^1 C_{F'}^{n-1} + C_F^2 C_{F'}^{n-2} + \ldots + C_F^n C_{F'}^0}{C_{C_{52-c}^5}^n}\text{, where}$$

E_n is the event “at least one of *n* opponents holds $\mathcal{F}$”, *F* is the number of favorable five-card combinations for one opponent to hold $\mathcal{F}$ and *F'* is the number of the rest of the combinations from all those possible to be dealt to that opponent.

So, if $\mathcal{F}$ = one pair, we can find the probability of at least one opponent holding one pair in any situation; if $\mathcal{F}$ = two pair we can find the similar probability, and so on.

If $\mathcal{F}$ = a higher formation than yours, to apply the formula above would mean computing *F* as the sum of all numbers of favorable combinations one opponent could hold as a higher formation than yours.

This would mean calculating *F* by its formula for each type of formation higher than yours, then summing the results and then inputting the total in the formula of $P(E_n)$.

Recall all the formulas of *F* you have seen throughout this book and you will see this calculation is practically impossible because of its length and complexity.

Therefore, this formula of $P(E_n)$ does not help us much.

Fortunately, there is a probability property that easily solves our problem – the inclusion-exclusion principle (see the chapter titled *The Supporting Mathematics*).

To apply it here, let us denote by *q* the probability that one opponent holds a higher formation than yours.

Denote $d = \frac{1}{C_{52-c}^5} = \frac{120}{(48-c)(49-c)(50-c)(51-c)(52-c)}$,

where *c* is the number of seen cards.

Let us find the probability of two specific opponents simultaneously holding a formation higher than yours.

Denote it by P_2. Denote by P_3 the probability of three specific opponents holding a formation higher than yours, by P_4 the same situation for four opponents and by P_5 the same situation for five opponents.

If *F* is the number of favorable card combinations one opponent could hold as a formation higher than yours, then:

$$P_2 = \frac{F}{C_{52-c}^5} \cdot \frac{F-1}{C_{52-c}^5 - 1} = q \cdot \frac{\frac{F}{C_{52-c}^5} - \frac{1}{C_{52-c}^5}}{1 - \frac{1}{C_{52-c}^5}} = q \cdot \frac{q-d}{1-d}$$

Similarly, we have:

$$P_3 = \frac{F}{C_{52-c}^5} \cdot \frac{F-1}{C_{52-c}^5 - 1} \cdot \frac{F-2}{C_{52-c}^5 - 2} = q \cdot \frac{q-d}{1-d} \cdot \frac{q-2d}{1-2d}$$

$$P_4 = \frac{F}{C_{52-c}^5} \cdot \frac{F-1}{C_{52-c}^5 - 1} \cdot \frac{F-2}{C_{52-c}^5 - 2} \cdot \frac{F-3}{C_{52-c}^5 - 3} = q \cdot \frac{q-d}{1-d} \cdot \frac{q-2d}{1-2d} \cdot \frac{q-3d}{1-3d}$$

$$P_5 = \frac{F}{C_{52-c}^5} \cdot \frac{F-1}{C_{52-c}^5 - 1} \cdot \frac{F-2}{C_{52-c}^5 - 2} \cdot \frac{F-3}{C_{52-c}^5 - 3} \cdot \frac{F-4}{C_{52-c}^5 - 4} =$$

$$= q \cdot \frac{q-d}{1-d} \cdot \frac{q-2d}{1-2d} \cdot \frac{q-3d}{1-3d} \cdot \frac{q-4d}{1-4d}$$

Now we can apply the inclusion-exclusion principle and find the probability of at least one opponent holding a higher formation than yours as being a function of *n*, *c* and *q*:

$$P(n,c,q) = nq - \frac{n(n-1)}{2} \cdot q \cdot \frac{q-d}{1-d} +$$

$$\frac{n(n-1)(n-2)}{6} \cdot q \cdot \frac{q-d}{1-d} \cdot \frac{q-2d}{1-2d} -$$

$$\frac{n(n-1)(n-2)(n-3)}{24} \cdot q \cdot \frac{q-d}{1-d} \cdot \frac{q-2d}{1-2d} \cdot \frac{q-3d}{1-3d} +$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)}{120} \cdot q \cdot \frac{q-d}{1-d} \cdot \frac{q-2d}{1-2d} \cdot \frac{q-3d}{1-3d} \cdot \frac{q-4d}{1-4d}$$

This formula gives us the overall probability of one or more opponents holding a higher formation than yours, when we know the probability of one opponent holding such kind of formation (*q*).

So, the calculus algorithm is as follows:

1) Establish *n* (the number of your opponents) and *c* (the number of seen cards).

2) Establish the types of formations higher than yours.

3) Calculate the probabilities of one opponent holding each type of formation from step 2.

4) Calculate *q* by the sum-formula of P(E) from the previous section (or by the descriptive table).

5) Replace *n*, *c* and *q* in the formula for P(n, c, q) and compute the numerical result.

Observe that for n = 1 the formula becomes $P(n, c, q) = nq = q$.

For n = 2 it becomes

$$P(n, c, q) = nq - \frac{n(n-1)}{2} \cdot q \cdot \frac{q-d}{1-d} = 2q - q \cdot \frac{q-d}{1-d} = q(2 - \frac{q-d}{1-d})$$

This particular expression is easy to compute.

