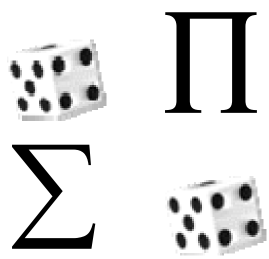
PROBABILITY GUIDE
TO
GAMBLING

The Mathematics of Dice, Slots, Roulette, Baccarat, Blackjack, Poker, Lottery and Sport Bets



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Introduction

Probability theory is a formal theory of mathematics like many others, but none of them raised so many questions about its interpretations and applicability in daily life as this theory does. Even though many of these questions have found no satisfactory answer yet, probability still remains the only theory that models hazards through mathematical methods, even if it operates on a minute part of what a hazard would mean.

Owing to its psychological impact on human concerns in daily life, probability theory has gained considerable popularity among ordinary people, regardless of whether they have a mathematical background. Moreover, people refer to probability and statistics anytime they need additional information about the occurrence of an event.

Laymen and scientists alike are fascinated by probability theory because it has multiple models in nature, it is a calculus tool for other sciences and the probability concept has major philosophical implications as well.

But no matter how paradoxically it may look, the popularity of probability theory is not that beneficial for average people. The relativities of the term probability, even if related only to the mathematical definition, may introduce a lot of errors into the qualitative and quantitative interpretation of probability, especially as a degree of belief.

These interpretation errors, as well as that false certainty psychologically introduced by the numerical result of measuring an event, turn probability calculus into a somewhat dangerous tool in the hands of persons having little or no elementary mathematical background. This affirmation is not at all hazardous, because probabilities are frequently the basis of decisions in everyday life. Due to a natural need that is more or less rigorously justified, humans consistently refer to statistics; therefore, probability has become a real decision-making tool.

Here is an example—otherwise unwanted—of making a decision based on probability is the following:

Your doctor communicates the stages of evolution of your disease: if you won't have an operation, you have a 70 percent chance of living, and if you'll have the operation, you have a 90 percent chance of a cure, but there is a 20 percent chance that you will die during the operation.

Thus, you are in a moment when you have to make a decision, based on personal criteria and also on communicated figures (their estimation was performed by the doctor according to statistics).

In most cases of probability-based decisions, the person involved performs the estimation or calculus. Here is a simple example:

You are in a phone booth and you must urgently communicate important information to one of your neighbors (let us say you left your front door open). You have only one coin, so you can make only one call. You have two neighboring houses. Two persons live in one of them and three persons live in the other. Both their telephones have answering machines. Which one of the two numbers will you call?

The risk is that nobody will be at the home you call and the coin will be lost when the answering machine starts. You could make an aleatory choice, but you could also make the following decision: "*Because the chances for somebody to be home are bigger in the case of house with three persons, I will call there*."

Thus, you have made a decision based on your own comparison of probabilities. Of course, the only information taken into account was the number of persons living in each house.

If other additional information—such the daily schedules of your neighbors—is factored in, the probability result might be different and, implicitly, you might make a different decision.

In the previous example, the estimation can be made by anyone because it is a matter of simple counting and comparison. But in most situations, a minimum knowledge of combinatorics and the calculus of probabilities are required for a correct estimation or comparison.

Millions of people take a chance in the lottery, but probably about 10 percent of them know what the winning probabilities really are. Let us take, for example, the *6 from 49* system (six numbers are drawn from 49 and one simple variant to play has six numbers; you win with a variant having a minimum of four winning numbers).

The probability of five numbers from your variant being drawn is about 1/53992, and the probability of all six numbers being drawn (the big hit!) is 1/13983816.

For someone having no idea of combinations, these figures are quite unbelievable because that person initially faces the small numbers 5, 6 and 49, and does not see how those huge numbers are obtained. In fact, this is the psychological element that a lottery company depends on for its system to work.

If a player knew those figures in advance, would he or she still play? Would he or she play less often or with fewer variants? Or would he or she play more variants in order to better the chances? Whatever the answers to these questions may be, those probabilities will influence the player's decision.

There are situations where a probability-based decision must be made—if wanted—in a relatively short time; these situations do not allow for thorough calculus even for a person with a mathematical background.

In gambling, such decisional situations are encountered all over: you ask yourself which combination of cards it is better to keep and which to replace in a five draw poker, if raise or not after flop in Hold'em, if ask for an additional card in blackjack when you have good points, etc.

If probability is used as a criterion in making gaming decisions, you have to know in advance probabilities of the events related to your own play, as well as probabilities related to opponents' play and compare them.

We estimate, approximate, communicate and compare probabilities daily, sometimes without realizing it, especially to make favorable decisions. The methods through which we perform these operations could not be rigorous or could even be incorrect, but the need to use probability as criterion in making decisions generally has a precedence. In addition, establishing a certain threshold from which the chance of an event occurring attributes to it the quality of being *probable* or *very probable* is a subjective choice.

Psychologically speaking, the tendency of novices to grant the word *probability* a certain importance is generally excessive in two ways: the word is granted too much importance—the figures come to represent the subjective absolute degree of trust in an event occurring—or too little importance—so many times, an equal sign is put between *probable* and *possible* and the information provided by numbers is not taken into account.

This could be explained by the fact that human beings automatically refer to statistics in any specific situation, and statistics and probability theory are related. We usually take a certain action as result of a decision because statistically, that action led to a favorable result in a number of previous cases. In other words, the probability of getting a favorable result after that action is acceptable. This decisional behavior belongs to a certain human psychology and the human action is generally not conditioned by additional knowledge.

Although statistics and even probability do not provide any precise information on the result of a respective action, the decision is made intuitively, without reliance on scientific proof that the decision is optimum.

What is, in fact, the motivation behind such general behavior of appealing to statistics? Is there a rational motivation or it is only a matter of human biological structures? The answer is somewhere at the middle and can be explained in large part by psychology.

All along, man felt safe as an individual only when he was grounded on something sure and perceptible. The human mind also submits to this principle. Practical statistics is a collection of *sure* results; namely, frequencies of events that *already happened*, and these happenings are a certainty. Unlike statistics, the prediction, by estimating a degree of belief, refers to events that have *not happened* yet and their occurrence is an uncertainty, so the human mind classifies them in the category of *unsafe* and tends to increase somehow their sureness by transferring sure things (the statistical results) upon them.

Although humans act in the real word in uncertain conditions, in an unsure environment, the human mind perceives this thing as an anomaly and tries to ameliorate it by elaborating degrees of belief that come from sure environments, such as practical statistics.

This psychic process of unconditioned migration to certainty environments is a reflex action of possibly ancestral origin and may be changed only through a profound study of notions of frequency, probability and degree of belief.

Beyond the psychological aspect, there is also a minimal rational theoretical motivation behind this process of appealing to statistics, which results from its connection to probability and is analogue to the motivation for probability-based decisions in games. We talk about this more in the chapter titled *The Probability–based Strategy.*

Appealing to practical statistics is not only an individual option, but it is also currently practiced in industry and even in scientific communities. Economic fields such as insurance, marketing, industrial testing of materials; research domains such as pharmacology and medicine; and disciplines such as psychology and sociology all use statistics as the main tool in making decisions related to their activities by transforming statistical results into degrees of belief and decisional criteria.

The psychological factor manifests not only in the interpretation and application processes of probability, but also in the estimation of numerical results. The calculation of mathematical probability itself in practical situations is the subject of subjective and even erroneous judgments in turn, especially if applied without minimal conceptual knowledge of probability theory.

Many times probabilities are intuited, even when a rigorous calculus is possible. In such situations, the respective person estimates or compares two probabilities without executing the entire process, which consists of framing the problem, studying it and applying a rigorous calculus, but bases the decision only on his or her own feeling.

Many times, sufficiently good approximations are obtained through intuition, but most of time the result does not conform to reality and may lead to wrong decisions.

The estimation of probabilities may be incorrect at the intuitive level because such an intuition is not in fact a quick rational process that skips some steps and reaches a correct final result, but is a group of reflex actions that combine known trivial results with a human's own subjective expectations.

The estimation errors can come from an unsubstantial observation, an inadequate or inexistent framing of the problem or they may be just technical errors in calculus.

Here is a simple example of false intuition, in which the error comes from an incorrect framing of the problem:

You have the following information: A person has two children, at least one a boy. What is the probability of the other child being a boy, too? You are tempted to answer 1/2, by thinking there are only two choices: boy and girl. In fact, the probability is 1/3, because the possible situations are three:

boy–girl (*BG*)
girl–boy (*GB*)
boy–boy (*BB*)

and one of them is favorable, namely (*BB*).

The initial information refers to both children as a group and not to a particular child from the group. In the case of estimation of 1/2 , the error comes from establishing the sample space (and, implicitly the field of events) as being $\{B, G\}$, when in, fact it, is a set of ordered pairs: $\{BG, GB, BB\}$.

The probability would be 1/2 if one of the two children had been fixed by hypothesis (as example: *the oldest is a boy* or *the tallest is a boy).*

For many people, the famous birthday problem is another example of contradiction with their own intuitions:

If you randomly choose twenty-four persons, what do you think of the probability of two or more of them having the same birthday (this means the same month and the same day of the year)?

Even if you cannot mentally estimate a figure, intuitively you feel that it is very low (if you do not know the real figure in advance). Still, the probability is 27/50, which is a little bit higher than 50 percent!

A simple method of calculus to use here is the step by step one:

The probability for the birthday of two arbitrary persons not to be the same is 364/365 (because we have one single chance from 365 for the birthday of the first person to match the birthday of the second). The probability for the birthday of a third person to be different from those of the other two is 363/365; for the birthday of a fourth person is 362/365, and so on, until we get to the last person, the 24th, with a 342/365 probability.

We have obtained twenty-three fractions, which all must be multiplied to get the probability of all twenty-four birthdays to be different. The product is a fraction that remains as 23/50 after reduction. The probability we are looking for is the probability of the contrary event, and this is 1 – 23/50 = 27/50.

This calculus does not take February 29 into account, or that birthdays have a tendency to concentrate higher in certain months rather than in others. The first circumstance diminishes the probability, while the second increases it.

If you bet on the coincidence of birthdays of twenty-four persons, on average you would loose twenty-three and win twenty-seven of each fifty bets over time. Of course, the more persons considered, the higher the probability. With over sixty persons, probability gets very close to certitude. For 100 persons, the chance of a bet on a coincidence is about 3000000 : 1. Obviously, absolute certitude can be achieved only with 366 persons or more.

One of the most curious behaviors based on false intuition is that of lottery players, where the winning probabilities are extremely low.

As a game of chance that offers the lowest winning odds, it is not predisposed to strategies. The player (regular or not) purely and simply tries his or her fortune, whether he or she knows the involved mathematical probabilities beforehand.

Still, too few players stop contributing to the lottery, even when they hear or find what the real probability figures are.

In a *6 from 49* lottery system, the probability of winning the 1st category with a single played variant (six numbers) is 1/13983816.

If playing weekly during a lifetime (let us assume eighty years of playing, respectively, 4320 draws), the probability for that player to finally win improves to 1/3236. Still assuming that the person plays ten or even 100 variants once, he or she has a probability of 1/323 or 1/32, which is still low for a lifetime. And we did not even take into account the amount invested. What exactly makes lottery players persevere in playing by ignoring these figures?

Beyond the addiction problems, there is also a psychological motivation of reference to community, having observation as a unique criterion.

A regular player may ask himself or herself the question: “If people all around me win the lottery, why can’t I have my day once, too?” Probability theory cannot completely answer to that question, but in exchange it can answer the question why that player has not won until the present moment: because the probability of winning is very close to zero.

Another example of false intuition is still related to the lottery.

Most players avoid playing the variant 1, 2, 3, 4, 5, 6. Their argument is intuitive: It is impossible for the first six numbers to be drawn. Indeed, it is almost impossible, in the sense that the probability of drawing that variant is 1/13983816. Still, this probability remains the same for any played variant (assuming the technical procedure of drawing is absolutely random). There are no preferential combinations, so that particular variant has not at all an inferior status from point of view of possibility of occurrence.

Moreover, if someone won by playing that variant, the amount won would be much higher than in the case of other played variant, because the winning fund will be divided (eventually) among fewer players. Thus, the optimal decision would be to play that particular variant instead of others. Of course, this decision remains optimal as long as most players are not acquainted with this information.

False intuition successfully manifests in several gaming situations in gambling. The so-called feeling of the player, which at a certain moments will influence a gaming decision, is very often a simple illusory psychical reaction that is not analytically grounded.

Probability represents one of the domains in which intuition may play bad tricks, even for persons with some mathematical education. Therefore, intuition must not be used as a calculus tool or for probability estimation.

A correct probability calculus must be based on minimal, but clear, mathematical knowledge and must follow the basic logical algorithm of the application process, starting with framing the problem, then establishing the probability field and the calculus itself.

This book is not a teaching material on probability theory basics and calculus, even it holds a mathematical chapter with the fundamental probability notions and results. For the readers interested in a course for beginners, we recommend the book *Understanding and Calculating the Odds: Probability Theory Basics and Calculus for Beginners, with Applications in Games of Chance and Everyday Life*, where they can find detailed explanations of the probability concept and all the supporting math explained at undergraduate level, along with all interpretations of probability.

This book is addressed primarily to gamblers, and therefore contains a large collection of precalculated probability results (both formulas and numerical values) for the major games of chance.

It aims to present all the real numerical probabilities attached to most gaming situations, especially those requiring decisions, to help gamblers in avoiding calculus errors, false intuitions and the wrong probability-based decisions.

It is not a gambling strategy book, even though some of the sections dealing with games contain recommendations, but these are only mathematical expectations based on probability figures.

Even though it helps gamblers, this book is not intended to encourage them to persevere in the games they practice. It just presents the mathematical facts, namely the probabilities attached to the gaming events they encounter. They will find that most of these probabilities are very low in comparison with their own previous convictions, which could make them quit some games, rather than encouraging them to continue gambling.

The structure and content of the major chapters follows:

Probability Theory Basics

This is the strictly mathematical chapter that contains all the rigorous definitions that establish the basis for the probability concept, starting from set operations, sequences of real numbers, convergence, Boole algebras, and measures, to field of events, probability, conditional probability and random variables.

We only insisted on the finite cases, because all probability applications in games of chance are running on finite fields of events and finite sample spaces.

The chapter contains only the main theoretical results, which are presented as enunciations without demonstrations, but also contains many examples.

You will find here all the math needed in performing probability calculus for finite applications.

The players only interested in the numerical results or the persons having no mathematical background may choose to skip the mathematical chapters, with no repercussions on the practical goal of using this guide, namely to pick the numerical probabilities for the desired gaming situations.

Combinatorics

Combinatorial analysis is an important calculus tool in probability applications and this is the reason why it has a dedicated chapter.

This chapter contains the definitions of permutations, combinations and arrangements, along with their calculus formulas and main properties.

The Mathematics of Games of Chance

Here we present these games not only as a good application field for probability theory, but also in terms of human actions where probability-based strategies can be tried to achieve favorable results.

Through suggestive examples, the reader can see what experiments, events and probability fields mean in games of chance and why and how probability formulas can be applied there.

The Guide of Numerical Results (for Dice, Slots, Roulette, Baccarat, Blackjack, Classical Poker, Texas Hold'em Poker, Lottery and Sport Bets)

This is the collection of numerical results that players can use in frequent gaming stages and situations.

Each section corresponds to a game of chance and starts with a description of the respective game and its rules, and then goes on with the general framing of probability problems within the respective game.

Next you will find some partially or completely solved applications and a collection of precalculated results that are structured on categories of gaming situations specific to respective game.

All numerical results are illustrated in tables and listings at the end of each section, from where they can be extracted and directly used by interested players.

The Probability-based Strategy

The preponderant usage of probability as a decision-making criterion naturally generates the need for theoretical motivation.

This chapter is an essay containing the definition of probability-based strategy and mathematical demonstrations showing that such strategy is theoretically optimal.

PROBABILITY THEORY BASICS

In this chapter we present the main set of notions and foundation results for the mathematical concept of probability and probability theory.

Because this guide is addressed principally to beginners, we have limited it to the notions leading to the rigorous definition of probability and the properties generating the formulas that are necessary to practical calculus as well, especially for discrete and finite cases.

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COMBINATORICS

Combinatorial analysis plays a major role in probability applications, from a calculus perspective, because many situations deal with permutations, combinations or arrangements.

The correct approach to combinatorial problems and the ease of handling combinatorial calculus are 50 percent of the probability calculus abilities for games of chance.

As in the previous chapter on mathematics, the theoretical discussions present only definitions and important results, without demonstrations.

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THE MATHEMATICS OF GAMES OF CHANCE

For a math person or the reader who skims the math chapter, the games of chance appear to be pure applications of probability calculus. And this is exactly what they are: experiments generating various types of aleatory events, the probability of which can be calculated by using the properties of probability on a finite field of events.

For a gambler interested in the mathematics of games of chance, these games are more than pure probability applications. They are a way of life, a kind of living companion a gambler interacts with in order to squeeze profit from them by using strategies that may or may not include mathematics.

Even though the randomness inherent in games of chance is would seem to ensure their fairness (at least with respect to the players around a table—shuffling a deck or spinning a wheel do not favor any player except if they are fraudulent), gamblers always search and wait for irregularities in this randomness that will allow them to win.

It has been mathematically proved that, in ideal conditions of randomness, no long-run regular winning is possible for players of games of chance; therefore, gambling is not a good option as a way of making a living. Most gamblers accept this premise, but still work on strategies to make them win over the long run. And this approach may not be quite as absurd as it appears, even if mathematics is taken into account—probability theory is built under idealized conditions and the law of large numbers theoretically works at *infinity.*

Play using a long-run strategy to achieve a cumulated positive result means ignoring the randomness and skipping the experiments giving negative results. This strategy is possible only if a player has access to some paranormal information—someone has to have prior knowledge and be able to tell the player when to play and when not to! Until this magic help becomes possible, probability theory remains the only tool that provides some information about gaming events, even as an idealized relative frequency.

Gamblers also may be interested in isolated winnings, running the risk in a single game or in short-run play, with or without a gaming strategy. No matter the chosen options and strategies, they will be interested in the amount of that risk and this means probability. Knowing the probabilities attached to gaming events is extra information all players need, even if they do not use it.

In the beginning, probability theory was inspired by games of chance. The observations on the occurrences of the various gaming events and on their frequencies led to the probability models that were built later.

And as probability theory developed, all works and presentations became packed with examples from games of chance. This happened because, as discussed earlier, these games offer, through their technical processes of working, a randomness that is hardly questionable. The famous unbiased die or dice have always been included in scholarly examples as random generators of measurable events. The term *lottery* was adopted in the common speech as an attribute of something very unpredictable.

Owing to this unquestionable randomness, any application of probability calculus in gambling has sufficient accuracy to lead to a numerical result that can stand for the measure of the physical possibility of a gaming event occurring.

Experiments, events, probability fields

The technical processes of a game stand for experiments that generate aleatory events.

Throwing the dice in craps is an experiment that generates events such as occurrences of certain numbers on the dice, obtaining a certain sum of the shown numbers, obtaining numbers with certain properties (less than a specific number, higher that a specific number, even, uneven, and so on).

The sample space of such an experiment is {1, 2, 3, 4, 5, 6} for rolling one die or {(1, 1), (1, 2), ..., (1, 6), (2, 1), (2, 2), ..., (2, 6), ..., (6, 1), (6, 2), ..., (6, 6)} for rolling two dice. The latter is a set of ordered pairs and counts 6 x 6 = 36 elements.

The events can be identified with sets, namely parts of the sample space. For example, the event *occurrence of an even number* is represented by the following set in the experiment of rolling one die: {2, 4, 6}.

Spinning the roulette wheel is an experiment whose generated events could be the occurrence of a certain number, of a certain color or a certain property of the numbers (low, high, even, uneven, from a certain row or column, and so on).

The sample space of the experiment involving spinning the roulette wheel is the set of numbers the roulette holds: {1, 2, 3, ..., 36, 0, 00} for the American roulette, or {1, 2, 3, ..., 36, 0} for the European.

The event *occurrence of a red number* is represented by the set {1, 3, 5, 7, 9, 12, 14, 16, 18, 19, 21, 23, 25, 27, 30, 32, 34, 36}.

These are the numbers inscribed in red on the roulette wheel and table.

Dealing cards in blackjack is an experiment that generates events such as the occurrence of a certain card or value as the first card dealt, obtaining a certain total of points from the first two cards dealt, exceeding 21 points from the first three cards dealt, and so on.

In card games we encounter many types of experiments and categories of events. Each type of experiment has its own sample space.

For example, the experiment of dealing the first card to the first player has as its sample space the set of all 52 cards (or 104, if played with two decks).

The experiment of dealing the second card to the first player has as its sample space the set of all 52 cards (or 104), less the first card dealt.

The experiment of dealing the first two cards to the first player has as its sample space a set of ordered pairs, namely all the 2-size arrangements of cards from the 52 (or 104).