For n = 3 it becomes

$$P(n, c, q) = nq - \frac{n(n-1)}{2} \cdot q \cdot \frac{q-d}{1-d} +$$

$$\frac{n(n-1)(n-2)}{6} \cdot q \cdot \frac{q-d}{1-d} \cdot \frac{q-2d}{1-2d} = 3q - 3q \cdot \frac{q-d}{1-d} + q \cdot \frac{q-d}{1-d} \cdot \frac{q-2d}{1-2d}$$

From n = 3 upward, the complexity of expression increases, so we recommend using a table with precalculated values.

Further, we present a table containing the numerical values of the expression for P(n, c, q), for all values of *c* and *n* and a large set of values of *q*.

These values of *q* go from 0.5% to 99.5% in increments of 0.5%.

The table has no sections for c = 5, c = 6 and so on because the values of *q* are pre-established for approximation and *q* is a function of *c*.

When we approximate the probability P(n, c, q) by using this table, we take the nearest value of *q* listed in the table.

If we calculate P(n, c, q) directly by its formula for the real (exact) value of *q*, we obtain different results for the same *n* and different values of *c*, but these results do not differ too much because of the structure of the formula and because *d* always has very low values.

In fact, the absolute value of the difference is less than 1/10000.

This also happens when we take pre-established values for *q*.

In essence, there is no need to list sections for c = 5, c = 6 and so on because the probability results written with five decimal digits would be the same.

Probability of at least one opponent holding a higher formation, function of *q*

q	n = 1	n = 2	n = 3	n = 4	n = 5
0.5%	0.50000%	0.99750%	1.49251%	1.98505%	2.47513%
1.0%	1.00000%	1.99000%	2.97010%	3.94040%	4.90100%
1.5%	1.50000%	2.97750%	4.43284%	5.86635%	7.27836%
2.0%	2.00000%	3.96000%	5.88080%	7.76319%	9.60793%
2.5%	2.50000%	4.93750%	7.31407%	9.63122%	11.89045%
3.0%	3.00000%	5.91000%	8.73271%	11.47073%	14.12661%
3.5%	3.50000%	6.87750%	10.13679%	13.28201%	16.31715%
4.0%	4.00000%	7.84000%	11.52641%	15.06536%	18.46275%
4.5%	4.50000%	8.79750%	12.90162%	16.82106%	20.56412%
5.0%	5.00000%	9.75000%	14.26251%	18.54939%	22.62193%
5.5%	5.50000%	10.69750%	15.60915%	20.25065%	24.63688%
6.0%	6.00000%	11.64000%	16.94161%	21.92512%	26.60963%
6.5%	6.50000%	12.57750%	18.25997%	23.57309%	28.54085%
7.0%	7.00000%	13.51000%	19.56431%	25.19482%	30.43120%
7.5%	7.50000%	14.43750%	20.85470%	26.79061%	32.28133%
8.0%	8.00000%	15.36000%	22.13121%	28.36073%	34.09189%
8.5%	8.50000%	16.27751%	23.39393%	29.90546%	35.86351%
9.0%	9.00000%	17.19001%	24.64291%	31.42507%	37.59683%
9.5%	9.50000%	18.09751%	25.87825%	32.91983%	39.29247%
10.0%	10.00000%	19.00001%	27.10002%	34.39003%	40.95104%
10.5%	10.50000%	19.89751%	28.30828%	35.83592%	42.57317%
11.0%	11.00000%	20.79001%	29.50312%	37.25779%	44.15945%
11.5%	11.50000%	21.67751%	30.68461%	38.65589%	45.71048%
12.0%	12.00000%	22.56001%	31.85282%	40.03050%	47.22686%
12.5%	12.50000%	23.43751%	33.00783%	41.38187%	48.70915%
13.0%	13.00000%	24.31001%	34.14972%	42.71027%	50.15796%
13.5%	13.50000%	25.17751%	35.27856%	44.01597%	51.57383%
14.0%	14.00000%	26.04001%	36.39442%	45.29922%	52.95735%
14.5%	14.50000%	26.89751%	37.49738%	46.56028%	54.30906%
15.0%	15.00000%	27.75001%	38.58752%	47.79941%	55.62952%
15.5%	15.50000%	28.59751%	39.66491%	49.01687%	56.91927%
16.0%	16.00000%	29.44001%	40.72962%	50.21290%	58.17886%
16.5%	16.50000%	30.27751%	41.78173%	51.38777%	59.40881%
17.0%	17.00000%	31.11001%	42.82132%	52.54172%	60.60965%
17.5%	17.50000%	31.93751%	43.84846%	53.67500%	61.78190%
18.0%	18.00000%	32.76001%	44.86322%	54.78786%	62.92607%
18.5%	18.50000%	33.57751%	45.86569%	55.88055%	64.04267%
19.0%	19.00000%	34.39001%	46.85592%	56.95332%	65.13221%
19.5%	19.50000%	35.19751%	47.83401%	58.00640%	66.19517%
20.0%	20.00000%	36.00001%	48.80003%	59.04004%	67.23205%
20.5%	20.50000%	36.79751%	49.75404%	60.05448%	68.24333%
21.0%	21.00000%	37.59001%	50.69613%	61.04996%	69.22949%
21.5%	21.50000%	38.37751%	51.62636%	62.02672%	70.19099%
22.0%	22.00000%	39.16001%	52.54483%	62.98498%	71.12831%
22.5%	22.50000%	39.93751%	53.45159%	63.92500%	72.04190%
23.0%	23.00000%	40.71001%	54.34673%	64.84700%	72.93221%
23.5%	23.50000%	41.47751%	55.23031%	65.75121%	73.79970%
24.0%	24.00000%	42.24001%	56.10243%	66.63787%	74.64480%
24.5%	24.50000%	42.99751%	56.96314%	67.50719%	75.46795%
25.0%	25.00000%	43.75001%	57.81253%	68.35942%	76.26958%
25.5%	25.50000%	44.49751%	58.65067%	69.19477%	77.05012%
26.0%	26.00000%	45.24001%	59.47763%	70.01347%	77.80998%
26.5%	26.50000%	45.97751%	60.29349%	70.81574%	78.54959%
27.0%	27.00000%	46.71001%	61.09833%	71.60180%	79.26933%
27.5%	27.50000%	47.43751%	61.89222%	72.37188%	79.96963%
28.0%	28.00000%	48.16001%	62.67523%	73.12618%	80.65087%
28.5%	28.50000%	48.87751%	63.44744%	73.86494%	81.31345%
29.0%	29.00000%	49.59001%	64.20893%	74.58836%	81.95775%
29.5%	29.50000%	50.29751%	64.95977%	75.29666%	82.58416%
30.0%	30.00000%	51.00001%	65.70003%	75.99004%	83.19305%

30.5%	30.50000%	51.69751%	66.42979%	76.66872%	83.78478%
31.0%	31.00000%	52.39001%	67.14913%	77.33292%	84.35973%
31.5%	31.50000%	53.07751%	67.85812%	77.98283%	84.91826%
32.0%	32.00000%	53.76001%	68.55683%	78.61866%	85.46071%
32.5%	32.50000%	54.43751%	69.24534%	79.24063%	85.98744%
33.0%	33.00000%	55.11001%	69.92373%	79.84892%	86.49879%
33.5%	33.50000%	55.77751%	70.59207%	80.44374%	86.99511%
34.0%	34.00000%	56.44001%	71.25043%	81.02530%	87.47672%
34.5%	34.50000%	57.09751%	71.89889%	81.59379%	87.94395%
35.0%	35.00000%	57.75001%	72.53753%	82.14941%	88.39713%
35.5%	35.50000%	58.39751%	73.16642%	82.69236%	88.83659%
36.0%	36.00000%	59.04002%	73.78563%	83.22282%	89.26262%
36.5%	36.50000%	59.67752%	74.39524%	83.74100%	89.67555%
37.0%	37.00000%	60.31002%	74.99533%	84.24708%	90.07567%
37.5%	37.50000%	60.93752%	75.58597%	84.74125%	90.46329%
38.0%	38.00000%	61.56002%	76.16723%	85.22370%	90.83871%
38.5%	38.50000%	62.17752%	76.73919%	85.69462%	91.20221%
39.0%	39.00000%	62.79002%	77.30193%	86.15419%	91.55407%
39.5%	39.50000%	63.39752%	77.85552%	86.60260%	91.89459%
40.0%	40.00000%	64.00002%	78.40003%	87.04003%	92.22403%
40.5%	40.50000%	64.59752%	78.93554%	87.46666%	92.54268%
41.0%	41.00000%	65.19002%	79.46213%	87.88267%	92.85079%
41.5%	41.50000%	65.77752%	79.97987%	88.28824%	93.14863%
42.0%	42.00000%	66.36002%	80.48883%	88.68354%	93.43646%
42.5%	42.50000%	66.93752%	80.98909%	89.06874%	93.71454%
43.0%	43.00000%	67.51002%	81.48073%	89.44403%	93.98311%
43.5%	43.50000%	68.07752%	81.96381%	89.80957%	94.24242%
44.0%	44.00000%	68.64002%	82.43843%	90.16553%	94.49271%
44.5%	44.50000%	69.19752%	82.90464%	90.51209%	94.73422%
45.0%	45.00000%	69.75002%	83.36253%	90.84940%	94.96718%
45.5%	45.50000%	70.29752%	83.81216%	91.17764%	95.19183%
46.0%	46.00000%	70.84002%	84.25363%	91.49697%	95.40838%
46.5%	46.50000%	71.37752%	84.68699%	91.80755%	95.61705%
47.0%	47.00000%	71.91002%	85.11233%	92.10955%	95.81807%
47.5%	47.50000%	72.43752%	85.52971%	92.40311%	96.01164%
48.0%	48.00000%	72.96002%	85.93923%	92.68841%	96.19798%
48.5%	48.50000%	73.47752%	86.34094%	92.96560%	96.37729%
49.0%	49.00000%	73.99002%	86.73492%	93.23482%	96.54977%
49.5%	49.50000%	74.49752%	87.12126%	93.49625%	96.71561%
50.0%	50.00000%	75.00002%	87.50002%	93.75002%	96.87502%
50.5%	50.50000%	75.49752%	87.87129%	93.99630%	97.02818%
51.0%	51.00000%	75.99002%	88.23512%	94.23522%	97.17527%
51.5%	51.50000%	76.47752%	88.59161%	94.46694%	97.31647%
52.0%	52.00000%	76.96002%	88.94082%	94.69161%	97.45198%
52.5%	52.50000%	77.43752%	89.28284%	94.90936%	97.58195%
53.0%	53.00000%	77.91002%	89.61772%	95.12034%	97.70657%
53.5%	53.50000%	78.37752%	89.94556%	95.32470%	97.82599%
54.0%	54.00000%	78.84002%	90.26642%	95.52256%	97.94039%
54.5%	54.50000%	79.29752%	90.58038%	95.71409%	98.04991%
55.0%	55.00000%	79.75002%	90.88752%	95.89939%	98.15473%
55.5%	55.50000%	80.19752%	91.18791%	96.07863%	98.25500%
56.0%	56.00000%	80.64002%	91.48162%	96.25192%	98.35085%
56.5%	56.50000%	81.07752%	91.76873%	96.41941%	98.44245%
57.0%	57.00000%	81.51002%	92.04932%	96.58122%	98.52993%
57.5%	57.50000%	81.93752%	92.32346%	96.73748%	98.61343%
58.0%	58.00000%	82.36002%	92.59122%	96.88832%	98.69310%
58.5%	58.50000%	82.77752%	92.85268%	97.03387%	98.76906%
59.0%	59.00000%	83.19002%	93.10792%	97.17425%	98.84145%
59.5%	59.50000%	83.59752%	93.35701%	97.30960%	98.91039%
60.0%	60.00000%	84.00002%	93.60002%	97.44002%	98.97601%
60.5%	60.50000%	84.39752%	93.83703%	97.56563%	99.03843%
61.0%	61.00000%	84.79002%	94.06812%	97.68657%	99.09777%
61.5%	61.50000%	85.17752%	94.29336%	97.80295%	99.15414%