In a game with one player, the event *the player is dealt a card of 10 points as the first dealt card* is represented by the set of cards {10♠, 10♣, 10♥, 10♦, J♠, J♣, J♥, J♦, Q♠, Q♣, Q♥, Q♦, K♠, K♣, K♥, K♦}.

The event *the player is dealt a total of five points from the first two dealt cards* is represented by the set of 2-size combinations of card values {(*A*, 4), (2, 3)}, which in fact counts 4 x 4 + 4 x 4 = 32 combinations of cards (as value and symbol).

In 6/49 lottery, the experiment of drawing six numbers from the 49 generate events such as drawing six specific numbers, drawing five numbers from six specific numbers, drawing four numbers from six specific numbers, drawing at least one number from a certain group of numbers, etc.

The sample space here is the set of all 6-size combinations of numbers from the 49.

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In the experiment of dealing the pocket cards in Texas Hold'em Poker:

– The event of dealing (3♣, 3♦) to a player is an elementary event;

– The event of dealing two 3's to a player is compound because is the union of events (3♣, 3♠), (3♣, 3♥), (3♣, 3♦), (3♠, 3♥), (3♠, 3♦) and (3♥, 3♦);

– The events *player 1 is dealt a pair of kings* and *player 2 is dealt a pair of kings* are nonexclusive (they can both occur);

– The events *player 1 is dealt two connectors of hearts higher than J* and *player 2 is dealt two connectors of hearts higher than J* are exclusive (only one can occur);

– The events *player 1 is dealt (7, K)* and *player 2 is dealt (4, Q)* are non-independent (the occurrence of the second depends on the occurrence of the first, while the same deck is in use).

These are a few examples of gambling events, whose properties of compoundness, exclusiveness and independency are easily observable. These properties are very important in practical probability calculus.

The complete mathematical model is given by the probability field attached to the experiment, which is the triple *sample space—field of events—probability function*.

For any game of chance, the probability model is of the simplest type—the sample space is finite, the field of events is the set of parts of the sample space, implicitly finite, too, and the probability function is given by the definition of probability on a finite field of events:

If $\{\Omega, \Sigma\}$ is a finite field of events, we call probability on Σ, a function $P : \Sigma \to R$ meeting the following conditions:

(1) $P(A) \geq 0$, for any $A \in \Sigma$;

(2) $P(\Omega) = 1$;

(3) $P(A_1 \cup A_2) = P(A_1) + P(A_2)$, for any $A_1, A_2 \in \Sigma$ with $A_1 \cap A_2 = \phi$.

From this definition and the axioms of a Boole algebra flow all the properties of probability that can be applied in the practical calculus in gambling.

In the assumption that elementary events are equally possible, this definition reverts to the classical definition of probability: Call probability of event A the ratio between the number of situations favorable for A to occur and the number of equally possible situations.

The *equally possible* idealization is the only approximation we have to make to apply probability theory in gambling experiments in order to have aleatory events attached to them.

But this idealization is much more accurate here than in any other field where probability theory is applied, owing to the technical construction of the gambling machines, the physical supplies used and the gaming processes. No one can question that, in nonfraudulent conditions, no certain numbers are favored during the spin of a roulette wheel, certain faces of a die at a roll or certain symbols on a slot machine spinning.

This idealization comes into question only in some interpretations of probability at the theoretical level, but the mathematical theory of probability remains rigorous even if based on this hypothesis. Otherwise, it would not exist and the probability concept would make no sense.

Returning to the classical definition of probability, it is used on a large scale in probability calculus applied to gambling because for any game of chance and for any experiment related to that game, we can find a field of events with equally possible elementary events.

This phenomenon is possible for the reasons explained before when we talked about the equally possible idealization in gambling.

For example:

– In the experiment of rolling three dice, the elementary events could be the occurrence of 3-size combinations of numbers from 1 to 6; these can be denoted by (*xyz*), with $x, y, z \in \{1, ..., 6\}$;

.... Missing part ....

We then have different sample spaces, different fields of events and, implicitly, different probability fields.

This is why the same event (literally defined) can have two different probabilities, one in each probability field.

For example, the event *the flop cards are suited*, which has the probability $4C_{13}^3 / C_{52}^3$ for a neutral observer, has a different probability for a player inside the game, depending on his or her own pocket cards.

Of course, we must always choose the probability field offering the maximal amount of information for the probability to be the most accurate as a measure of possibility.

In our example, if you are a player, you must take into account all the seen cards. This is true for any card game.

After framing the problem by establishing the probability field and the elementary events, the probability calculus follows.

Probability calculus

Probability calculus actually means to apply selected formulas and obtain a final result by replacing the numerical values within the formulas and working the numerical calculations.

These formulas are the properties of the probability function and the Boolean properties of the field of events.

As mentioned earlier, the probability fields attached to games of chance allow decomposing any event to be measured into equally possible elementary events.

That is why the classical definition of probability is the main formula for use in gambling probability applications.

Theoretically, any probability gambling problem, no matter how difficult, can be solved by using this basic formula several successive times. Occasionally, this approach can be very laborious and impractical in a reasonable time period.

Depending on the complexity of the event to be measured, other formulas can be used, and the most important of them are discussed in the following paragraphs.

Properties of probability

$P\left(A \cup B\right) = P(A) + P(B)$, for any $A, B \in \Sigma$ with $A \cap B = \phi$.

$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$, for any finite family of mutually exclusive events $\left(A_i\right)_{i=1}^{n}$ (finite additivity in condition of incompatibility)

$P(A^C) = 1 - P(A)$ (probability of contrary event)

$P\left(A \cup B\right) = P(A) + P(B) - P\left(A \cap B\right)$ (general formula of probability of union of two events)

$$P\left(A_1 \cup A_2 \cup \ldots A_n\right) = \sum_{i=1}^{n} P(A_i) - \sum_{j<i} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) + \ldots + (-1)^{n-1} P(A_1 \cap A_2 \cap \ldots \cap A_n)$$

(inclusion-exclusion principle or the general formula of probability of finite union of events)

This formula is applied as follows:

We have *n* events and want to calculate the probability of their union.

We consider successively all 1-size combinations, 2-size combinations and so on until n-size combinations of the *n* events (we have a single 1-size combination and a single *n*-size combination).

We add the probabilities of unions of each group of same-size combinations.

For the groups of combinations having an even number as a dimension, the total result is add then with minus (subtracted) and for those with an uneven number as a dimension the total result is add with plus (addition).

$P(A \cap B) = P(A) \cdot P(B)$, if events A and B are independent (the definition of independent events).

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ (the definition of conditional probability)

$P(A) = \sum_{i \in I} P(A_i) P(A|A_i)$, if $(A_i)_{i \in I} \subset \Sigma$ is a complete system of events with $P(A_i) \neq 0, \forall i \in I$ (the formula of total probability)

$P(A_i|A) = \frac{P(A_i) \cdot P(A|A_i)}{\sum_{i \in I} P(A_i) \cdot P(A|A_i)}$, if $(A_i)_{i \in I} \subset \Sigma$ is a complete system of events with $P(A_i) \neq 0, \forall i \in I$ (Bayes's theorem).

Formulas of the classical probability repartitions:

Bernoulli scheme

n independent experiments are performed. In each an event A occurs with probability p and does not occur with probability $q = 1 - p$.

The probability for event A to occur exactly m times in the n experiments is $P_{m,n} = C_n^m p^m q^{n-m}$

Poisson scheme

n independent experiments are performed. In each an event A occurs with probability $p_i, (i = 1, 2, ..., n)$ and does not occur with probability $q_i = 1 - p_i$.

The probability for event A to occur exactly m times in the n experiments ($P_{m,n}$) is the coefficient of z^m from the product

$$\phi_n(z) = \prod_{i=1}^{n}(p_i z + q_i)$$ (the generating function of probabilities $P_{m,n}$)

The probability for A to occur at least m times in the n experiments is $P(C_m) = \sum_{k=m}^{n} P_{k,n}$.

.... Missing part ....

Combinations

Games of chance are also good examples of combinations, permutations and arrangements, which are met at every step: combinations of cards in a player's hand, on the table or expected in any card game; combinations of numbers when rolling several dice once; combinations of numbers in lottery and bingo; combinations of symbols in slots; permutations and arrangements in a race to be bet on, and the like.

Combinatorial calculus is an important part of gambling probability applications.

In games of chance, most of the gambling probability calculus in which we use the classical definition of probability reverts to counting combinations. The gaming events can be identified with sets, which often are sets of combinations. Thus, we can identify an event with a combination.

For example, in a five draw poker game, the event *at least one player holds a four of a kind formation* can be identified with the set of all combinations of (*xxxxy*) type, where *x* and *y* are distinct values of cards.

This set has $13C_4^4(52-4) = 624$ combinations, so it is too big to be unfolded here.

Possible combinations are (3♠ 3♣ 3♥ 3♦ J♣) or (7♠ 7♣ 7♥ 7♦ 2♣).

These can be identified with elementary events that the event to be measured consists of.

The combinations must be counted very carefully to avoid double counting or missing one or more combinations. A proper count may use the combinatorial formulas and properties directly or may use additional procedures such as partitioning.

Expectation

Games of chance are not merely pure applications of probability calculus and gaming situations are not just isolated events whose numerical probability is well established through mathematical methods; they are also games whose progress is influenced by human action.

In gambling, the human element has a striking character. The player is not only interested in the mathematical probability of the various gaming events, but he or she has expectations from the games while a major interaction exists. To obtain favorable results from this interaction, gamblers take into account all possible information, including statistics, to build gaming strategies.

While humans appeal to past statistical results to get a subjective probability as a degree of belief, there is also a reverse psychological process: the prediction of future statistical results based on a given probability. Such prediction behavior manifests preponderantly in gambling, where probabilities are associated with stakes in order to predict an average future gain or loss.

This predicted future gain or loss is called *expectation* or expected value and is the sum of the probability of each possible outcome of the experiment multiplied by its payoff (value). Thus, it represents the average amount one expects to win per bet if bets with identical odds are repeated many times.

A game or situation in which the expected value for the player is zero (no net gain nor loss) is called a *fair game.* The attribute *fair* refers not to the technical process of the game, but to the chance balance house (bank)–player.

For example, an American roulette wheel has 38 equally possible outcomes. Assume a bet placed on a single number pays 35–to–1 (this means that you are paid 35 times your bet and your bet is returned, so you get back 36 times your bet). So, the expected value of the profit resulting from a one dollar repeated bet on a single number is:

$$\left(-\$1\times\frac{37}{38}\right)+\left(\$35\times\frac{1}{38}\right), \text{ which is about } -\$0.05.$$

Therefore one expects, on average, to lose over five cents for every dollar bet.