62.0%	62.00000%	85.56002%	94.51282%	97.91488%	99.20766%
62.5%	62.50000%	85.93752%	94.72658%	98.02247%	99.25843%
63.0%	63.00000%	86.31002%	94.93472%	98.12585%	99.30657%
63.5%	63.50000%	86.67752%	95.13730%	98.22512%	99.35217%
64.0%	64.00000%	87.04002%	95.33442%	98.32040%	99.39535%
64.5%	64.50000%	87.39751%	95.52613%	98.41178%	99.43619%
65.0%	65.00000%	87.75001%	95.71252%	98.49939%	99.47479%
65.5%	65.50000%	88.09751%	95.89365%	98.58332%	99.51125%
66.0%	66.00000%	88.44001%	96.06961%	98.66367%	99.54565%
66.5%	66.50000%	88.77751%	96.24048%	98.74056%	99.57809%
67.0%	67.00000%	89.11001%	96.40631%	98.81409%	99.60865%
67.5%	67.50000%	89.43751%	96.56720%	98.88435%	99.63741%
68.0%	68.00000%	89.76001%	96.72321%	98.95143%	99.66446%
68.5%	68.50000%	90.07751%	96.87443%	99.01545%	99.68987%
69.0%	69.00000%	90.39001%	97.02091%	99.07649%	99.71371%
69.5%	69.50000%	90.69751%	97.16275%	99.13464%	99.73607%
70.0%	70.00000%	91.00001%	97.30001%	99.19001%	99.75700%
70.5%	70.50000%	91.29751%	97.43277%	99.24267%	99.77659%
71.0%	71.00000%	91.59001%	97.56111%	99.29273%	99.79489%
71.5%	71.50000%	91.87751%	97.68510%	99.34026%	99.81197%
72.0%	72.00000%	92.16001%	97.80481%	99.38535%	99.82790%
72.5%	72.50000%	92.43751%	97.92032%	99.42809%	99.84273%
73.0%	73.00000%	92.71001%	98.03171%	99.46856%	99.85651%
73.5%	73.50000%	92.97751%	98.13905%	99.50685%	99.86932%
74.0%	74.00000%	93.24001%	98.24241%	99.54303%	99.88119%
74.5%	74.50000%	93.49751%	98.34187%	99.57718%	99.89218%
75.0%	75.00000%	93.75001%	98.43751%	99.60938%	99.90235%
75.5%	75.50000%	93.99751%	98.52940%	99.63970%	99.91173%
76.0%	76.00000%	94.24001%	98.61761%	99.66823%	99.92038%
76.5%	76.50000%	94.47751%	98.70222%	99.69502%	99.92833%
77.0%	77.00000%	94.71001%	98.78331%	99.72016%	99.93564%
77.5%	77.50000%	94.93751%	98.86095%	99.74371%	99.94234%
78.0%	78.00000%	95.16001%	98.93521%	99.76575%	99.94846%
78.5%	78.50000%	95.37751%	99.00617%	99.78633%	99.95406%
79.0%	79.00000%	95.59001%	99.07391%	99.80552%	99.95916%
79.5%	79.50000%	95.79751%	99.13849%	99.82339%	99.96380%
80.0%	80.00000%	96.00001%	99.20001%	99.84000%	99.96800%
80.5%	80.50000%	96.19751%	99.25852%	99.85541%	99.97181%
81.0%	81.00000%	96.39001%	99.31411%	99.86968%	99.97524%
81.5%	81.50000%	96.57751%	99.36684%	99.88287%	99.97833%
82.0%	82.00000%	96.76001%	99.41681%	99.89503%	99.98110%
82.5%	82.50000%	96.93751%	99.46407%	99.90621%	99.98359%
83.0%	83.00000%	97.11001%	99.50870%	99.91648%	99.98580%
83.5%	83.50000%	97.27751%	99.55079%	99.92588%	99.98777%
84.0%	84.00000%	97.44001%	99.59040%	99.93447%	99.98951%
84.5%	84.50000%	97.59751%	99.62762%	99.94228%	99.99105%
85.0%	85.00000%	97.75001%	99.66250%	99.94938%	99.99241%
85.5%	85.50000%	97.89751%	99.69514%	99.95580%	99.99359%
86.0%	86.00000%	98.04001%	99.72560%	99.96158%	99.99462%
86.5%	86.50000%	98.17751%	99.75397%	99.96679%	99.99552%
87.0%	87.00000%	98.31001%	99.78030%	99.97144%	99.99629%
87.5%	87.50000%	98.43751%	99.80469%	99.97559%	99.99695%
88.0%	88.00000%	98.56001%	99.82720%	99.97926%	99.99751%
88.5%	88.50000%	98.67751%	99.84791%	99.98251%	99.99799%
89.0%	89.00000%	98.79001%	99.86690%	99.98536%	99.99839%
89.5%	89.50000%	98.89751%	99.88424%	99.98785%	99.99872%
90.0%	90.00000%	99.00001%	99.90000%	99.99000%	99.99900%
90.5%	90.50000%	99.09751%	99.91426%	99.99186%	99.99923%
91.0%	91.00000%	99.19001%	99.92710%	99.99344%	99.99941%
91.5%	91.50000%	99.27751%	99.93859%	99.99478%	99.99956%
92.0%	92.00000%	99.36000%	99.94880%	99.99590%	99.99967%
92.5%	92.50000%	99.43750%	99.95781%	99.99684%	99.99976%
93.0%	93.00000%	99.51000%	99.96570%	99.99760%	99.99983%