For a one dollar bet on heads at a coin flip (win \$1 and your bet is returned if heads comes up, and loose the bet if not), the expected value is: $\left(-\$1\times\frac{1}{2}\right)+\left(\$1\times\frac{1}{2}\right)=0$, so this is a fair game.

This prediction also has a rigorous model in probability theory, namely, the mathematical expectation:

If X is a discrete random variable with values x_i and corresponding probabilities p_i, $i\in I$, the sum or sum of series (if convergent) $M(X)=\sum_{i\in I}x_i\cdot p_i$ is called mathematical expectation, expected value or mean of variable X.

So, the mathematical expectation is a weighted mean, in the sense of the definition given above. It terms of gambling, this value means the amount (positive or negative) a player should expect, if performing same type of experiment (game or gaming situation) in identical conditions and by making the same bet, via mathematical probability. In fact, it is information about the fairness of that game.

………………… Missing part ……………………

Relative frequency. Law of Large Numbers

Let us see what relative frequency means.

Staying with the example experiment of rolling the die and taking the same event A (occurrence of number 5), let us assume we have a sequence of independent experiments (tests) $E_1, E_2, E_3, ..., E_n, ...$, each generating a certain outcome.

We can choose this sequence as being the chronological sequence of tests of rolling the die, performed by same person over time, or the chronological sequence of such tests performed by whatever number of established persons.

We can also choose the sequence as being the chronological sequence of such tests performed by all the people on Earth who make up this experiment.

No matter the set of chosen tests, as long as they are well defined and form a sequence (an infinite enumeration can be attached to them). Obviously, any of these choices is hypothetical.

Within this sequence of independent tests $(E_n)_{n\geq 1}$, we define the relative frequency of occurrence (producing) of event A.

Let us assume the die rolls a 2 on the first test (E_1). At this moment (after one test), the number of occurrences of 5 is 0.

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Relativity of probability results

The Law of Large Numbers is the only qualitative result probability theory provides us about the occurrence of an event, either in gambling or daily life. This is the only practical prediction of an event, even if it only works at infinity.

Still, it must be applied with prudence because the mathematical model that it comes from does not entirely reproduce the real phenomenon, and this is due to *infinity*.

In fact, infinity grounds the concept of probability, which makes no sense in the absence of an infinite collective. And this becomes a major relativity of probability when talking about its applications in daily life, where all real experiments have a finite nature.

The mathematical fact stated by the Law of Large Numbers, namely, that the sequence of relative frequencies converges toward probability (which stand for the definition of probability as a limit), is many times transposed into practice without analyzing in detail if the theorem's conditions about the infinite collective (the sequence of independent experiments is infinite) are met.

Knowing the Law of Large Numbers and the probability of rolling a certain number on a die (let us say, number 3) is 1/6, a gambler expects the die to show the number 3 once in six rolls, at least cumulatively, in a large number of rolls.

But the sequence of experiments of rolling a die in which the gambler participates is not infinite, for the hypothesis of the Law of Large Numbers to be met and its conclusion to be applied.

Even if the gambler accumulates all the experiments of this type in which he participated until that moment, he still gets a finite number of these.

Nevertheless, experience has proved statistically that the relative frequency of occurrence of number 3 on a die oscillates around 1/6 for very long successions of experiments.

If the gambler bets 5 to 1 on the number 3 (that is, he will lose the stake if he does not roll a 3 and will win five times the stake if he does), he does so because he is 1/6 convinced of the occurrence of the number 3 and is 5/6 convinced of the contrary.

These degrees of belief are, in fact, the expression of the relative frequency: in a long succession of rolls, the number 3 will occur one-sixth of the time.

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THE GUIDE OF NUMERICAL RESULTS

This chapter contains a large collection of numerical probability results covering a very large part of the gaming events and situations encountered in games of chance.

The results are grouped into sections along with a description of each game and solutions to a few of the probability applications involved in each game.

The solved applications can be skipped by gamblers interested only in numerical results because the partial results are used again at the end of each subsection and listed, mostly in tables, from where they can be easily selected to match the desired situation.

The numerical probability results are shown in a double format: fraction and decimal numbers as percentages; the second format always includes five or three decimal places for each irrational number.

The games of chance covered by this chapter are: Dice, Slots, Roulette, Baccarat, Blackjack, Classical Poker, Texas Hold'em Poker, Lottery and Sport Bets.

Gamblers using this collection of results must bear in mind that knowing the odds only results in additional information and not in immediate or even short-term winnings.

These results are just the numerical applications of mathematical probability in gambling.

Using them in a properly built strategy is not only a matter of mathematics but also of unavoidable subjective choice.

We recommend that all readers with some mathematical background read the last chapter titled *The Probability-based Strategy,* to see how to incorporate the role of probability theory in making decisions.

Dice

A die is a cube with faces numbered from 1 to 6 and is used as random generator of values in some casino, society or strategic games.

Games may use one or more dice.

For any game using dice, the probabilities of obtaining certain combinations of numbers on the dice are the same.

Obtaining a certain number (from 1 to 6) in one roll of a die has the same probability for every number, that is 1/6.

The probability of rolling a certain number is $P = 1/6 = 16.66666\%$**.**

Two dice

The occurrence of a certain two numbers on two dice represents two independent events, so the probability of obtaining any ordered 2–size combination of numbers is $1/6 \times 1/6 = 1/36$.

For example, the event *2 on the first die and 5 on the second die* has a 1/36 probability.

However, in games the order of numbers in the combination

(5, 2) does not count (no matter which die shows 5 and which shows 2).

Therefore, the probability of obtaining (5, 2) (unordered) is 1/36 + 1/36 = 1/18.

The same probability holds for any other combination of two different numbers, except in doubles, which have a 1/36 probability each.

The probability of obtaining a double is 6 x 1/36 = 1/6.

Let us calculate the probability of obtaining a certain number on at least one die.

Let us take number 1 (for any other number the probability will be the same) and let *A – the first die shows number 1* and *B – the second die shows number 1* be the events involved.

The intersection $A \cap B$ has a single element, namely the variant (1, 1), which has a 1/36 probability of occurrence.

Hence, we have $P(A \cup B)$ = 1/6 + 1/6 – 1/36 = 11/36.

Probability of obtaining a certain combination of two different numbers is P = 1/18 = 5.55555%**.**

Probability of obtaining a certain double is P = 1/36 = 2.77777%**.**

Probability of obtaining a double is P = 1/6 = 16.66666%.

Probability of obtaining a certain number on at least one die is P = 11/36 = 30.55555%**.**

In some games, the sum of the two numbers shown also counts.

Let us calculate the probability of obtaining the sum S = 2.

This happens only in the case of the occurrence of a variant (1, 1), so we have a 1/36 probability.

For S = 3, the favorable variants are (1, 2) and (2, 1), so P = 2/36 = 1/18.

For S = 4, the favorable variants are (1, 3), (2, 2) and (3, 1), so P = 3/36 = 1/12.

Probability of obtaining sum 2 is P = 1/36 = 2.77777%.
Probability of obtaining sum 3 is P = 1/18 = 5.55555%.
Probability of obtaining sum 4 is P = 1/12 = 8.33333%.

If we want to calculate the probability of obtaining the number 4 on at least one die or obtaining the sum 4, we observe that the events *obtaining 4 on at least one die* and *obtaining the sum 4* are incompatible (at a variant with a (4, x) form, we have $4 + x \neq 4$), so $P = 11/36 + 1/12 = 7/18$.

Probability of obtaining number 4 on at least one die or obtaining sum 4 is $P = 7/18 = 38.88888\%$**.**

The probabilities of all events specific to games that use two dice are shown in the table below.

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Slots

A slot machine is a device with three, four or five reels that is actuated mechanically or electrically after one or more coins or paid chips are inserted. The reels have on their external side several graphical symbols, with the same symbols on each reel, and they spin independently at the same time, after the player activates the handle or the button. After the reels stop spinning, a certain combination of collinear symbols occurs by the side of a central mark (the winning line).

Any winnings are differentiated by the combinations of symbols achieved. Generally, a player wins when three or four identical symbols line up, or with other combinations that hold two identical symbols.

The slot machine is also called a *flipper, fruit machine* or *one-armed bandit.* There are also electronic variants of this system that randomly generate symbols on a display.

We now present the probabilities of obtaining winning combinations for slot machines having three and four reels and one winning line, which are the most common.

Three reels

At a three-reel slot machine, the winning combinations are three identical symbols (triple), combinations with a certain symbol and two certain identical symbols and two identical symbols (double).

The problem of calculating the probabilities of occurrence of the various combinations is most similar to the one from the dice case.

Let n be the number of symbols on each reel. The total number of possible combinations that can occur after the machine is actuated is n^3. The probability of obtaining a certain triple is $(1/n)(1/n)(1/n) = 1/n^3$, and the probability of obtaining a triple is $n \cdot (1/n^3) = 1/n^2$ because the events *a triple of symbol* s_i *occurs,* $i = 1, \ldots, n,$ are mutually exclusive.

Probability of obtaining a certain triple is $P = 1/n^3$**.**

Probability of obtaining a triple is $P = 1/n^2$**.**

Let us calculate the probability of obtaining a certain double. Let s be a symbol and let:

A_1 – s only appears on reels 1 and 2 – variants (ssx), $x \neq s$

A_2 – s only appears on reels 1 and 3 – variants (sxs), $x \neq s$

A_3 – s only appears on reels 2 and 3 – variants (xss), $x \neq s$

be the events involved. $P(A_1) = P(A_2) = P(A_3) = 3(n-1)/n^3$

The intersections of events A_i are empty; therefore, $P(A_1 \cup A_2 \cup A_3) = 3(n-1)/n^3$. The probability of obtaining a double is $n \cdot 3(n-1)/n^3 = 3(n-1)/n^2$ because the events *a double of symbol* s_i *occurs,* $i = 1, \ldots, n$, are mutually exclusive (a variant containing two symbols s_i cannot contain other two symbols s_j, if $i \neq j$).

Probability of obtaining a certain double is $P = 3(n-1)/n^3$**.**

Probability of obtaining a double is $P = 3(n-1)/n^2$**.**

Let us calculate the probability of obtaining a certain double (of symbol s) and a certain symbol (v).

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The following tables list the probabilities of occurrence of the various winning combinations for slot machines having three and four reels, which have 5, 6, 7, 8, 9 or 10 symbols.