93.5%	93.50000%	99.57750%	99.97254%	99.99822%	99.99988%
94.0%	94.00000%	99.64000%	99.97840%	99.99870%	99.99992%
94.5%	94.50000%	99.69750%	99.98336%	99.99908%	99.99995%
95.0%	95.00000%	99.75000%	99.98750%	99.99938%	99.99997%
95.5%	95.50000%	99.79750%	99.99089%	99.99959%	99.99998%
96.0%	96.00000%	99.84000%	99.99360%	99.99974%	99.99999%
96.5%	96.50000%	99.87750%	99.99571%	99.99985%	99.99999%
97.0%	97.00000%	99.91000%	99.99730%	99.99992%	99.99999%
97.5%	97.50000%	99.93750%	99.99844%	99.99996%	99.99999%
98.0%	98.00000%	99.96000%	99.99920%	99.99998%	99.99999%
98.5%	98.50000%	99.97750%	99.99966%	99.99999%	99.99999%
99.0%	99.00000%	99.99000%	99.99990%	99.99999%	99.99999%

Where you see equal values of probability in the same row or in the same column (it is about 99.99999%), this is only in appearance because the decimal digits after the fifth are missing.

The listed values are approximations and the exact values are distinct.

When we talked earlier about the use of partial tables, we said we can improve any approximation by lowering or increasing the listed probability values, as the real q is closer to a certain head of an interval of adjacent listed values than to another head.

For example, if your calculated q is 5.558% and n = 3, we frame q between the listed values 5.5% and 6.0%, having the attached probabilities 15.60915% and 16.94161%, respectively (for n = 3).

We observe that 5.558% is closer to 5.5% than to 6.0%, so we may take the approximation 15.7% (for example) for P(c, n, q).

The formula and the table from this section can also be used for predictions about opponents' hands before the first distribution (with no seen cards, c = 0). In this situation, you must take the values of q from the last column of the table at the end of the chapter titled *Initial Probabilities of the First Card Distribution for Your Own Hand*.

Examples

In this chapter we present some concrete examples to follow to see practical methods for using all the results, formulas, tables and calculus algorithms described in this book.

Players both with and without some mathematical background may take them as pure applications of framing the problem and performing the final calculus in order to find the probability involved in respective situations.

Anyone can follow these examples because the presentation of the solution is algorithmic and only involves arithmetical calculations.

The detailed calculations of *F* were skipped in these examples and the solutions focused on the use of the tables.

The situations are taken at random, with no intended order. Nor do they fully cover the possible predictions or selectivity of the hypotheses.

They are just exercises in gaming situations picked up randomly from the millions possible in Draw Poker.

1) You are playing against five opponents and you are dealt (2♣ 6♦ 8♠ 3♥ J♥).

You hold no valuable formation (nothing).

Before the second distribution, you wonder about the probability of your opponents holding one pair and two pair at that moment in the game.

We have c = 5 seen cards, with the value distribution 1-1-1-1-1.

By searching the table in the section titled *Odds of opponents holding one pair* in the chapter titled *Prediction Probabilities for Opponents' Hands*, we find 93.467% as the probability of at least one of your opponents holding one pair.

For two pair, we look in the table in the section *Odds of opponents holding two pair* in the same chapter, and find 21.313% as the probability of at least one opponent holding two pair.

Now assume you want to discard. You hold three cards from a straight (2♣ 3♥ 6♦) and you wonder about the probability of succeeding if playing for a straight. There is one combination of values that completes your straight, namely (45).

By searching in the chapter titled *Prediction Probabilities After the First Card Distribution and Before the Second for Your Own Hand*, in sections 2, 3, 9, 11, 13 or 14, we find 1.480% as the probability of hitting a straight after replacements.

2) You are playing against three opponents and you were dealt (A♣ 3♦ 7♥ Q♥ 7♠).

You hold one pair (77) and a high card (*A*).

You want to keep the pair and wonder if is better to also keep the ace. You want to know the probabilities of hitting two pair and three of a kind after replacements.

From the chapter titled *Prediction Probabilities After the First Card Distribution and Before the Second for Your Own Hand*, section *1) One pair + high card*, we find that:

The probability of achieving two pair is 15.985% if you keep only the pair, and 17.206% if you keep the pair and the ace.

The probability of achieving three of a kind is 11.433% if you keep only the pair and 7.770% if you keep the pair and the ace.

The probabilities of achieving two pairs differ from each other by about 1.2% in favor of keeping the pair and the ace, while the probabilities of achieving three of a kind differ from each other by about 3.7% in favor of keeping only the pair.

Now assume you kept only the pair, you discard the rest and the replacements come (7♣ 2♦ 10♦).

You hold three of a kind: (7♥ 7♠ 7♣ 2♦ 10♦).

You want the probability of at least one opponent holding a straight.

We have c = 8 seen cards with the exact value distribution (1110003001010) (from *A*, 2, 3 to *K*)

By doing the calculus algorithm described in the section titled *Odds of holding a straight*, we find F = 3680. By searching in the table of the same section, we find 1.013% as the probability of at least one opponent holding a straight.

The odds of at least one opponent holding a flush, a full house, four of a kind or a straight flush are each lower or much lower than this value. Without calculating the probabilities, we can say that the overall probability will not exceed 3%.

In addition, the probability of at least one opponent holding a higher three of a kind formation is 3.808% (see the section titled *Three of a kind against higher three of a kind*).

All these values give an approximate image of the probability of your opponents holding a higher formation without doing an exact calculation of this overall probability.

3) You are playing against three opponents and you were dealt (A♣ 2♦ 4♠ 5♦ 5♥).

You hold one pair and four cards from a straight.

If you want to know the probabilities of your opponents holding the various formations in this moment of the game, you take the results from the chapter titled *Prediction Probabilities for Opponents' Hands*, according to the value and symbol distributions of the seen cards (there are c = 5 seen cards and n = 3 opponents).

The value distribution is 2-1-1-1.

The symbol distribution is 2-1-1-1.

The exact value distribution is (1101200000000).

For one opponent we have:

higher pair: 34.187%

two pair: 4.804%

three of a kind: 2.156%

straight: 0.433%

flush: 0.185%

full house: 0.148%

four of a kind: 0.025%

straight flush: 0.002%

The probability of one opponent holding a higher formation than yours in this moment of the game is given by the corresponding formula in the chapter titled *The Overall Probability of Someone Holding a Higher Formation than Yours*, in the section titled *One opponent*:

P(*higher one pair*) + P(*two pair*) + P(*three of a kind*) + P(*straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*) – P(*straight flush*) = 41.937%

The probability of at least one opponent holding a higher formation than yours is given by the formula in the chapter titled *The Overall Probability of Someone Holding a Higher Formation than Yours*, in the section titled *At least one opponent*, for q = 41.937% and n = 3. It returns 80.425%.

Now, let us come back to your own hand.

You are in situation 4) (one pair + four from a straight) from the chapter titled *Prediction Probabilities After the First Card Distribution and Before the Second for Your Own Hand*.

The suggestions are:

A) Keep the pair (5♦ 5♥) and discard the rest (A♣ 2♦ 4♠), playing for

two pair: 15.98519%
three of a kind: 11.43385%
full house: 1.01757%
four of a kind: 0.24668%,

totaling 28.68329%.

B1) Keep the straight cards (A♣ 2♦ 4♠ 5♦) and discard the remaining card (5♥), playing for a straight: 8.51063%.

Let us say you ignore both suggestions by keeping (5♦ 5♥ A♣) and discarding (2♦ 4♠).

Let us say your replacements come (5♠ 7♣). You hold three of a kind: (5♦ 5♥ 5♠ 7♣ A♣)

At this moment, c = 7 (the seen cards are 5♦ 5♥ 5♠ 2♦ 4♠ 7♣ A♣) and the distributions are:

The value distribution is 3-1-1-1-1.

The symbol distribution is 2-2-2-1.

The exact value distribution is (1101301000000).

The probabilities for one opponent are:

higher three of a kind: 1.887%
straight: 0.416%
flush: 0.178%
full house: 0.160%
four of a kind: 0.027%
straight flush: 0.001%

The probability of one opponent holding a higher formation than yours in this moment of the game is:
P(*higher three of a kind*) + P(*straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*) – P(*straight flush*) = 2.667%

The probability of at least one opponent holding a higher formation than yours in this moment of the game, for q = 2.667%, c = 7 and n = 3 is 7.789%.

4) You are playing against four opponents and you were dealt (6♣ 6♦ 8♦ 10♥ K♠).

You wonder about the probability of at least one of your opponents holding a higher pair in that moment before the second distribution.

By doing the calculation for *F* and searching the table in the section titled *Pair against higher pair*, you find 69.893%.

Now you want to discard and decide to run the risk, playing for a straight (you keep 6♦ 8♦ 10♥). The probability of hitting a straight is only 1.480%, according to section *3) (one pair + three from a straight)* from the chapter titled *Prediction Probabilities After the First Card Distribution and Before the Second for Your Own Hand*.

Assume your replacements come (8♣ J♦). You hold a pair of 8's.

The probability of at least one opponent holding a higher pair at this moment is 58.574% (see the section titled *Pair against higher pair*), which is still quite high.

The overall probability of at least one opponent holding a higher formation than yours will be even higher.

For an exact calculation, we find first the probabilities for one opponent:

higher pair: 19.773%
two pair: 4.851%
three of a kind: 2.207%
straight: 0.343%
flush: 0.188%
full house: 0.153%
four of a kind: 0.027%
straight flush: 0.001%,

from the corresponding sections of the chapter titled *Prediction Probabilities for Opponents' Hands*.