3 reels, n = 5

Event	***P* (/)**	***P* (%)**
certain triple	1/125	0.8
triple	1/25	4
certain double	12/125	9.6
double	12/25	48
certain double and certain symbol	3/125	2.4

3 reels, n = 6

Event	***P* (/)**	***P* (%)**
certain triple	1/216	0.46296
triple	1/36	2.77777
certain double	5/72	6.94444
double	5/12	41.66666
certain double and certain symbol	1/72	1.38888

3 reels, n = 7

Event	***P* (/)**	***P* (%)**
certain triple	1/343	0.29154
triple	1/49	2.04081
certain double	18/147	12.24489
double	18/49	36.73469
certain double and certain symbol	3/343	0.87463

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Roulette

Roulette is a simple, easy to learn game. It offers a wide variety of bets and combinations of bets.

The Roulette wheel has 36 numbers from 1 to 36, a “0”, and usually a “00”. Most U.S. casinos have a “00” as well as the “0”, so they have 38 numbers. This is called American roulette.

Most European casinos have only “0”, without “00”, so they have 37 numbers. This is called European roulette.

The players place bets on numbers or groups of numbers; a player’s goal is to predict the winning number or other of properties of that number (colour, evenness, size, or place on the roulette table).

Each game consists of placing bets and waiting for a number, which is randomly generated by a spinning ball coming to rest inside a disk on which numbers are inscribed (the roulette wheel) that is also spinning, but in the opposite direction.

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Depending on how the chips should be placed on roulette table, these bets are of two categories: inside and outside bets.

The next two tables note all possible bets, along with their brief description and corresponding payouts.

Inside bet	Description	Payout
Straight Up	A bet directly on any single number	35 to 1
Split Bet	A bet split between any two numbers	17 to 1
Street Bet	A bet on a row of 3 numbers	11 to 1
Corner Bet	A bet on 4 numbers	8 to 1
Line Bet	A bet on 6 numbers over 2 rows	5 to 1

Outside bet	Description	Payout
Column Bet	A bet covering 12 numbers from a column	2 to 1
Dozen Bet	A bet covering a set of 12 numbers low (1-12) mid (13-24) and high (25-36)	2 to 1
Colour Bet	A bet on either red or black	1 to 1
Even/Odd Bet	A bet on either even or odd	1 to 1
Low/High Bet	A bet on either 1-18 or 19-36	1 to 1

The payout is written in odds format and represents the coefficient to multiply the stake of a won bet.

For example, if you bet $3 on a column (payout 2 to 1) and a number from that column wins, you will receive $3 x 2 = $6, together with your initial $3 stake.

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Simple bets

When betting in roulette, we are interested in the winning or losing probability, and in the amount of profit we can gain or lose. These depend, of course, on the stake we put in. In fact, they depend on the basic stake we put on each placement.

The winning probability, losing probability, and possible profit and loss are objective criteria for a player when deciding the type of bet to make at a certain moment or what betting system to run.

Beside these objective criteria, there are also subjective criteria related to a player's personal gambling behavior. However, in this book we deal only with objective criteria, which are strictly related to mathematics.

We call a *simple bet* a bet that is made through a unique placement of chips on the roulette table. So, if we place chips in any number or amount in a single place on the table, we have made a simple bet.

If the outcome after the spin is a number we have bet on, we win the bet and the profit on that bet, which is the stake multiplied by the payout. If the outcome is not favorable, we lose the stake. This applies to any simple bet made.

The table below notes the winning probabilities for each category of simple bet, for both European and American roulette:

Simple bet	Probability (odds) European roulette	Probability (odds) American roulette	Payout
Straight Up	1/37 = 2.70% (36 : 1)	1/38 = 2.63% (37 : 1)	35 to 1
Split Bet	2/37 = 5.40% (17.5 : 1)	2/38 = 5.26% (18 : 1)	17 to 1
Street Bet	3/37 = 8.10% (11.3 : 1)	3/38 = 7.89% (11.6 : 1)	11 to 1
Corner Bet	4/37 = 10.81% (8.2 : 1)	4/38 = 10.52% (8.5 : 1)	8 to 1
Line Bet	6/37 = 16.21% (5.1 : 1)	6/38 = 15.78% (5.3 : 1)	5 to 1
Column Bet	12/37 = 32.43% (2 : 1)	12/38 = 31.57% (2.1 : 1)	2 to 1
Dozen Bet	12/37 = 32.43% (2 : 1)	12/38 = 31.57% (2.1 : 1)	2 to 1
Colour Bet	18/37 = 48.64% (1.0 : 1)	18/38 = 47.36% (1.1 : 1)	1 to 1
Even/Odd Bet	18/37 = 48.64% (1.0 : 1)	18/38 = 47.36% (1.1 : 1)	1 to 1
Low/High Bet	18/37 = 48.64% (1.0 : 1)	18/38 = 47.36% (1.1 : 1)	1 to 1

The probabilities of winning the simple bets are approximately equal in both American and European roulette (the difference varies from 1/1406 in the case of one number bet to about 1/78 in the case of a colour bet).

The probabilities of specific events attached to the experiment of spinning the roulette wheel are easily calculable—each probability is given by the ratio between the number of favorable numbers and 37 or 38.

The table shows 10 categories of simple bets, each containing several specific placements.

We can state that we have 154 possible simple bets. This statement is rigorous if we identify a bet with its placement, but a bet is determined not only by the placement of the chips, but also on the amount of the bet. In fact, the number of all possible simple bets is infinite (under the idealization that any chip division in smaller amounts is possible, even when only limited stakes are allowed).

Let us denote by R the set of all roulette numbers. Any placement for a bet is then a subset of R, or an element of $\mathcal{P}(R)$.

Denote by $\mathcal{A}$ the set of the groups of numbers from R allowed for a bet made through a unique placement.

For example, $\{2\} \in \mathcal{A}$ (straight-up bet), $\{16, 17\} \in \mathcal{A}$ (split bet), $\{11, 12, 14, 15\} \in \mathcal{A}$ (corner bet), $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35\} \in \mathcal{A}$ (odd bet), $\{0, 19\} \notin \mathcal{A}$ (the numbers 0 and 19 cannot be covered by an allowed unique placement).

$\mathcal{A}$ has 154 elements; the number of unique placements allowed.

As mentioned earlier, a simple bet is determined solely by the placement and the stake.

We can define a simple bet as being a pair (A, S), where $A \in \mathcal{A}$ and $S > 0$ is a real number.

A is the placement (the set of numbers covered by the bet) and S is the basic stake (the money amount in chips).

Because each simple bet has a payout defined by the rules of roulette, we can also look at a simple bet as at a triple (A, p_A, S), where p_A is a natural number (the coefficient of multiplication of the stake in case of winning), which is determined solely by A.

We have that $p_A \in \{1, 2, 5, 8, 11, 17, 35\}$, according to the rules of roulette.

The probability of winning a simple bet becomes $P(A) = \frac{|A|}{|R|}$, where $|A|$ means the cardinality of the set A. Of course, $|R|$ could be 38 or 37, depending on the roulette type (American or European, respectively).

.... Missing part ....

$W_B(e)$ can be also written as:

$$W_B(e) = 1_A(e) p_A S - [1 - 1_A(e)]S = [1_A(e)(p_A + 1) - 1]S$$

Function W_B is called the *profit* of bet B, applying the convention that profit can also be negative (a loss).

The variable e is the outcome of the spin. If $e \in A$ (the player wins bet B), then the player makes the positive profit $p_A S$, and if

$e \notin A$ (the player loses the bet B), then the player makes a negative profit of $-S$ (losing an amount equal to S as result of that bet).

.... Missing part ....

Complex bets

Roulette allows players to spread their chips anywhere on the table by following the rules of placement. In other words, a player can make simultaneously multiple placements of various stakes.

We can call *complex bets* these simultaneous placements. A complex bet consists of several placements of various stakes, so a complex bet is a family of simple bets.

The number of possible multiple placements is huge. If we identify a placement with the set of numbers it covers, this number is in fact the number of all subsets of the set $\mathcal{A}$.

While $\mathcal{A}$ has 154 elements, the number of its subsets is 2^{154}, which is a 47-digit number.

Mathematically, a complex bet can be described by the following:

<u>Definition</u>:

We call a *complex bet* any finite family of pairs $B = (A_i, s_i)_{i \in I}$ with $A_i \in \mathcal{A}$ and $s_i > 0$ real numbers, for every $i \in I$ (I is a finite set of consecutive indexes starting from 1).

Each pair (A_i, s_i) is a simple bet, whose placement covers the set of numbers A_i and whose basic stake is s_i.

Thus, we define a complex bet as a family of simple bets.

Of course, if $|I| = 1$, then B becomes a simple bet.

We can also denote bet B by the triple (A_i, p_i, s_i), where p_i is the payout of each simple bet A_i.

Denote by $\mathcal{B}$ the set of all possible bets (simple or complex).

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Betting on a colour and on numbers of the opposite colour

This complex bet consists of a colour bet (payout 1 to 1) and several straight-up bets (payout 35 to 1) on numbers of the opposite colour.

To generalize the example from the previous section, let us denote by S the amount bet on each number, by cS the amount bet on the colour and by n the number of bets placed on single numbers (the number of straight-up bets).

S is a positive real number (measurable in any currency), the coefficient c is also a positive real number and n is a non-negative natural number (between 1 and 18 because there are 18 numbers of one colour).

The possible events after the spin are:

A – winning the bet on colour

B – winning a bet on a number

C – not winning any bet.

These events are mutually exclusive and exhaustive, so:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 1$$

Now let us find the probability of each event and the profit or loss in each case:

A. The probability of a number of a certain colour winning is $P(A) = 18/38 = 9/19 = 47.368\%$.

In the case of winning the colour bet, the player wins $cS - nS = (c - n)S$, using the convention that if this amount is negative, that will be called a loss.

B. The probability of one of n specific numbers winning is $P(B) = n/38$.

In the case of winning a straight-up bet, the player wins $35S - (n - 1)S - cS = (36 - n - c)S$, using the same convention from event A.

C. The probability of not winning any bet is

$$P(C) = 1 - P(A) - P(B) = 1 - \frac{9}{19} - \frac{n}{38} = \frac{20-n}{38}.$$

In the case of not winning any bet, the player loses $cS + nS = (c + n)S$.

As we can see, the overall winning probability is

$$P(A) + P(B) = \frac{18+n}{38}.$$

With this formula, increasing the probability of winning would be done by increasing n.

But this increase should be done under the constraint of the bet being non-contradictory. Of course, this reverts to a constraint on the coefficients c.

It is natural to put the condition of a positive profit in both cases A and B, which results in: $n < c < 36 - n$.

This condition gives a relation between parameters n and c and restrains the number of subcases to be studied.