Totaling them by the rule
P(*higher pair*) + P(*two pair*) + P(*three of a kind*) + P(*straight*) + P(*flush*) + P(*full house*) + P(*four of a kind*) – P(*straight flush*)
results in an overall probability of 27.540% of one opponent holding a higher formation than yours.

The probability of at least one opponent holding a higher formation than yours in this moment of the game, for q = 27.540%, c = 7 and n = 4 is 72.433%, which is high.

5) You were dealt (4♣ 6♦ 10♥ 10♠ K♥). One of your opponents discards one card and you wonder if he is playing for a full house (he holds two pair).

For the probability of one opponent holding two pair at that moment, we must take into account the current value distribution, which is 2-1-1-1 (c = 5).

The table in the section *Odds of holding two pair* gives us 4.804% for this probability.

Now assume you discarded (4♣ 6♦ K♥) and replaced them with (3♣ 3♦ 5♠). You hold two pair.

In the light of this new information, you may evaluate again the prediction for that opponent's hand.

The new value distribution is 2-2-1-1-1-1 (c = 8).

You want the probability of that opponent holding either two pair or a full house.

The tables for the respective sections give us 4.797% and 0.148% respectively.

Therefore, the chances are small for that opponent to hold two pair initially.

6) You are playing against five opponents and you were dealt (3♣ 4♦ 5♣ 6♠ Q♥). You hold an open-ended straight and you discard the queen, playing for a straight (17.021% probability of hitting a straight).

Let us say you receive 2♦ and hit the straight.

Now you want to find the probability of at least one of your opponents holding a higher straight or a flush.

The table in the section titled *Straight against higher straight* gives us a 1.964% probability for a higher straight, while the table in the section titled *Odds of holding a flush* gives us a 0.911% probability for a flush.

The probabilities for full house, four of a kind and straight flush are much lower than those above, so we can approximate a less than 4% overall probability for a higher formation without doing all the calculations.

7) You were dealt (2♥ 5♣ 10♦ J♠ Q♣). One of your opponents chooses not to discard and you wonder whether he is bluffing.

You evaluate the predictions for him holding a full valuable formation: straight, full house or flush (for four of a kind, he may choose to discard the unpaired card to confuse the other players).

Before you discard, your seen cards have the value distribution 1-1-1-1-1 and the symbol distribution 2-1-1-1.

The tables for the corresponding sections give us the probabilities 0.397% for a straight, 0.185% for a flush and 0.138% for a full house.

We can say there is a very big chance that opponent is bluffing.

........................missing part..........................

11) (**one low pair**) You were initially dealt (3♣ 3♦ 6♥ 8♥ K♠).

At this moment, the odds of at least one opponent holding the various formations higher than yours are:

For n = 1:
higher pair: 37.694% (1.65 : 1)
two pair: 4.804% (19.82 : 1)
three of a kind: 2.156% (45.38 : 1)
straight: 0.396% (251.29 : 1)
flush: 0.185% (539.50 : 1)
full house: 0.148% (674.45 : 1)
four of a kind: 0.025% (3962.67 : 1)
straight flush: 0.002% (61356.56 : 1)
a higher formation than yours: 45.407% (1.20 : 1)

For n = 2:
higher pair: 61.180% (0.63 : 1)
two pair: 9.376% (9.67 : 1)
three of a kind: 4.226% (22.44 : 1)
straight: 0.791% (125.40 : 1)
flush: 0.370% (269.50 : 1)
full house: 0.296% (336.97 : 1)
four of a kind: 0.050% (1981.08 : 1)

straight flush: 0.003% (30678.02 : 1)
a higher formation than yours: 70.196% (0.42 : 1)

For n = 3:
higher pair: 75.813% (0.32 : 1)
two pair: 13.729% (6.28 : 1)
three of a kind: 6.330% (14.80 : 1)
straight: 1.184% (83.43 : 1)
flush: 0.554% (179.50 : 1)
full house: 0.443% (224.48 : 1)
four of a kind: 0.076% (1320.55 : 1)
straight flush: 0.005% (20451.84 : 1)
a higher formation than yours: 83.729% (0.19 : 1)

For n = 4:
higher pair: 84.930% (0.18 : 1)
two pair: 17.873% (4.59 : 1)
three of a kind: 8.350% (10.98 : 1)
straight: 1.576% (62.45 : 1)
flush: 0.738% (134.50 : 1)
full house: 0.591% (168.24 : 1)
four of a kind: 0.101% (990.29 : 1)
straight flush: 0.007% (15338.75 : 1)
a higher formation than yours: 91.117% (0.10 : 1)

For n = 5:
higher pair: 90.610% (0.10 : 1)
two pair: 21.818% (3.58 : 1)
three of a kind: 10.326% (8.68 : 1)
straight: 1.966% (49.86 : 1)
flush: 0.922% (107.50 : 1)
full house: 0.738% (134.49 : 1)
four of a kind: 0.126% (792.13 : 1)
straight flush: 0.008% (12270.90 : 1)
a higher formation than yours: 95.151% (0.05 : 1)

Of course, *n* is the number of your opponents.

........................missing part..........................

25) (playing for straight, nothing hit by one replacement) You were initially dealt (3♣ 5♥ 6♠ 8♥ 9♠). You discard the 3♣, playing for a straight (probability of succeeding 8.510%, odds 10.75 : 1) and you receive J♦.

You still hold nothing: (5♥ 6♠ 8♥ 9♠ J♦).