These formulas return the next tables of values, in which n increases from 1 to 17 and c increases by increments of 0.5.

S is left as a variable for players to replace with any basic stake according to their own betting behaviors and strategies.

<u>Observation</u>:

The same formulas and tables also hold true for the following complex bets: Even/Odd bet and straight-up bets on odd/even numbers; High/Low bet and straight-up bets on low/high numbers.

This happens because the bets are equivalent, respectively, if they have the same stakes.

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	***c***	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
1	1.5	47.36%	0.5 S	2.63%	33.5 S	50%	2.5 S
1	2	47.36%	S	2.63%	33 S	50%	3 S
1	2.5	47.36%	1.5 S	2.63%	32.5 S	50%	3.5 S
1	3	47.36%	2 S	2.63%	32 S	50%	4 S
1	3.5	47.36%	2.5 S	2.63%	31.5 S	50%	4.5 S
1	4	47.36%	3 S	2.63%	31 S	50%	5 S
1	4.5	47.36%	3.5 S	2.63%	30.5 S	50%	5.5 S
1	5	47.36%	4 S	2.63%	30 S	50%	6 S
1	5.5	47.36%	4.5 S	2.63%	29.5 S	50%	6.5 S
1	6	47.36%	5 S	2.63%	29 S	50%	7 S
1	6.5	47.36%	5.5 S	2.63%	28.5 S	50%	7.5 S
1	7	47.36%	6 S	2.63%	28 S	50%	8 S
1	7.5	47.36%	6.5 S	2.63%	27.5 S	50%	8.5 S
1	8	47.36%	7 S	2.63%	27 S	50%	9 S
1	8.5	47.36%	7.5 S	2.63%	26.5 S	50%	9.5 S
1	9	47.36%	8 S	2.63%	26 S	50%	10 S
1	9.5	47.36%	8.5 S	2.63%	25.5 S	50%	10.5 S
1	10	47.36%	9 S	2.63%	25 S	50%	11 S
1	10.5	47.36%	9.5 S	2.63%	24.5 S	50%	11.5 S
1	11	47.36%	10 S	2.63%	24 S	50%	12 S
1	11.5	47.36%	10.5 S	2.63%	23.5 S	50%	12.5 S
1	12	47.36%	11 S	2.63%	23 S	50%	13 S
1	12.5	47.36%	11.5 S	2.63%	22.5 S	50%	13.5 S
1	13	47.36%	12 S	2.63%	22 S	50%	14 S
1	13.5	47.36%	12.5 S	2.63%	21.5 S	50%	14.5 S
1	14	47.36%	13 S	2.63%	21 S	50%	15 S
1	14.5	47.36%	13.5 S	2.63%	20.5 S	50%	15.5 S
1	15	47.36%	14 S	2.63%	20 S	50%	16 S
1	15.5	47.36%	14.5 S	2.63%	19.5 S	50%	16.5 S
1	16	47.36%	15 S	2.63%	19 S	50%	17 S
1	16.5	47.36%	15.5 S	2.63%	18.5 S	50%	17.5 S
1	17	47.36%	16 S	2.63%	18 S	50%	18 S
1	17.5	47.36%	16.5 S	2.63%	17.5 S	50%	18.5 S
1	18	47.36%	17 S	2.63%	17 S	50%	19 S
1	18.5	47.36%	17.5 S	2.63%	16.5 S	50%	19.5 S
1	19	47.36%	18 S	2.63%	16 S	50%	20 S
1	19.5	47.36%	18.5 S	2.63%	15.5 S	50%	20.5 S
1	20	47.36%	19 S	2.63%	15 S	50%	21 S
1	20.5	47.36%	19.5 S	2.63%	14.5 S	50%	21.5 S
1	21	47.36%	20 S	2.63%	14 S	50%	22 S

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	***c***	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
1	21.5	47.36%	20.5 S	2.63%	13.5 S	50%	22.5 S
1	22	47.36%	21 S	2.63%	13 S	50%	23 S
1	22.5	47.36%	21.5 S	2.63%	12.5 S	50%	23.5 S
1	23	47.36%	22 S	2.63%	12 S	50%	24 S
1	23.5	47.36%	22.5 S	2.63%	11.5 S	50%	24.5 S
1	24	47.36%	23 S	2.63%	11 S	50%	25 S
1	24.5	47.36%	23.5 S	2.63%	10.5 S	50%	25.5 S
1	25	47.36%	24 S	2.63%	10 S	50%	26 S
1	25.5	47.36%	24.5 S	2.63%	9.5 S	50%	26.5 S
1	26	47.36%	25 S	2.63%	9 S	50%	27 S
1	26.5	47.36%	25.5 S	2.63%	8.5 S	50%	27.5 S
1	27	47.36%	26 S	2.63%	8 S	50%	28 S
1	27.5	47.36%	26.5 S	2.63%	7.5 S	50%	28.5 S
1	28	47.36%	27 S	2.63%	7 S	50%	29 S
1	28.5	47.36%	27.5 S	2.63%	6.5 S	50%	29.5 S
1	29	47.36%	28 S	2.63%	6 S	50%	30 S
1	29.5	47.36%	28.5 S	2.63%	5.5 S	50%	30.5 S
1	30	47.36%	29 S	2.63%	5 S	50%	31 S
1	30.5	47.36%	29.5 S	2.63%	4.5 S	50%	31.5 S
1	31	47.36%	30 S	2.63%	4 S	50%	32 S
1	31.5	47.36%	30.5 S	2.63%	3.5 S	50%	32.5 S
1	32	47.36%	31 S	2.63%	3 S	50%	33 S
1	32.5	47.36%	31.5 S	2.63%	2.5 S	50%	33.5 S
1	33	47.36%	32 S	2.63%	2 S	50%	34 S
1	33.5	47.36%	32.5 S	2.63%	1.5 S	50%	34.5 S
1	34	47.36%	33 S	2.63%	1 S	50%	35 S
1	34.5	47.36%	33.5 S	2.63%	0.5 S	50%	35.5 S
2	2.5	47.36%	0.5 S	5.26%	31.5 S	47.36%	4.5 S
2	3	47.36%	1 S	5.26%	31 S	47.36%	5 S
2	3.5	47.36%	1.5 S	5.26%	30.5 S	47.36%	5.5 S
2	4	47.36%	2 S	5.26%	30 S	47.36%	6 S
2	4.5	47.36%	2.5 S	5.26%	29.5 S	47.36%	6.5 S
2	5	47.36%	3 S	5.26%	29 S	47.36%	7 S
2	5.5	47.36%	3.5 S	5.26%	28.5 S	47.36%	7.5 S
2	6	47.36%	4 S	5.26%	28 S	47.36%	8 S
2	6.5	47.36%	4.5 S	5.26%	27.5 S	47.36%	8.5 S
2	7	47.36%	5 S	5.26%	27 S	47.36%	9 S
2	7.5	47.36%	5.5 S	5.26%	26.5 S	47.36%	9.5 S
2	8	47.36%	6 S	5.26%	26 S	47.36%	10 S
2	8.5	47.36%	6.5 S	5.26%	25.5 S	47.36%	10.5 S
2	9	47.36%	7 S	5.26%	25 S	47.36%	11 S

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	***c***	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
2	9.5	47.36%	7.5 S	5.26%	24.5 S	47.36%	11.5 S
2	10	47.36%	8 S	5.26%	24 S	47.36%	12 S
2	10.5	47.36%	8.5 S	5.26%	23.5 S	47.36%	12.5 S
2	11	47.36%	9 S	5.26%	23 S	47.36%	13 S
2	11.5	47.36%	9.5 S	5.26%	22.5 S	47.36%	13.5 S
2	12	47.36%	10 S	5.26%	22 S	47.36%	14 S
2	12.5	47.36%	10.5 S	5.26%	21.5 S	47.36%	14.5 S
2	13	47.36%	11 S	5.26%	21 S	47.36%	15 S
2	13.5	47.36%	11.5 S	5.26%	20.5 S	47.36%	15.5 S
2	14	47.36%	12 S	5.26%	20 S	47.36%	16 S
2	14.5	47.36%	12.5 S	5.26%	19.5 S	47.36%	16.5 S
2	15	47.36%	13 S	5.26%	19 S	47.36%	17 S
2	15.5	47.36%	13.5 S	5.26%	18.5 S	47.36%	17.5 S
2	16	47.36%	14 S	5.26%	18 S	47.36%	18 S
2	16.5	47.36%	14.5 S	5.26%	17.5 S	47.36%	18.5 S
2	17	47.36%	15 S	5.26%	17 S	47.36%	19 S
2	17.5	47.36%	15.5 S	5.26%	16.5 S	47.36%	19.5 S
2	18	47.36%	16 S	5.26%	16 S	47.36%	20 S
2	18.5	47.36%	16.5 S	5.26%	15.5 S	47.36%	20.5 S
2	19	47.36%	17 S	5.26%	15 S	47.36%	21 S
2	19.5	47.36%	17.5 S	5.26%	14.5 S	47.36%	21.5 S
2	20	47.36%	18 S	5.26%	14 S	47.36%	22 S
2	20.5	47.36%	18.5 S	5.26%	13.5 S	47.36%	22.5 S
2	21	47.36%	19 S	5.26%	13 S	47.36%	23 S
2	21.5	47.36%	19.5 S	5.26%	12.5 S	47.36%	23.5 S
2	22	47.36%	20 S	5.26%	12 S	47.36%	24 S
2	22.5	47.36%	20.5 S	5.26%	11.5 S	47.36%	24.5 S
2	23	47.36%	21 S	5.26%	11 S	47.36%	25 S
2	23.5	47.36%	21.5 S	5.26%	10.5 S	47.36%	25.5 S
2	24	47.36%	22 S	5.26%	10 S	47.36%	26 S
2	24.5	47.36%	22.5 S	5.26%	9.5 S	47.36%	26.5 S
2	25	47.36%	23 S	5.26%	9 S	47.36%	27 S
2	25.5	47.36%	23.5 S	5.26%	8.5 S	47.36%	27.5 S
2	26	47.36%	24 S	5.26%	8 S	47.36%	28 S
2	26.5	47.36%	24.5 S	5.26%	7.5 S	47.36%	28.5 S
2	27	47.36%	25 S	5.26%	7 S	47.36%	29 S
2	27.5	47.36%	25.5 S	5.26%	6.5 S	47.36%	29.5 S
2	28	47.36%	26 S	5.26%	6 S	47.36%	30 S
2	28.5	47.36%	26.5 S	5.26%	5.5 S	47.36%	30.5 S
2	29	47.36%	27 S	5.26%	5 S	47.36%	31 S

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	***c***	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
2	29.5	47.36%	27.5 S	5.26%	4.5 S	47.36%	31.5 S
2	30	47.36%	28 S	5.26%	4 S	47.36%	32 S
2	30.5	47.36%	28.5 S	5.26%	3.5 S	47.36%	32.5 S
2	31	47.36%	29 S	5.26%	3 S	47.36%	33 S
2	31.5	47.36%	29.5 S	5.26%	2.5 S	47.36%	33.5 S
2	32	47.36%	30 S	5.26%	2 S	47.36%	34 S
2	32.5	47.36%	30.5 S	5.26%	1.5 S	47.36%	34.5 S
2	33	47.36%	31 S	5.26%	1 S	47.36%	35 S
2	33.5	47.36%	31.5 S	5.26%	0.5 S	47.36%	35.5 S
3	3.5	47.36%	0.5 S	7.89%	29.5 S	44.73%	6.5 S
3	4	47.36%	1 S	7.89%	29 S	44.73%	7 S
3	4.5	47.36%	1.5 S	7.89%	28.5 S	44.73%	7.5 S
3	5	47.36%	2 S	7.89%	28 S	44.73%	8 S
3	5.5	47.36%	2.5 S	7.89%	27.5 S	44.73%	8.5 S
3	6	47.36%	3 S	7.89%	27 S	44.73%	9 S
3	6.5	47.36%	3.5 S	7.89%	26.5 S	44.73%	9.5 S
3	7	47.36%	4 S	7.89%	26 S	44.73%	10 S
3	7.5	47.36%	4.5 S	7.89%	25.5 S	44.73%	10.5 S
3	8	47.36%	5 S	7.89%	25 S	44.73%	11 S
3	8.5	47.36%	5.5 S	7.89%	24.5 S	44.73%	11.5 S
3	9	47.36%	6 S	7.89%	24 S	44.73%	12 S
3	9.5	47.36%	6.5 S	7.89%	23.5 S	44.73%	12.5 S
3	10	47.36%	7 S	7.89%	23 S	44.73%	13 S
3	10.5	47.36%	7.5 S	7.89%	22.5 S	44.73%	13.5 S
3	11	47.36%	8 S	7.89%	22 S	44.73%	14 S
3	11.5	47.36%	8.5 S	7.89%	21.5 S	44.73%	14.5 S
3	12	47.36%	9 S	7.89%	21 S	44.73%	15 S
3	12.5	47.36%	9.5 S	7.89%	20.5 S	44.73%	15.5 S
3	13	47.36%	10 S	7.89%	20 S	44.73%	16 S
3	13.5	47.36%	10.5 S	7.89%	19.5 S	44.73%	16.5 S
3	14	47.36%	11 S	7.89%	19 S	44.73%	17 S
3	14.5	47.36%	11.5 S	7.89%	18.5 S	44.73%	17.5 S
3	15	47.36%	12 S	7.89%	18 S	44.73%	18 S
3	15.5	47.36%	12.5 S	7.89%	17.5 S	44.73%	18.5 S
3	16	47.36%	13 S	7.89%	17 S	44.73%	19 S
3	16.5	47.36%	13.5 S	7.89%	16.5 S	44.73%	19.5 S
3	17	47.36%	14 S	7.89%	16 S	44.73%	20 S
3	17.5	47.36%	14.5 S	7.89%	15.5 S	44.73%	20.5 S
3	18	47.36%	15 S	7.89%	15 S	44.73%	21 S
3	18.5	47.36%	15.5 S	7.89%	14.5 S	44.73%	21.5 S

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	***c***	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
3	19	47.36%	16 S	7.89%	14 S	44.73%	22 S
3	19.5	47.36%	16.5 S	7.89%	13.5 S	44.73%	22.5 S
3	20	47.36%	17 S	7.89%	13 S	44.73%	23 S
3	20.5	47.36%	17.5 S	7.89%	12.5 S	44.73%	23.5 S
3	21	47.36%	18 S	7.89%	12 S	44.73%	24 S
3	21.5	47.36%	18.5 S	7.89%	11.5 S	44.73%	24.5 S
3	22	47.36%	19 S	7.89%	11 S	44.73%	25 S
3	22.5	47.36%	19.5 S	7.89%	10.5 S	44.73%	25.5 S
3	23	47.36%	20 S	7.89%	10 S	44.73%	26 S
3	23.5	47.36%	20.5 S	7.89%	9.5 S	44.73%	26.5 S
3	24	47.36%	21 S	7.89%	9 S	44.73%	27 S
3	24.5	47.36%	21.5 S	7.89%	8.5 S	44.73%	27.5 S
3	25	47.36%	22 S	7.89%	8 S	44.73%	28 S
3	25.5	47.36%	22.5 S	7.89%	7.5 S	44.73%	28.5 S
3	26	47.36%	23 S	7.89%	7 S	44.73%	29 S
3	26.5	47.36%	23.5 S	7.89%	6.5 S	44.73%	29.5 S
3	27	47.36%	24 S	7.89%	6 S	44.73%	30 S
3	27.5	47.36%	24.5 S	7.89%	5.5 S	44.73%	30.5 S
3	28	47.36%	25 S	7.89%	5 S	44.73%	31 S
3	28.5	47.36%	25.5 S	7.89%	4.5 S	44.73%	31.5 S
3	29	47.36%	26 S	7.89%	4 S	44.73%	32 S
3	29.5	47.36%	26.5 S	7.89%	3.5 S	44.73%	32.5 S
3	30	47.36%	27 S	7.89%	3 S	44.73%	33 S
3	30.5	47.36%	27.5 S	7.89%	2.5 S	44.73%	33.5 S
3	31	47.36%	28 S	7.89%	2 S	44.73%	34 S
3	31.5	47.36%	28.5 S	7.89%	1.5 S	44.73%	34.5 S
3	32	47.36%	29 S	7.89%	1 S	44.73%	35 S
3	32.5	47.36%	29.5 S	7.89%	0.5 S	44.73%	35.5 S
4	4.5	47.36%	0.5 S	10.52%	27.5 S	42.10%	8.5 S
4	5	47.36%	1 S	10.52%	27 S	42.10%	9 S
4	5.5	47.36%	1.5 S	10.52%	26.5 S	42.10%	9.5 S
4	6	47.36%	2 S	10.52%	26 S	42.10%	10 S
4	6.5	47.36%	2.5 S	10.52%	25.5 S	42.10%	10.5 S
4	7	47.36%	3 S	10.52%	25 S	42.10%	11 S
4	7.5	47.36%	3.5 S	10.52%	24.5 S	42.10%	11.5 S
4	8	47.36%	4 S	10.52%	24 S	42.10%	12 S
4	8.5	47.36%	4.5 S	10.52%	23.5 S	42.10%	12.5 S
4	9	47.36%	5 S	10.52%	23 S	42.10%	13 S
4	9.5	47.36%	5.5 S	10.52%	22.5 S	42.10%	13.5 S
4	10	47.36%	6 S	10.52%	22 S	42.10%	14 S

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	***c***	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
4	10.5	47.36%	6.5 S	10.52%	21.5 S	42.10%	14.5 S
4	11	47.36%	7 S	10.52%	21 S	42.10%	15 S
4	11.5	47.36%	7.5 S	10.52%	20.5 S	42.10%	15.5 S
4	12	47.36%	8 S	10.52%	20 S	42.10%	16 S
4	12.5	47.36%	8.5 S	10.52%	19.5 S	42.10%	16.5 S
4	13	47.36%	9 S	10.52%	19 S	42.10%	17 S
4	13.5	47.36%	9.5 S	10.52%	18.5 S	42.10%	17.5 S
4	14	47.36%	10 S	10.52%	18 S	42.10%	18 S
4	14.5	47.36%	10.5 S	10.52%	17.5 S	42.10%	18.5 S
4	15	47.36%	11 S	10.52%	17 S	42.10%	19 S
4	15.5	47.36%	11.5 S	10.52%	16.5 S	42.10%	19.5 S
4	16	47.36%	12 S	10.52%	16 S	42.10%	20 S
4	16.5	47.36%	12.5	10.52%	15.5 S	42.10%	20.5 S
4	17	47.36%	13 S	10.52%	15 S	42.10%	21 S
4	17.5	47.36%	13.5 S	10.52%	14.5 S	42.10%	21.5 S
4	18	47.36%	14 S	10.52%	14 S	42.10%	22 S
4	18.5	47.36%	14.5 S	10.52%	13.5 S	42.10%	22.5 S
4	19	47.36%	15 S	10.52%	13 S	42.10%	23 S
4	19.5	47.36%	15.5 S	10.52%	12.5 S	42.10%	23.5 S
4	20	47.36%	16 S	10.52%	12 S	42.10%	24 S
4	20.5	47.36%	16.5 S	10.52%	11.5 S	42.10%	24.5 S
4	21	47.36%	17 S	10.52%	11 S	42.10%	25 S
4	21.5	47.36%	17.5 S	10.52%	10.5 S	42.10%	25.5 S
4	22	47.36%	18 S	10.52%	10 S	42.10%	26 S
4	22.5	47.36%	18.5 S	10.52%	9.5 S	42.10%	26.5 S
4	23	47.36%	19 S	10.52%	9 S	42.10%	27 S
4	23.5	47.36%	19.5 S	10.52%	8.5 S	42.10%	27.5 S
4	24	47.36%	20 S	10.52%	8 S	42.10%	28 S
4	24.5	47.36%	20.5 S	10.52%	7.5 S	42.10%	28.5 S
4	25	47.36%	21 S	10.52%	7 S	42.10%	29 S
4	25.5	47.36%	21.5 S	10.52%	6.5 S	42.10%	29.5 S
4	26	47.36%	22 S	10.52%	6 S	42.10%	30 S
4	26.5	47.36%	22.5 S	10.52%	5.5 S	42.10%	30.5 S
4	27	47.36%	23 S	10.52%	5 S	42.10%	31 S
4	27.5	47.36%	23.5 S	10.52%	4.5 S	42.10%	31.5 S
4	28	47.36%	24 S	10.52%	4 S	42.10%	32 S
4	28.5	47.36%	24.5 S	10.52%	3.5 S	42.10%	32.5 S
4	29	47.36%	25 S	10.52%	3 S	42.10%	33 S
4	29.5	47.36%	25.5 S	10.52%	2.5 S	42.10%	33.5 S
4	30	47.36%	26 S	10.52%	2 S	42.10%	34 S