At the moment after the second distribution, the odds of at least one opponent holding the various formations are:

For n = 1:
one pair: 41.950% (1.38 : 1)
two pair: 4.641% (20.55 : 1)
three of a kind: 2.019% (48.54 : 1)
straight: 0.353% (282.45 : 1)
flush: 0.183% (545.55 : 1)
full house: 0.135% (738.35 : 1)
four of a kind: 0.021% (4661.43 : 1)
straight flush: 0.001% (80631.59 : 1)
a higher formation than yours: 49.301% (1.03 : 1)

For n = 2:
one pair: 66.302% (0.51 :1)
two pair: 9.066% (10.03 : 1)
three of a kind: 3.997% (24.02 : 1)
straight: 0.704% (140.97 : 1)
flush: 0.366% (272.53 : 1)
full house: 0.270% (368.92 : 1)
four of a kind: 0.043% (2330.46 : 1)
straight flush: 0.002% (40315.53 : 1)
a higher formation than yours: 74.296% (0.35 : 1)

For n = 3:
one pair: 80.439% (0.24 : 1)
two pair: 13.286% (6.53 : 1)
three of a kind: 5.935% (15.85 : 1)
straight: 1.055% (93.82 : 1)
flush: 0.548% (181.52 : 1)
full house: 0.405% (245.78 : 1)
four of a kind: 0.064% (1553.48 : 1)
straight flush: 0.004% (26876.84 : 1)

a higher formation than yours: 86.968% (0.15 : 1)

For n = 4:
one pair: 88.645% (0.13 : 1)
two pair: 17.310% (4.78 : 1)
three of a kind: 7.834% (11.77 : 1)
straight: 1.404% (70.24 : 1)
flush: 0.730% (136.01 : 1)
full house: 0.540% (184.21 : 1)
four of a kind: 0.086% (1164.98 : 1)
straight flush: 0.005% (20157.50 : 1)
a higher formation than yours: 93.393% (0.07 : 1)

For n = 5:
one pair: 93.408 (0.07 : 1)
two pair: 21.147% (3.73 : 1)
three of a kind: 9.694% (9.32 : 1)
straight: 1.752% (56.09 : 1)
flush: 0.911% (108.71 : 1)
full house: 0.674% (147.27 : 1)
four of a kind: 0.107% (931.88 : 1)
straight flush: 0.006% (16125.89 : 1)
a higher formation than yours: 96.650% (0.03 : 1)

Compare this situation with simulation 16). Observe that the additional card (J♦) does not change the odds too much (they vary within a 1% range, respectively).

.........................missing part..........................

Using a Software Program

As we said earlier, this book has a didactic side but is mainly practical.

The didactic side is addressed to anyone having a minimal mathematical background who wants to learn and apply the probability calculus in poker and other card games.

The practical side is addressed to poker players who want to improve their game by using probability in their decisions.

For all readers, we organized the structure of the book with this prevailing practical goal: anyone can pick the needed numerical results from a table or listing according to a desired situation.

Often, however, some calculations are required to get a partial result.

No matter which category readers fall into, they will all face easy, hard or very hard calculations in this book.

Doing these calculations correctly until their end is a matter of skill as well as arithmetic and algebraic knowledge, which are not at everybody's hand.

Moreover, some of the complex calculations are very long and require some time to perform.

Most of these calculations come from the complex expressions that represent some formulas for F, the number of favorable combinations for the various formations.

Other hard calculations come from applications of the probability schemes and the inclusion-exclusion principle.

This book offers many tables of values that can be used directly to avoid having to use entire long calculations or at least parts of them.

Where the information could not be listed because of its volume or to avoid extra mathematical presentations, we worked out approximations to cover all the probabilities involved.

Even though the numerical results in the tables cover all gaming situations, not all the tables can be used directly.

Many hold the possible values of F and not the possible distributions of the seen cards.

To use those tables, the reader must first compute the value of *F* for a given situation.

Below is a resume of all the listings presenting numerical probabilities, along with the details of the calculus requirements.

.......................missing part..........................

This is why Infarom Publishing developed a software program based on the formulas from this book. It is the *Draw Poker Odds Calculator*, the first poker software using compact probability formulas instead of partial simulations.

Draw Poker Odds Calculator is a hand analyzer and an odds calculator suitable for the several variations of draw poker that submit to the set of game rules assumed in this book.

The user can simulate his or her play and determine the numerical probabilities of the gaming events and predictions involved at every stage of the game.

The software analyzes your initial hand and final hand by recognizing valuable formations or parts of them, and provides suggestions on discarding along with all odds associated with your own play.

At every stage of the game, the software also provides the odds of your opponents holding valuable formations as well as a higher formation than yours.

This combined information helps the player to evaluate his or her hand in front of the opponents at every stage of the game and to make decisions regarding his or her play or betting dialogue.

Draw Poker Odds Calculator can be used while playing draw poker on line by simulating your own play independent of the playing software. It can be also used off line, as a learning tool: players can simulate and memorize as many as gaming situations as possible along with the associated odds for use in making decisions during a real game.

In fact, this software helps the player who learns at home much more than the book because no calculations are required. All the odds associated with a specific gaming situation are displayed instantly.

All the tables of values from this book are practically contained in the returns of this software.

And the inputs are not value or symbol distributions, nor values of *F*, but specific card distributions – your real own hand plus replacements.

Obviously, this kind of display helps to memorizing odds, because the association process uses images of real cards.

Check publisher's website http://probability.infarom.ro/software.html for more details about this software.

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