		Winning the bet on colour		Winning a bet on a number		Not winning any bet	
n	***c***	**Odds**	**Profit**	**Odds**	**Profit**	**Odds**	**Loss**
4	30.5	47.36%	26.5 S	10.52%	1.5 S	42.10%	34.5 S
4	31	47.36%	27 S	10.52%	1 S	42.10%	35 S
4	31.5	47.36%	27.5 S	10.52%	0.5 S	42.10%	35.5 S
5	5.5	47.36%	0.5 S	13.15%	25.5 S	39.47%	10.5 S
5	6	47.36%	1 S	13.15%	25 S	39.47%	11 S
5	6.5	47.36%	1.5 S	13.15%	24.5 S	39.47%	11.5 S
5	7	47.36%	2 S	13.15%	24 S	39.47%	12 S
5	7.5	47.36%	2.5 S	13.15%	23.5 S	39.47%	12.5 S
5	8	47.36%	3 S	13.15%	23 S	39.47%	13 S
5	8.5	47.36%	3.5 S	13.15%	22.5 S	39.47%	13.5 S
5	9	47.36%	4 S	13.15%	22 S	39.47%	14 S
5	9.5	47.36%	4.5 S	13.15%	21.5 S	39.47%	14.5 S
5	10	47.36%	5 S	13.15%	21 S	39.47%	15 S
5	10.5	47.36%	5.5 S	13.15%	20.5 S	39.47%	15.5 S
5	11	47.36%	6 S	13.15%	20 S	39.47%	16 S
5	11.5	47.36%	6.5 S	13.15%	19.5 S	39.47%	16.5 S
5	12	47.36%	7 S	13.15%	19 S	39.47%	17 S
5	12.5	47.36%	7.5 S	13.15%	18.5 S	39.47%	17.5 S
5	13	47.36%	8 S	13.15%	18 S	39.47%	18 S
5	13.5	47.36%	8.5 S	13.15%	17.5 S	39.47%	18.5 S
5	14	47.36%	9 S	13.15%	17 S	39.47%	19 S
5	14.5	47.36%	9.5 S	13.15%	16.5 S	39.47%	19.5 S
5	15	47.36%	10 S	13.15%	16 S	39.47%	20 S
5	15.5	47.36%	10.5 S	13.15%	15.5 S	39.47%	20.5 S
5	16	47.36%	11 S	13.15%	15 S	39.47%	21 S
5	16.5	47.36%	11.5 S	13.15%	14.5 S	39.47%	21.5 S
5	17	47.36%	12 S	13.15%	14 S	39.47%	22 S
5	17.5	47.36%	12.5 S	13.15%	13.5 S	39.47%	22.5 S
5	18	47.36%	13 S	13.15%	13 S	39.47%	23 S
5	18.5	47.36%	13.5 S	13.15%	12.5 S	39.47%	23.5 S

………………… Missing part ……………………

Initial value 5

	Favorable values	Probabilities						
		$n(x) = 0$ $v = 2$	$n(x) = o$ $v = 3$	$n(x) = 1$ $v = 2$	$n(x) = 1$ $v = 3$	$n(x) = 2$ $v = 2$	$n(x) = 2$ $v = 3$	$n(x) = 3$ $v = 3$
1 + 4	**2**	2/25 8 %	4/49 8.16326 %	–	3/49 6.12244 %	–	–	–
	3	2/25 8 %	4/49 8.16326 %	–	3/49 6.12244 %	–	–	–
	4	–	–	–	3/49 6.12244 %	–	2/49 4.08163 %	–
Total		4/25 16 %	8/49 16.32653 %	–	9/49 18.36734 %	–	2/49 4.08163 %	–
Simple arithmetic mean : 13.69387 %								

	Favorable values	Probabilities						
		$n(x) = 0$ $v = 2$	$n(x) = o$ $v = 3$	$n(x) = 1$ $v = 2$	$n(x) = 1$ $v = 3$	$n(x) = 2$ $v = 2$	$n(x) = 2$ $v = 3$	$n(x) = 3$ $v = 3$
2 + 3	**2**	–	–	3/50 6 %	3/49 6.12244 %	–	2/49 4.08163 %	–
	3	–	–	3/50 6 %	3/49 6.12244 %	–	2/49 4.08163 %	–
	4	2/25 8 %	4/49 8.16326 %	–	3/49 6.12244 %	–	–	–
Total		2/25 8 %	4/49 8.16326 %	3/25 12 %	9/49 18.36734 %	–	4/49 8.16326 %	–
Simple arithmetic mean : 10.93877 %								

Initial value 6

	Favorable values	Probabilities						
		$n(x) = 0$ $v = 2$	$n(x) = o$ $v = 3$	$n(x) = 1$ $v = 2$	$n(x) = 1$ $v = 3$	$n(x) = 2$ $v = 2$	$n(x) = 2$ $v = 3$	$n(x) = 3$ $v = 3$
1 + 5	**1**	–	–	3/50 6 %	3/49 6.12244 %	–	2/49 4.08163 %	–
	2	2/25 8 %	4/49 8.16326 %	–	3/49 6.12244 %	–	–	–
	3	2/25 8 %	4/49 8.16326 %	–	3/49 6.12244 %	–	–	–
Total		4/25 16 %	8/49 16.32653 %	3/50 6 %	9/49 18.36734 %	–	2/49 4.08163 %	–
Simple arithmetic mean : 12.1531 %								

	Favorable values	Probabilities						
		$n(x) = 0$ $v = 2$	$n(x) = o$ $v = 3$	$n(x) = 1$ $v = 2$	$n(x) = 1$ $v = 3$	$n(x) = 2$ $v = 2$	$n(x) = 2$ $v = 3$	$n(x) = 3$ $v = 3$
2 + 4	**1**	2/25 8 %	4/49 8,16326 %	–	3/49 6.12244 %	–	–	–
	2	–	–	3/50 6 %	3/49 6.12244 %	–	2/49 4.08163 %	–
	3	2/25 8 %	4/49 8,16326 %	–	3/49 6.12244 %	–	–	–
Total		4/25 16 %	8/49 16,32653 %	3/50 6 %	9/49 18.36734 %	–	2/49 4.08163 %	–
Simple arithmetic mean : 12.1531 %								

………………… Missing part ……………………

THE PROBABILITY-BASED STRATEGY

As we said several times at the beginning of this book, the mathematical probability is frequently chosen as a decision criterion in daily life.

The motivation for such a choice generally has a psychological, therefore, subjective nature, and any denying of it may find solid arguments among the relativities of probability.

But there is a particular type of decisional behavior in which choosing probability as unique criterion of decision can be mathematically justified. It is about using the probability-based strategy.

This kind of strategy is generally suitable for games, but it can be extended by analogy to certain situations in daily life as well.

The term *strategy* assumes, directly or indirectly, a certain repetitive action, even though that strategy is applied in a singular context.

When a player acts in a certain way in a gaming situation as a result of a previously adopted strategy, this action is repeated the next time a similar situation is encountered (because that strategy is considered *good* or *optimum* with respect to some player's objective or subjective criteria).

Thus, the strategy *I act in way x in situation y,* even if applied in an isolated situation, assumes intrinsically a collective, even infinite context: *I will act in way x in any hypothetical situation of type y from now on.*

If we refer to the probability-based strategy, the infinite collective is still much more present in this concept, because the mathematical definition of probability uses such a collective.

What does a probability-based strategy actually mean?

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Let us assume that in a certain game, the player reaches a situation requiring a decision. He or she expects the occurrence of a certain event *E*, which gives the player the advantage.

The player must choose between a finite number of playing variants $A_1, A_2, ..., A_m$, as result of which event *E* may occur with probabilities $p_1, p_2, ..., p_m$ respectively.

Assume that $p_1 < p_2 < ... < p_m$.

Denote $p_i = P_{A_i}(E)$, $i = 1, ..., m$ (the probability of the occurrence of expected event E, on condition that the playing variant A_i is chosen, is p_i).

Applying a probability-based strategy means choosing the playing variant A_m, the one offering the player the highest probability of the occurrence of event *E*.

Why must a player always choose this particular playing variant?

If we answer this question, the choice of the probability-based strategy as optimum is theoretically justified.

Probability theory does not provide any information about the occurrence or nonoccurrence of event *E* in an isolated gaming situation in which a certain playing variant is chosen, or in a succession of situations in which the playing variants A_i are chosen arbitrarily.

In addition, the law of large numbers cannot be applied in this last case, because we do not deal with experiments performed in identical conditions.

Still, as the only result hinting of the practical aspect of a sequence of experiments, even if the information is one of limit, we must continue to rely on the law of large numbers in the proofs that look for the answer to the optimality problem.

To simplify, we consider the case in which there are only two playing variants *A* and *B* ($m = 2$; the proof can easily be extended to a finite number *m* of playing variants).

The main condition of the hypothesis is $P_A(E) > P_B(E)$.

We will prove that playing variant *A* is the optimum choice, in the sense of a definition to be stated later.

The proof uses only the notions of elementary mathematical analysis on R and the law of large numbers.

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A sequence $\mathcal{V} = (A, B, A, A, B, A, B, A, A, ...)$ means the choice of an order and a number of experiments of type A and B and represents the player's particular strategy chosen for the succession of games (more exactly, for the succession of occurrences of the given gaming situation).

Such a sequence is called a sequence of (A, B) experiments and the whole set of these sequences is denoted by ***S***.

For an arbitrary sequence $\mathcal{V}$ of (A, B) type, we make the following denotations:

$v(n)$ = the number of occurrences of expected event E after the first n experiments (the first n terms of the sequence);

n_A = the number of experiments of type A from the first n experiments (the number of terms A from the first n terms of the sequence);

n_B = the number of experiments of type B from the first n experiments (the number of terms B from the first n terms of the sequence);

Obviously, $n = n_A + n_B$.

The ratio $\frac{v(n)}{n}$ represents the relative frequency of occurrence of event E within the sequence of experiments $\mathcal{V}$.

Note that this relative frequency cannot be the subject of the law of large numbers because although the sequence consists of independent experiments, they are not of same type.

We are allowed to apply this theorem only in the particular cases in which the sequence contains only experiments of A type or of B type.

We introduce the relations $\geq$, respectively $>$, on the set **S,** as follows:

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The model created and the proof presented give us theoretical motivation for choosing the probability-based strategy as the optimum strategy in the countable case.

How do we transfer these results to the finite or even isolated case (of a single experiment)?

How do we motivate the choice of experiment of type *A* or of a finite group of type *A* in the case—obviously, practical—in which a player participates in only a limited number of experiments?

Returning to a suggestive example of a decisional situation from the introductory chapter, how do we theoretically motivate the person in the phone booth to consider calling the house with more people living in it the optimum choice?

.... Missing part ....

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